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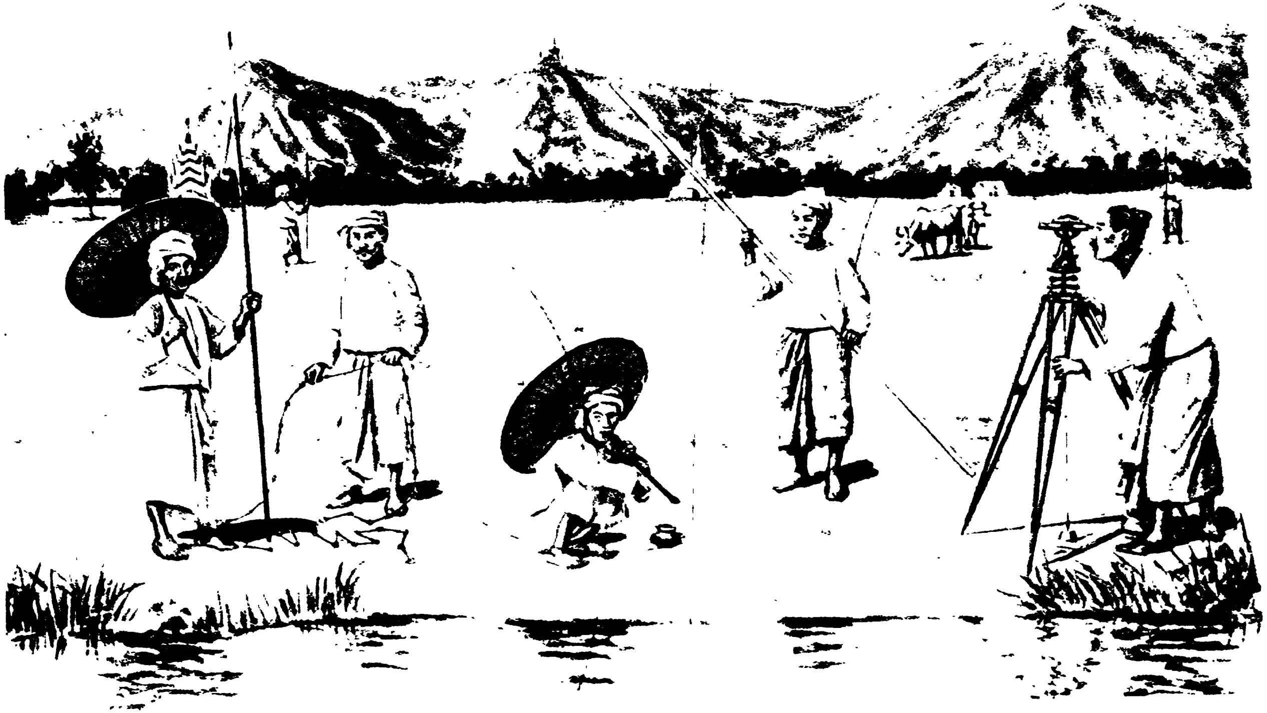

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Assistant Director of Land Records and Agriculture, Burma
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Dedication of the First Edition

To

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GEORGE FREDERICK SAMUEL, MARQUIS OF RIPON

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VICEROY AND GOVERNOR-GENERAL OF INDIA

WHO HAS AT HEART THE PROMOTION AND SPREAD
OF SCIENTIFIC EDUCATION IN THE EAST

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IS, WITH PERMISSION, GRATEFULLY DEDICATED BY HIS EXCELLENCY'S
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J. C. CLANCEY

PREFACE TO THE THIRD EDITION

THIS Edition (third) is practically a reprint of the last, with merely slight alterations in places where needed. Time does not much affect technical publications of the present kind, though improvements in methods, when they occur, are taken advantage of and duly introduced into the book. The work is quite up-to-date in all respects.

For convenience of reference, the tabular portion appears separately, as before, in a companion volume entitled "Calculating Tables," Part I (fifth edition).

The press proofs of the text have on this occasion been passed by Mr. J. HADDOCK, F.R.G.S., and subsequently checked by Mr. H. G. ANDREW, while the final proofs and the examination of the litho sheets were attended to by the undersigned.

J. C. C.

RANGOON,
The 1st August 1906.

PREFACE TO THE SECOND EDITION

THE present work on Land-Surveying has been prepared with the view of meeting fully the requirements of Officers of the Engineering and Survey Departments, and of District Officials, Private and Municipal Surveyors, Planters, and others, who desire to obtain a complete guide to practical surveying, and who form such a large and important class of Her Majesty's subjects throughout India, Burma, and elsewhere. For the use of students in surveying classes and for junior land-revenue officials and land-measurers generally, a small edition of the book, containing only very elementary matter, has been specially prepared and published. Both the large and small editions have received substantial recognition from the Government, and have been adopted as text-books.

The book contains altogether 16 Chapters, all of which have been considered indispensable to the surveyor—whether Trigonometrical, Topographical, Revenue, or Engineering—and to the student; while, for facility of reference, an exhaustive index has been added to the end of the treatise.

The entire work has been designed to lead a beginner systematically through a thorough course of practical Land-Surveying, and with this object there is embodied a collection of examples in the various subjects treated of, which will, it is hoped, prove of service. The only knowledge the student must of necessity possess before undertaking a study of surveying, is that of elementary mathematics, and this knowledge is here presumed.

The work is illustrated with 510 diagrams on sheets of foolscap-size which have been, for convenience sake, put together in a body immediately after the Index.

The manual embraces a course of surveying such as will be needed for all revenue and general purposes; and the more advanced portions of Astronomy, Trigonometry, Geodesy, etc., have been purposely excluded, as beyond its scope. The matter dealt with is intended for practical survey work alone, and falls more in the domain of the useful than of the beautiful.

The aim throughout the book has been to make each Chapter as thoroughly practical and complete in itself as possible, and, while it is hoped that this object has been accomplished, it must be borne in mind that it is utterly impossible to learn surveying from books alone, and nothing but practice can make a quick and accurate surveyor.

In writing this treatise, no pretence is made to originality as regards the principles of surveying, nor even to certain portions of the work itself which are of necessity borrowed from other authors to whom, with a few exceptions only, a general acknowledgment is made hereafter. The manual, it is gratifying to note, has been found serviceable hitherto, and if continued to be utilized by those for whom it is intended, its object will have been still further achieved.

The following works have been consulted, and from several of them extracts have been made:—Manual of Surveying for India, Thuillier; Outlines of a Trigonometrical Survey, Frome; Roorkee Treatise on Surveying, Firebrace; Practical Surveying for India, Kelly; Aid to Survey-Practice, Jackson; Notes on Surveying and Practical Geometry, Scott; Mathematical Instruments, Stanley;

Surveying and Levelling, Brees; Practical Geodesy, Williams; Navigation, Pryde; Plane Astronomy, Main; Practical Mathematics, Chambers; and Levelling, Bruff.

The main alterations made in the present edition consist of the omission of the Chapter on Tables and the elimination of the Exercises for practice which appeared in the previous edition (1882), and of other unimportant matter; also, a contraction of the illustrations. The text and diagrams, on the other hand, have been much augmented with new matter, and considerable portions of the old matter greatly modified in places, the whole being condensed, re-arranged and rendered quite up-to-date in all particulars. The logarithmic, traverse and other tables, which existed in the former edition, now find a place in the "Calculating Tables" (4th edition), so that the latter work must necessarily be consulted in connection with the calculations contained in the present treatise.

I have to express my indebtedness to Colonel J. Hill, R.E., and Mr. W. G. Beverley of the Great Trigonometrical Survey of India, and some other friends who very kindly assisted in reviewing the original manuscript, and helping to accuracy in the work where needed; as also to those who have contributed in various ways to the completion of the book generally.

Suggestions for the improvement of future editions of the book are invited and will be gratefully acknowledged.

J. C. C.

CLAPHAM, LONDON, S.W.,
The 1st May 1898.

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[No. 12.]

ERRATA ET CORRIGENDA

(To the "Aid to Land-Surveying", Large Edition, 3rd).

Text.

Page 235, line 37, *after* " × tan." *insert* the sign " = ".
Page 252, line 28, *after* " limb " *insert* an asterisk (*)
Page 252, footnote, *after* line 3 *insert* the following:—

" * In occultations of stars by the Moon, disappearances take place from New to Full at the Dark Limb, and reappearances from Full to New."

Illustrations.

Page 33, Fig. 489, from the point R.A. $0^h 0^m$, Dec. 85° N., continue the dotted line separating the constellations URSA MINOR and CEPHEUS to the point R.A. $8^h 0^m$, Dec 84° N, first in a slightly outward and then in a slightly inward curve.

Page 33, Fig. 489, continue the dotted line indicating the path of the pole from the N. Polar point to the point R.A $12^h 5^m$, Dec. 80° N., in an eliptical curve.

G. B. C. P. O.—No. 335, C. S. & L. R. 12-5-13—775—A. D. C.

[No. 11.]

ERRATA ET CORRIGENDA

(To the "Aid to Land-Surveying," Large Edition, 3rd).

Text.

Page 274, at the end *insert* the following:—

"82. *Abridged Nautical Almanac.*—An abridged edition of the Nautical Almanac is now published for the use of seamen. It supplies the seamen with all the astronomical data he requires for finding his position, etc., at sea by observations of the Sun, Moon, Planets, or Stars; or for rating his chronometer on shore by observations of the Sun or Stars. Quantities are given to a degree of accuracy comparable with that obtainable in the data by sextant observations; as a general rule to 0'·1 of arc and $0^{s}\cdot 1$ of time. The Almanac gives, at Greenwich mean noon throughout the year and in certain cases for greater convenience at every even hour of Greenwich mean time, the positions, with reference to the equator, of all the heavenly bodies the seaman makes use of, in terms of Declination and Right Ascension, together with the Equation of Time. The values at any other Greenwich mean time may be found, either by the ordinary methods of interpolation, or by making use of certain auxiliary tables, or in certain cases by inspection only. All quantities in this Almanac are computed for astronomical mean time at the meridian of Greenwich. The Almanac also gives the mean times of the occurrences of certain events (such as Meridian Passages, etc.) at the meridian of Greenwich

For every month, Page I gives the Declination of Sun, Semi-diameter, Equation of Time, Right Ascension of Mean Sun (*Sidereal Time*). Page II gives the Transit of the First point of Aries, Semi-diameter of Moon, Horizontal Parallax, Meridian Passage, Age. Pages III—VI give the Right Ascension of Mean Sun, Declination of Sun, Equation of Time. Pages VII—X give the Right Ascension of Moon, Declination, Phases. Page XI gives the Ephemeris of Venus, Mars. Page XII gives the Ephemeris of Jupiter, Saturn. Then follow the Apparent Places of Stars, Acceleration and Retardation, Pole Star Tables, Rising and Setting Tables, Azimuth of Pole Star, Eclipses, Observatories, Tides, Table of Proportional Parts, Explanation.

Table of Rising or Setting. The quantity given in the Table when latitude and declination are of the same name, or subtracted from twelve hours when latitude and declination are of different names, is the hour-angle from the meridian when the body is in the true horizon, no allowance being made for refraction, etc.

In the case of the Sun, the hour-angle is the *Apparent* time of *setting*, and subtracted from twelve hours it is the Apparent (Civil) time of *rising*.

In the case of the moon, planets, or stars, the hour-angle, *added* to the M. T. of meridian passage, is the M. T. of *setting*; *subtracted* from the M. T. of meridian passage, it is the M. T. of *rising*.

The resulting time may be only approximate in the case of the Moon, owing to its rapid change of place in the heavens. To obtain a more exact time, the declination must be corrected for the G. Date corresponding to this approximate time and a new value of the hour-angle taken from the Table and still further corrected by adding the correction due to the daily variation of the time of Moon's passage over the meridian, or $\frac{\text{H.A.}}{24^{h}} \times$ diff., which correction may be taken from the "Table of Proportional Parts". The time of setting or rising is then obtained, as before, by adding or subtracting the corrected hour-angle to or from the M. T. of meridian passage.

Example.—To find the time at which the Moon rises and sets on the night of February 10th, in lat. 60°N., long. 150°W.

M. T. of Mer. Pass at Greenwich, February 10th	12^{h}	32^{m}
Corr. for long. 150° W. and "diff." 52^{m}	+	22
Local M. T. of Mer. Pass., February 10th	12	54
long. W.	10	0
G. Date of Mer. Pass., February 10th	22	54
At G Date ☾'s Dec. 12' N.		
Approx. H. A. from Table	7	26
Approx. G. Date of rising, February 10th	15	28
Approx. G. Date of setting, February 11th	6	20

Rising.		*Setting.*	
At G. Date, ☾'s Dec. ...	14° N.	At G. Date, ☾'s Dec.	10° N.
H. A. from Table ...	$7^h\ 42^m$	H. A. from Table	$7^h\ 11^m$
Corr. for $7^h\ 42^m$ and "diff." 52^m ..	+ 17	Corr. for $7^h\ 11^m$ and "diff." 52^m ...	+ 16
	$7^h\ 59^m$		$7^h\ 27^m$
M. T. of Pass., February 10th ..	12 54	M. T. of Pass., February 10th .	12 54
M. T. of rising, February 10th ...	4 55	M. T. of setting, February 10th ...	20 21

The hour-angle, obtained from the Table of rising and setting, is that when the true altitude of the centre of the body is 0°. Owing, however, to refraction and height of observer's eye, the centre of the Sun will at that time appear to be 34′ (the refraction) + dip (for height of eye) *above* the visible horizon.

To find the hour-angle when the Sun's centre will appear to be in the horizon, a correction in minutes must be *added* to the tabular hour-angle equal to, *viz.* :—

$$\text{Sec. lat. Sec. decl. Cosec. H. A.} \times \frac{34' + \text{dip}}{15}$$

To find the hour-angle when the Sun's upper limb will appear to be in the horizon, a correction must be *added* to the tabular hour-angle equal to, *viz.* :—

$$\text{Sec. lat. Sec. decl. Cosec. H. A.} \times \frac{34' + 16' + \text{dip}}{15}$$

In the case of the Moon a similar correction must be *subtracted* from the tabular hour-angle equal to, *viz.* :—

$$\text{Sec. lat. Sec. decl. Cosec. H. A.} \times \frac{\text{Hor. Par.} - 34' - \text{dip}}{15}$$

in order to find the hour-angle when the Moon's centre will appear to be in the horizon.

Table of Proportional Parts.—With the arguments, difference in two hours at the top of page and interval from nearest even hour of G. M.T. at left-hand side of page, the number in the Table is the proportional part required.

With arguments, difference in twenty four hours at the top of page and interval from nearest G. M. noon at right-hand side of page, the number in the Table is the proportional part required.

Lunar Distances.—The method of finding Greenwich Mean Time by Lunar Distances—that is to say, by measurement of the distance between the Moon and another heavenly body—is explained in all treatises on Navigation. The Lunar Distances are no longer printed in the *Nautical Almanac*, because the necessity for their use could only arise in exceptional circumstances; nevertheless, there is probably no better method of becoming an expert observer with the sextant than by practising this observation.

The accuracy of the concluded Greenwich time is greater or less according as the distance of the Moon from the star is changing more or less rapidly. This rate of change of distance varies with the Moon's velocity of motion at the instant, over which the navigator has no control, and with the cosine of the angle that the great circle distance between the Moon and the Star makes with the tangent to the Moon's path. The navigator should therefore choose his star so that this angle is less than thirty degrees.

The following method is recommended in order to secure that the star is suitable: Ascertain from the Almanac the declination of the Moon when during the week before or after the observation its R.A. is equal to that of the star under consideration; and take the difference between this declination and that of the star. Then the star is suitable provided that this difference of declination does not exceed five degrees for each fifteen degrees of distance or hour of difference of R. A.

The Sun and the four bright planets, Venus, Mars, Jupiter, and Saturn, are usually suitable when visible simultaneously with the Moon. One or other of the following stars is also nearly certain to be suitable :—*a* Arietis, *a* Tauri (Aldebaran), *a* Orionis (Betelguese), *a* Geminorum (Castor), *a* Canis Minoris (Procyon), *β* Geminorum (Pollux), *a* Leonis (Regulus), *a* Virginis (Spica), *a* Scorpii (Antares), *a* Aquilæ (Altair), *a* Piscis Australis (Fomalhaut), *a* Pegasi (Markab).

In the case of the Sun, it is important to remember that it is the true and not the mean Sun which is observed. It is therefore the R. A. of the true or apparent Sun that is required. It is obtained by applying the Equation of Time with its sign as given in the Almanac to the R. A. M. S.

The process of calculating a Lunar Distance is illustrated by the following example, which admits of three modifications, according as the Nautical Tables in use contain Natural Versines, Natural Haversines, or neither. All tables contain Log Haversines, or, as they are sometimes called, "Log Sine Square."

Example.—

D = true distance between Moon and heavenly body.
P = diff. of R. A. of Moon and heavenly body $= 4^h\ 9^m\ 21^s$.
d = declination of Moon $= 7^\circ\ 56'\ 20''$N.
d' = declination of heavenly body $= 29^\circ\ 32'\ 30''$ S.
$d \backsim d'$ = diff. of declinations, if of same name,
= sum of declinations, if of different names, $= 37^\circ\ 28'\ 50''$.

Log. hav. P	9·427922
Log. cos. d	9·995818
Log. cos. d'	9·939518
Log. hav. θ	9·363258

$$\theta = 57^\circ\ 25'\ 37''$$

Using Nat.	*Versines.*	*Using Nat.*	*Haversines.*	*Using Log.*	*Haversines.*
Nat. Vers. θ	461624	Nat. Hav. θ	23081	Log. Hav. θ	9·363258 = Nat. No ·23081
Nat. Verr. $(d \backsim d')$	206439	Nat. Hav. $(d \backsim d')$	10322	Log. Hav. $(d \backsim d')$	9·013701 = Nat. No. ·10322
Nat. Vers. D	668063	Nat. Hav. D	33403	Log. Hav D	9·523785 = Nat. No. ·33403
$D = 70^\circ\ 36'\ 49''$		$D = 70^\circ\ 36'\ 49''$		$D = 70^\circ\ 36'\ 48''$	

It may be noticed from the above that Nat. Versine = 2 Nat. Haversine, and that the Nat. number corresponding to Log. Hav. is the Nat. Hav.

Page 195, 2nd footnote, at the end, *add*:—

"The three great periods in ancient Astronomy are—

(i) From B. C. 2700 to B. C. 700, when the constellation *Taurus* led the Zodiac.

(ii) From B. C. 700 to B. C. 129 when the constellation *Aries* was considered to lead the Zodiac The Vernal Equinox was reckoned as being at the 8th degree of Aries, the position held by the star *Hamal* at the beginning of the period.

(iii) From B. C. 129 to the present day, when the Vernal Equinox has been taken as the fiducial point from which the signs of the Zodiac are reckoned: it is the 'First Point of Aries.' This fiducial point is fixed as regards the Signs, but moves as regards the constellations of actual stars."

G. B. C. P. O.—No. 269, C. L. S. R., 23-12-1912—627—A. J. DeS.

[No. 10.]

ERRATA ET CORRIGENDA

(To the "Aid to Land-Surveying," Large Edition, 3rd.)

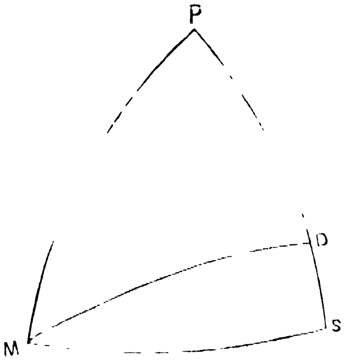
Text.

Page 93, last footnote, line 2, *for* "in" *read* "is."

Page 259, line 28, *after* "Art. 78" *add* "the true Greenwich Time when the distance was observed can be ascertained."

Page 259, *below* line 23, *insert* the following :—

"*Another Method.*



In let M be the Moon, S the Star, and P the Pole.

MD is perpendicular to PS.

Then,
$$\cos MS = \cos PM \cos PS + \sin PM \sin PS \cos P$$
$$= \cos PM \cos PS\ (1 + \tan PM \tan PS \cos P)\ ;$$

but, in the right-angled triangle PMD,
$$\tan PD = \tan PM \cos P :$$

therefore,
$$\cos MS = \cos PM \cos PS\ (1 + \tan PD \tan PS)$$
$$= \cos PM \cos (PS - PD) \sec PD.$$

Example I.—Sun and Moon.

To find the *true distance* between the Sun and Moon at noon, Greenwich Mean Time, on March 12, 1912.

From the *Nautical Almanac.*

Right Ascension.

	h	m	s
Sun	23	29	0·0
Moon	18	44	20·8
diff.	4	44	39·2

„ (in arc) 71° 9′ 48″

Declination.

	S.	3°	20′	54′
	S.	28	9	56
PS	=	93	20	51
PM	=	118	9	56
P	=	71	9	48

tan PM	− 0·271304		
cos P	+ 9·509030		
tan PD	− 9·780334		
PD	148°	54′	33″
PS	93	20	54
diff.	55	33	39
cos diff.	+ 9·752456		
cos PM	− 9·673961		
sec PD	− 0·067349		
cos MS	+ 9·493766		

MS 71° 50′ 13″ = true distance.

Therefore 71° 50′ 13″ is the *true distance* between the Sun and the Moon at noon on March 12, 1912.

Example II.—Star and Moon.

To find the *true distance* between Regulus and the Moon at 4h p.m., Greenwich Mean Time, on February 28, 1912.

From the *Nautical Almanac.*

Right Ascension.	h	m	s
Moon	7	15	4·8
Star	10	3	42·8
diff.	2	48	38·0
„ (in arc)	42°	9′	30″

Declination.			
N.	27°	29′	22″
N.	12	23	48
PS =	77	36	12
PM =	62	30	38
P =	42	9	30

tan PM	+ 0·283719
cos P	+ 9·869990
tan PD	+ 0·153709
PD	54° 56′ 2″
PS	77 36 12
diff.	22 40 10
cos diff.	+ 9·965081
cos PM	+ 9·664252
sec PD	+ 0·240694
cos MS	+ 9·870027

MS 42° 9′ 11″ = true distance.

Therefore 42° 9′ 11″ is the *true distance* between Regulus and the Moon at 4 p.m. on February 28, 1912."

Page 261, line 22, 1st column, *for* "Cassiopea" *read* "Cassiopeia."
Page 261, line 32, 3rd column, *for* "Cham leon" *read* "Chamæleon."
Page 261, line 47, 3rd column, *for* "Cœlum" *read* "Cæolum."
Page 261, line 50, 3rd column. *for* "Pixis" *read* "Pyxis"
Page 262, line 31, *for* "Cassiopea" *read* "Cassiopeia."
Page 262, line 34, *for* "Cassiopea" *read* "Cassiopeia."
Page 262, line 35, *for* "Cassiopea" *read* "Cassiopeia."

Illustrations.

Page 33, fig. 489, from R.A. 11h. 5m., Dec. 5° 0′ S., to R.A. 10h. 25m., Dec. 5° 0′ S., draw a dotted divisional line between the constellations VIRGO and SAXTANS.

Page 34, fig. 488*a*, between R.A. 8h. and 10h., Dec. 2° N., *insert* the word "EQUATOR" parallel to and just below the Equinoctial line.

Page 34, fig. 488*a*, between R.A. 8h. and 10h., Dec. 15° N., *insert* the word "ECLIPTIC" parallel to and just above the line of the Ecliptic.

Page 34, fig. 489*a*, from R.A. 12h. 30m., Dec. 3° 0′ N. to R.A. 12h 30m., Dec. 4° 0′ S., *continue* the dotted divisional line separating the constellations LEO and VIRGO to the constellation CRATER.

Page 34, fig. 489*a*, between R.A. 13h. and 15h., Dec. 2° N., *insert* the word "EQUATOR" parallel to and just below the Equinoctial line.

Page 34, fig. 489*a*, between R.A. 18h. and 20h., Dec. 25° S., *insert* the word "ECLIPTIC" parallel to and just below the line of the Ecliptic.

G. B. C. P. O.—No. 313, C. S. L. R., 1-4-1915—500—J. A. P.

[No. 9.]

ERRATA ET CORRIGENDA

(*To the "Aid to Land-Surveying," Large Edition, 3rd*).

Illustrations.

P. 3, fig., 123, *for* "C" *read* "O".

G. B. C. P. O.—No 7, C of S. & L. R. 1(a), 13-5-1912—270—H. A. W.

[No. 8.]

ERRATA ET CORRIGENDA

(*To the "Aid to Land-Surveying," Large Edition, 3rd.*)

Text.

Pp. 174-175. Between the pages, *insert* as p. 174*a*, the following:—

174*a*

EXAMPLE IV(*a*).—*Reduction of the Co-ordinates of Stations from Rectangular to Spherical.*

Latitude of Origin (λo) *17° 00′ 00″ N., Longitude of Origin* (L°) *96° 00′ 00″ E.*

Name of Station (S) ...	⊙52	⊙368	1 {λ′ 17° 30′ N. L. 96 15 E.	2 {λ′ 17 15′ N. L. 96 15 E.
Y in chains (± if N/S) ...	N. 1,285·09	N. 1,650·09	N. 2,752·04	N. 1,376·77
Log "	3·108923	3·217508	3·439655	8 138801
Log 60	1·819544	1 819544	1·819544	1·819544
Sum = log Y in feet ...	4 928477	5·037052	5·259199	4·958405
	′ ″	′ ″	′ ″	′ ″
$\frac{\text{Y in feet}}{100}$ = Y in arc approximately = Y*a* †.	+0 14 08	+0 18 09	+ 0 30 16	+ 0 15 09
± ½ Y*a*	+0 7 04	+0 9 05	+ 0 15 08	+ 0 7 54
λo	17 00 00	17 00 00	17 00 00	17 00 00
Algebraical Sum ...	17 07 04	17 09 05	17 15 08	17 07 54
A. C. of log from Table LIII * for Algebraical Sum.	$\bar{3}$·996320	$\bar{3}$·996320	$\bar{3}$·996315	$\bar{3}$·996301
Log Y in feet ...	4·928477	5·037051	5·259199	4·958405
Sum = log Y in seconds ...	2·984797	3·033371	3·255513	2·954706
	′ ″	′ ″	′ ″	′ ″
Y in arc † ...	+0 14 01	+0 18 00	+ 0 30 01	+ 0 15 01
λo ± Y in arc λ′*a* ...	17 14 01	17 18 00	17 30 01	17 15 01
X in chains (± if E/W)	+1,497·72	+ 1,232·97	+ 1,319·61	+ 1,321·40
Log " ...	3·175431	3·090953	3·120446	3·121034
Log 66	1·8195	1·8195	1·8195	1·8195
A. C. of log from Table LIV * for λ′*a*.	$\bar{2}$·013624	$\bar{2}$·013780	$\bar{2}$·014253	$\bar{2}$·013663
Sum ...	3·0086	2·9242	2·9542	2·9542
Twice " ...	6·0172	5·8484	5·9084	5·9084
Log from Table LV * for λ′*a* .	$\bar{7}$·8389	$\bar{7}$·8404	$\bar{7}$·8447	$\bar{7}$·8393
Sum = log Arc-versine ...	$\bar{1}$·8561	$\bar{1}$·6888	$\bar{1}$·7531	$\bar{1}$·7477
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Arc-versine ...	0 0 1	0 0 1	0 0 1	0 0 1
λ′*a*—Arc-versine = Lat. of (S)	17 14 00	17 17 59	17 30 00	17 15 00
Log X in chains ..	3·175431	3·090953	3·120445	3·121034
,, 66	1·819544	1·819544	1·819544	1·819544
A. C. of log from Table LIV * for Lat. of (S).	$\bar{2}$·013623	$\bar{2}$·013780	$\bar{2}$·014253	$\bar{2}$·013664
Sum = log X in seconds	3·008598	2 924277	2·954242	2·954242
	° ′ ″	′ ″	° ′ ″	° ′ ″
X in arc † ...	+0 17 00	+0 14 00	+ 0 15 00	+ 0 15 00
L*o*	96 00 00	96 00 00	96 00 00	96 00 00
Algebraical Sum = long. of (S)	96 17 00	96 14 00	96 15 00	96 15 00

† X in arc and Y in arc have the same signs as they have in chains.

* Of the Calculating Tables, 5th Edition.

P. 180, Art. 4, line 6, *for* " +863" *read* " +8·63".

P. 180, Art. 4, line 11, *for* "AG." *read* "DG".

P. 186, Art. 29, line 18, col. 8 of form, *for* "0·75" *read* "0·78".

P. 186, Art. 29, line 19, col. 6 of form, *for* "5·34" *read* "5·54".

P. 192, Art. 1, line 1, *against* the 2nd "*Astronomy*" *place* 2 asterisks (**).

P. 192, at the bottom, *add* the following as a foot-note "Astronomy is divided into two branches—gravitational, or astronomy of position, and physical, which concerns itself with the composition of the heavenly bodies".

P. 205, line 41, *insert* two asterisks (**).

P. 205, at bottom, *insert* the following foot-note :—

"* In the case of a penumbral lunar eclipse it may happen that though predicted in the N.A. there may be no indication of the phenomenon, either by the appearance of a definite shadow on any portion of the moon's surface or only of a dimming of the moon's light. The explanation of this is that when the moon only enters a slight distance into the penumbra, there is little or nothing to observe. The N.A. gives the prediction because it is not easy to know where to draw the line. Any contact with the penumbra has of late been given. The policy is no doubt questionable."

P. 62, 1st footnote, *add* : "Sheets of the millionth scale (15·783 miles = 1 inch) series embrace 4° long. × 4° lat. ; the included Degree Sheets = 1° long. × 1° lat., and the Standard Sheets = 15′ long. × 15′ lat., the size of the older series being 30′ long. × 15′ lat. The scale of the Indian Atlas is 4 miles = 1 inch."

Illustrations.

P. 8, Fig. 216, *insert* "B" at top of the circle.

P. 32, Fig. 475, on the arc E W, below γ, *insert* a dot (·); and slightly to left of γ *insert* an arrow-head (<—), facing W.

P. 32, Fig. 478, on the arc *e*L, *insert* an arrow-head (<—), facing L.

G. B. C. P. O.—No. 312, C. of S., L. R., 1(a), 27-3-1915—500—J. A. P.

No. 3.]

ERRATA ET CORRIGENDA

(*To the "Aid to Land-Surveying," Large Edition, 3rd.*)

Text.

160, para 174, line 13, *after* "of" *insert* "any."

162, Table VIII, line 9, *after* "be" *insert* "the same as those in 1908 (or 1904), *viz.*,".

171, line 12, *for* "1·634377" *read* "1·634376".

171, line 13, *for* "1·640566" *read* "1·640565".

171, line 13, *for* "28·85" *read* "43·71".

P. 196, *to* the 5th footnote *add* "The red appearance of the sun when on the horizon is due to the absorption of the blue rays of the refracted light by the atmosphere."

P. 203, line 45, *after* "*partial*" *add* "according to the distance of the shadow of the earth from the moon's node".

C. P. O.—No. 54. C. S. & L. R. 1(*a*). 25-7-1911—824—J. A. P.

No. 2.

ERRATA ET CORRIGENDA

(To the "Aid to Land-Surveying," Large Edition, 3rd).

(i) Text.

P. 119, 1st footnote, line 3, ***after*** "sun" ***insert*** **:** "The longer the focal length of the object glass of a telescope and the shorter the focal length of the eye-piece, the greater the magnification".

P. 119, 2nd footnote, at end, ***after*** "equator" ***add*** **:** "There has always been a kind of rivalry between the refracting and reflecting telescopes Where accurate measurements are required the refractor is generally to be preferred. Where the object observed is extremely faint, the reflector has the advantage, for in proportion to size reflecting telescopes are of much shorter focal length, thus producing a smaller but brighter picture In the equatorial telescope, the telescope is perpendicular to, and can turn about, the declination axis The declination axis is perpendicular to, and turns about, the polar axis A ***position-micrometer*** is placed in the focal plane of the telescope by means of which the distance between two very near stars, which can be seen at the same time in the field of view, or the diameter of one of the planets may be measured. This distance is measured in linear measure, say, in fractions of an inch; the angle between the two positions is found by dividing this position by the focal length of the telescope. With a large telescope angles to nearly one minute of arc may be measured—or less than $\frac{1}{30}$th of the diameter of the Sun or Moon For angles comparable in size with the Sun's diameter the most accurate visual observations are made with an instrument called a *heliometer*, which is an equatorially mounted telescope having its object glass divided into two halves which can be moved by means which admit of the distance from the central position being accurately determined. The telescope is seen in its simplest form when it is used for photography. A photographic telescope is merely a large photographic lens of correspondingly long focus. A photographic plate is placed in the focal plane of the object glass or reflector, and gives a permanent record of the image formed there. Like the heliometer, a photograph can be used for finding distances and relative directions of stars whose distance apart is less than one or two degrees".

P. 119, 3rd footnote, line 3, ***after*** "glass" ***insert:*** "(a dense glass with large refracting power)".

P. 119, 3rd footnote, line 4, ***after*** "glass" ***insert:*** "(a lighter glass of less refractive power)".

P. 141, footnote, ***add*** the following para.: "The mean of the times at which the star crosses the wires gives, subject to some small corrections which are easily calculated, the ***exact moment*** at which the star crosses the meridian, *i.e.*, the *right ascension*. To obtain the star's ***declination*** it is necessary to determine the *altitude* at which the telescope is pointed With a good instrument, a good clock and a good observer, the right ascension and declination of a star can be determined with an error which is seldom more than 2". To form an idea of 2" comparison may be made with the angular diameter of the Sun, which is 30' or 1,800".

P. 197, para. 44, line 8, ***after*** "clock" ***insert*** 2 asterisks (**).

P. 197, at bottom, ***after*** 5th footnote, ***insert*** **:** "** The time indicated by a sidereal clock (supposed to be correct) gives the right ascensions of all stars which are at that moment on the meridian".

P. 199, footnote, line 6 from bottom, ***after*** "46·5", before previous additions, ***insert*** **:** "Except for a limited number of stars the parallactic effect, *i e*, the change produced in the relative positions of distant objects when viewed from different points, is practically insensible among the stars owing to their immense distance".

P. 199, footnote, line 6 from bottom, ***after*** previous additions, ***insert*** **:** "Of the number, constitution, characteristics, distance, mass, volume, density, proper motion, velocity, luminosity, etc, of the stars our knowledge is still extremely limited. In the words of Laplace '*Ce que nous connaissons est peu de chose, ce que nous ignorons est immense*'".

P. 201, footnote, at end, ***after*** previous addition, ***insert*** **:** "The point of the sky towards which the solar system is moving has R A. 275°, Dec 30° N, and is not far from the star Vega. The velocity with which the solar system is moving through space is found to be about 12½ miles per second".

P. 202, 1st footnote, at end, ***after*** "earth-light" ***add*** **:** "The Sun's axis is inclined at an angle of 7° to the perpendicular to the ecliptic. From 3rd June to 5th December the plane of the ecliptic is north of the Sun's equator, and thus for this half of the year the Sun's north pole is on the visible side of the Sun, and in the other half of the year its south pole is visible"

P. 203, line 21, ***after*** "point" ***insert*** 3 asterisks (***).

P. 203, line 35, ***after*** "Comets" ***insert*** 4 asterisks (****).

P. 203, line 35, ***after*** "meteors" ***insert*** 5 asterisks (*****).

P. 203, line 17 from bottom, ***after*** "intervals" ***insert*** "as in 1910,".

P. 203, line 16 from bottom, ***after*** "lunar" ***insert*** "or 3 solar and 4 lunar".

P. 203, line 15 from bottom, ***after*** "annually" ***insert*** "as in 1911,".

P. 203, line 14 from bottom, ***after*** "3" ***insert*** "or 4".

P. 203, ***after*** the 4th footnote, ***insert*** the 3 following footnotes:

"*** The existence of a radiant point shows that all meteors are moving in parallel lines. If meteors were falling perpendicular to the earth, their paths when drawn on a globe would all pass through the zenith. The position of the radiant point gives the direction in which the meteors are moving relative to the earth.

**** The swarm of meteors, known as the Leonids, has its radiant point in the constellation Leo; they move in nearly the same orbit round the Sun This orbit and the Earth's intersect at the point where the earth is situated on 13-14 November, when the meteors become visible These meteors move in an eccentric elliptic orbit which stretches beyond Uranus, with a period of 33⅓ years

***** A comet may be said to be made up of a loose collection of meteorites, gathered possibly a larger central nucleus. When the comet approaches the Sun, heat and other radiative influences tend to disintegrate the mass, and to drive off gaseous constituents which form the tails"

P. 205, last line from bottom, ***after*** "eclipses" ***insert*** an asterisk (*).

P. 205, at bottom, ***insert*** the following footnote: "* A solar eclipse should be viewed through the medium of a dark or smoked glass; it may also be seen, by reflection, inverted, in a basin of still water".

P. 245, line 7 from bottom, ***for*** "to" ***read*** "of".

P. 261, footnote, line 2, ***after*** "English" ***insert*** "doggerel".

P. 261, footnote, line 5, ***for*** comma (,) ***after*** "Scales" ***put*** semi-colon (;).

P. 261, footnote, line 6, ***for*** "Sea-goat" ***read*** "He-goat".

G. B. C. P O.— No. 210, C. of S. & L. R., 1 (*a*), 19-11-1910—1,100—F. X. deC.

o 1.

ERRATA ET CORRIGENDA

(To the "Aid to Land-Surveying," Large Edition, 3rd).

(i) Text.

. 11, Art. 18, *insert* the following footnote:—

"*Problem*: Find the difference between the radii of any two concentric circles whose circumferences differ in length by 6 feet.

Let R and r be the radii of the larger and smaller circles, respectively.

Then, ⊙ce of smaller ⊙ $= 2\pi r$;

$$\therefore r = \frac{2\pi r}{2\pi}.$$

And ⊙ce of larger ⊙ $= 2\pi R = 2\pi r + 6$;

$$\therefore R = \frac{2\pi r + 6}{2\pi}.$$

$$\text{Hence, } R - r = \frac{2\pi r + 6}{2\pi} - \frac{2\pi r}{2\pi}$$

$$= \frac{6}{2\pi} = \frac{21}{22} \text{ Ft. } \textit{Ans.}"$$

. 119, Art. 28, line 14, *for* "l" *read* "*l*".

. 119, Art. 28, line 32, *for* "l" *read* "*l*".

. 143, line 14, *after* "plotting" *add* "They consist of the plane-table sheets on which the traverse stations are coördinated and plotted and on which the detailed mapping is subsequently entered."

. 160, Art. 157, line 18, *after* "originals" *add* "For drawing and printing on tracing-cloth, indian-ink should alone be used; the ink is improved for writing with on either the glazed or the dull surface of the cloth by a slight admixture of indigo-blue paint, which may be effected by rubbing the cake of the paint in the colour saucer containing the prepared ink. Writing ink, blue-black or other, is unsuitable for map work on tracing-cloth."

P. 161, line 4, *for* "*Ab*" *read* "A*b*".

P. 162, Art. 7, left col., line 35, *for* "91·13" *read* "9·13".

P. 162, Art. 7, left col., line 35, *for* "8·1" *read* "8·10".

P. 163, Art. 10, line 8, *after* "apply" *insert* "arbitrarily".

P. 165, col. 6, line 15, *for* "°281 40′" *read* "281° 40′".

P. 165, col. 15, line 21, *for* "2" *read* "1".

P. 196, Art. 39, *to* the 5th footnote *add*:—

"When a ray of light passes from a rarer into a denser medium the refraction is *towards* the perpendicular, or the angle of refraction is less than the angle of incidence. On the contrary, when a ray of light passes from a denser into a rarer medium, the refraction is *from* the perpendicular, or the angle of refraction is greater than the angle of incidence. Astronomical or atmospheric refraction is the apparent angular elevation of the heavenly bodies above their true places, caused by the refraction of the rays of light in their passage through the earth's atmosphere, so that in consequence of this refraction the heavenly bodies appear higher than they really are."

(ii) Illustrations.

. 22, Fig. 411, on limb, the 30′ divisions from 0° to 10° should be half their length.

. B. C. P. O.—No. 45, C. of S. & L. R., 1(*a*), 5-8-1910—1,020—F. X. deC.

ERRATA ET CORRIGENDA

(i) Text.

P. 19, Art. 42, line 2, *after* "given" place 2 asterisks (**).

P. 19, Art. 42, line 5, *transpose* "7" and "17".

P. 19, Art. 42, line 6, *transpose* "7" and "17".

P. 19, at bottom, *add* a 3rd footnote: "*To find the periphery (approximately) of an ellipse. Rule:* Multiply the square root of half the sum of the squares of the two diameters by $\frac{22}{7}$. Thus, in the example, periphery $= \frac{22}{7}\sqrt{\frac{1}{2}(17^2+7^2)} = \frac{22}{7}\sqrt{169} = \frac{22}{7} \times 13 = 41$. *To find the length of any ordinate (x), the transverse (a) and conjugate (b) axes, and the distance of the ordinate from the centre (d) being known. Rule:* Take the square root of the difference between the square of the semi-conjugate axis and the square of the product of the semi-conjugate axis with the distance of the ordinate from the centre divided by the semi-transverse axis; *i.e.*, $x = \sqrt{b^2 - \left(\frac{b \times d}{a}\right)^2}$. Thus, in the example, taking d equal to 5, $x = \sqrt{b^2 - \left(\frac{b \times d}{a}\right)^2} = \sqrt{3{\cdot}5^2\left(\frac{3{\cdot}5 \times 5}{8{\cdot}5}\right)^2} = \sqrt{12{\cdot}25 - 4{\cdot}24} = \sqrt{8{\cdot}01} = 2{\cdot}83$.

P. 89, *insert* the following *after* the footnote, under 'OBSERVATION', *viz.:* "If h = the height of the observer in *feet*, and d = the distance of the horizon in *miles*; then, $h = \frac{2}{3}d^2$. Because if r = the radius of the earth [3,963 *miles*], $(r+h)^2 = r^2 + d^2$; $d^2 = h^2 + 2hr$. But as h^2 is comparatively very small it may be taken as negligible; so $d^2 = 7{,}920 \times \frac{h}{5{,}280} = \frac{3}{2}h$. Roughly, at 10 feet high, the horizon is 4 miles off; at 20 ft., 5½ mls; at 30 ft., 6¾ mls; at 40 ft., 7¾ mls.; at 50 ft., 8½ mls

P. 160, Art 157, last line, *after* "originals" *insert* "For drawing and printing on tracing-cloth, indian ink should alone be used; the ink is improved for writing with on either the glazed or the unglazed surface of the cloth by a slight admixture of indigo blue paint, which may be effected by rubbing the cake of the paint in the colour-saucer containing the prepared ink. Writing ink, blue-black or other, is unsuitable for map work on tracing-cloth."

P. 196, *to* the 4th footnote *add* the following: "Latest value of the Solar parallax.—From the reduction of the photographs of Eros taken at the Cambridge University Observatory, Mr. A. H. Hinks announced at the Astrographic Conference at Paris that the resulting value of the solar parallax was 8″·807".

(iv) Illustrations.

P. lxxiv, item 7, line 1, *delete* the lower of the two parallel lines of the diagram.

ERRATA ET CORRIGENDA

(i) Text.

P. 32, Art. 8, line 14, at end, *delete* the dagger (†)

P. 32, Art. 8, *after* line 14, *insert*:

2·327 410	3·532 488
+ 2·411 240	+ 1·842 264
4·738 650	5·374 752 †

P. 33, first footnote, line 2, *for* "1·2" *read* "4·2".

P. 34, Art. 15, line 7, *for* "[illegible]" *read* "41°".

P. 34, Art. 15, line 10, *for* "[illegible]" *read* "15".

P. 35, Art. 16, line 3 from bottom, *for* "L. cot" *read* "L. cot. 73°28′".

P. 36, line 8 from bottom, *for* "10·23841" *read* "10·238415"

P. 81, Art. 8, line 2, *after* "tested" *add* "Stout steel chains are more reliable than thin iron ones"

P. 98, Art. 24, line 32, *after* "Star" *insert* "Fig. 442)"

P. 99, line 22, *after* "transit the" *insert* "star"

P. 99, second footnote, line 7, *for* "XXXII" *read* "XXXVII"

P. 103, line 1, *for* "X. Compass" *read* "XI. Plane-table".

P. 117, Art. 19, line 1, *for* "29" *read* "19".

P. 118, Art. 25, line 1, *delete* "using".

P. 118, Art. 25, line 1, *for* "Never employ violent force" *read* "Force must never be employed".

P. 118, Art. 27, for "Lifting the instrument" *read* "Placing the theodolite".

P. 119, Art. 28, line 6 from bottom, *after* "Needle" *insert* 2 asterisks (**).

P. 119, Art. 28, line 9 from bottom, *after* "Object Glass" *place* two asterisks (**).

P. 119, first footnote, line 3, *after* "sun" *add* "The 'field of view' of a telescope signifies the small circular patch of the sky which is visible when looking through a telescope pointed to the heavens. In an ordinary telescope with a magnifying power of 1,000, the field of view would be about 2 minutes in angular measure. The higher the power the less is the field of view, and the greater the difficulty, consequently, in following a star in its diurnal motion. This difficulty is, however, overcome by an equatorial mounting of the telescope, so fitted, in respect to the polar and declination axes, that by means of a mechanical contrivance the telescope may be made to follow a star in its diurnal motion. A telescope for astronomical purposes has usually a smaller telescope, called the 'finder' fastened to the lower end of its tube, by which the object may be first found and brought directly into the field of view of the larger telescope. The small telescope is provided with a low magnifying power, and has therefore a larger field of view."

P. 119, at bottom, *insert* a third footnote: "The object-glass of a refracting telescope consists of a combination of a convex lens of crown glass and a concave lens of flint glass by which the dispersion of rays is neutralized. By the action of the objective, in thus bringing rays to a focus, the image of a distant object is formed in the focal plane, which is a plane that passes through the focus at right angles to the axis or line of sight of the telescope."

P. 119, at bottom of footnotes, *add* "** In some of the new-class instruments it is attached to the horizontal plate".

P. 121, Art. 44, line 5, *for* "Levelling" *read* "Horizontal levelling".

P. 121, Art. 46, line 2, *for* "your" *read* "the".

P. 121, Art. 47, line 8, in 2 places, *for* "your" *read* "the".

P. 121, Art. 48, line 1, *for* "Levelling" *read* "Horizontal levelling".

P. 122, line 10, *after* "level" *insert* 2 asterisks (**)

P. 122, Art. 52, line 2, *for* "vernier" *read* "graduations"

P. 122, Art. 53, line 5, *add* "This should be done once for all by the surveyor at the beginning of survey operations so as to suit his own vision"

P. 122, footnote, *after* line 1, *insert* "** In some of the later type of instruments there are two horizontal levels fixed to the vernier plate at right angles to each other. By this means the necessity for turning the telescope a quarter round is obviated. The bubble of the principal level is brought to the centre of its run by the two foot-screws parallel with the level, and that of the smaller level by a similar adjustment with the third foot-screw".

P. 123, line 2, *for* "your" *read* "the"

P. 123, line 3, *for* "your" *read* "the".

P. 124, Art. 61, line 1, *for* "How to Change" *read* "Change of".

P. 124, Art. 61, line 2 from bottom, *for* "it" *read* "the compass box".

P. 124, Art. 62, line 2, *for* "the" *read* "a".

P. 124, Art. 62, line 8, *delete* "as before".

P. 127, line 16, *for* "Face" *read* "Faces"

P. 128, Art. 67, line 1, *against* "Surveying" *insert* two asterisks (**)

P. 128, Art. 68, line 1, *for* "453" *read* "457".

P. 128, Art. 68, form at bottom, last col., line 5, *for* "100·09" *read* "100·20"

P. 128, footnote, line 5, *for* "is" *read* "are".

P. 128, at bottom, *insert* the following additional footnote: "*** Satisfactory geographical map-making must necessarily be based on two scientific functions, *i.e.*, triangulation, which means the accurate fixing by observation and mathematical deduction of certain points and features on the earth's surface; and topography, which means the accurate delineation on the spot of the features of the country relating to these triangulated points."

P. 129, Art. 68, form at bottom, last col., line 2, *for* "100·66" *read* "100·06".

P. 143, Art. 86, line 1, *for* "Manner of observing" *read* "Traversing".

P. 143, Art. 86, line 9, *for* "yourself" *read* "oneself".

P. 143, Art. 86, line 10, *for* "observed at" *read* "observed to".

P. 143, Art. 86, line 11, *for* "your" *read* "the".

P. 143, Art. 86, line 13, *for* "your" *read* "the".

P. 143, Art. 86, line 15, *for* "your" *read* "the".

P. 143, Art. 86, line 22, *for* "your" *read* "the".

P. 143, Art. 86, line 28, *after* "the" *insert* "given".

P. 145, Art. 87, line 3, *for* "kwins" *read* "villages".

P. 145, Art. 88, line 2, *for* "latter" *read* "taking of offsets".

P. 145, Art. 90, line 6, *after* "work" *add* "Steel chains are preferable to iron ones, and stout chains to thin ones; the latter are apt to bend and break and their rings to open out"

P. 145, Art 91, line 1, *for* "While" *read* "When".

P. 145, Art 93, line 5, *for* "camp" *read* "section"

P. 145, Art 93, line 6, *for* "camp" *read* "section"

P. 145, Art 93, line 8 from bottom, *for* "These" *read* "Stations thus marked"

P. 145, Art 93, lines 4 to 2 from bottom, *for* "by the * * * camp" *read* "to the traverse surveyor"

P. 146, line 4, *for* "mark" *read* "symbol".

P. 146, line 5, *for* "entire" *read* "centre".

P. 146, line 6, *for* "mark" *read* "symbol"

P. 146, line 10, *for* "are" *read* "should be"

P. 146, line 13, *for* "camp" *read* "section"

P. 146, line 16, *after* "also" *insert* "One or more sub-traverses should be run through the village and as many of the field surveyor's temporary stations picked up as possible. It is imperative that the theodolite traverse should be done in the same season as the field-surveyor's work, so that the field-surveyor's temporary stations may be traceable on the ground, and the demarcation lines may still be found open"

P. 146, line 20, *after* "mapped" *insert* "geographically fixed".

P. 146, Art, 94, line 5, *after* "places" *insert* "which".

P. 146, Art 94, line 12, *for* "was" *read* "were"

P. 146, Art 94, line 14, *for* "given in" *read* "of"

P. 146, Art. 94, line 15, *for* "of" *read* "in"

P. 146, Art. 96, line 4, *after* "circuits" *insert* "or sub-circuits"

P. 147, line 2, *for* "purdal s" *read* "purdas".

P. 147, Art. 97, line 3, *for* "at" *read* "to".

P. 148, Art. 105, line 1, *for* "These" *read* "Traverse lines".

P. 148, Art. 105, line 5, *for* "was" *read* "is".

P. 148, Art. 105, line 13, *after* "apart" *insert* "and the lines should, if possible, be zig zag to give more points for connection to the detail survey".

P. 148, Art 105, last line *for* "detailed" *read* "detail".

P. 149, Art 108, last line, *for* "camp" *read* "section".

P. 150, Art 111, line 4, *after* "circuits" *insert* "or traverses"

P. 150, Art 114, line 17, *after* "recorded" *insert* heading for new para "114*a* Apportionment of Work among Traverse-surveyors".

P. 151, line 10, *after* "observed" *insert* heading for new para "114*b*. Jungle Blocks"

P. 151, Art 115 line 3, *after* "instrument" *add* "and vernier A alone read for sub-traverses. Where all the verniers are to be recorded it is not sufficient at the start to set vernier A to zero and leave the other verniers unread though recorded; it is imperative that all the verniers be read at the commencement as well as at the succeeding observations in the measurement of the angles".

P. 152, Art 119, line 5, *for* "forward" *read* "back".

P. 152, Art. 119, line 6, *for* "back" *read* "forward".

P. 152, Art. 123, line 9, *for* "your" *read* "the".

P. 153, line 21, *after* "only" *add* "of the theodolite".

P. 153, line 28, *for* "board of" *read* "board at".

P. 153, line 33, *for* "to" *read* "of".

P. 154, *after* line 43, *insert* a new para "123*a*. Measurements over Bad Ground. The use of a long well stretched rope or tape may be resorted to with advantage, when a subtense bar is not available, for measuring lines over ravines and rocky ground, provided always that it is immediately compared with an accurate chain. When chaining up or down a slope for short distances, it is usual to keep the measuring line horizontal by using portions of chains, say, 40 links or 25 links. If the slope is long and regular, angles of elevation or depression may be taken to enable the horizontal distance to be computed. Reduction of hypotenusal to horizontal distances may be effected by Table X of the 'Calculating Tables' (5th edition)".

P. 154, Art 124, line 3, *after* "miles" *add* "Where villages are scattered they may be geographically connected by closed or unclosed traverses with main circuits, or with any older traverses in the neighbourhood".

P. 154, *to* second footnote, *add* "Another instrument, known as the 'Tacheometer', is used for a similar purpose by Engineer Surveyors engaged on railway and route surveys. It is constructed nearly as the theodolite, and is fitted with a diaphragm in the eye-piece provided with cleps, or webs, horizontally, by means of which the readings upon a stadium, or graduated vertical staff, are obtained, and the required distance then determined by calculation." "The two functions employed are: $H = 100\,S \times \sin.\theta \times \cos.\theta = 100\,S.\,\frac{\sin.2\theta}{2}$, $D = 100\,S \times \cos.^2\theta = 100\,S\left(\frac{1+\cos.2\theta}{2}\right)$; where S represents the stadia reading on the staff = difference between higher and lower readings of outside wires, H the height above ground at the point which cuts the central web, and D the distance. With the ordinary telescope the constants to be added to H and D, respectively, are $C \times \sin.\theta$ and $C \times \cos.\theta$; where C = focal length of object glass + distance of centre of object glass from the horizontal axis of tacheometer."

P. 156, *after* line 4 *insert* a new para. "131*a*. Receipts for Survey Marks. As soon as a traverse-surveyor has completed his traverse work in a village, he should personally hand over all the permanent survey marks put down therein to the charge of the village headman, and take a receipt from him which should be forwarded to the Camp Officer. Afterwards, when the 2" plots are ready in the camp office notices in duplicate, containing a copy of the village plot, should be prepared and sent to the Officer in charge of the survey for service on the village headmen concerned. One copy of the notice would be retained by the village headman and the other, duly signed by him, returned to the office of issue".

P. 156, Art. 133, line 10, *after* "minimized" *add* "In the course of survey care should be taken that no damage is done to standing crops".

P. 156, Art. 134, line 7, *after* "principle" *insert* "to be followed".

P. 156, Art. 134, line 13, *after* "this" *insert* "The plotting should be done as the survey is proceeded with, direct on the board, in pencil. Before beginning the detail survey a 'kaka' or rough plan on Bally paper should be prepared with all the survey stations duly shown thereon, and all the quadrilaterals, termed 'morabba', laid out by the surveyor with the lengths of all the sides measured and the distances along the various lines crossed by field boundaries. The angles of the quadrilaterals into which the village has been divided up, other than the theodolite stations, should be marked by a small hole dug in the ground encircled by a shallow drain cut with about a foot radius. Such temporary marks are useful for the after 'partals'. In laying out the quadrilateral lines, deep lakes and tanks should be avoided, if possible. If this cannot be done, a rope may sometimes be carried round and stretched across, or the distance may be obtained by laying off 4 right angles with the optical square and the perpendiculars or sides of the rectangle measured. No man should be sent with a chain into water beyond his depth. Except along the quadrilateral lines, no measurements are recorded. On completion of the measurements along the main lines and after recording them on the 'kaka' or graphical record, the measured distances are to be plotted on the 16" skeleton maps to scale, small differences being proportionately distributed, where required, on the affected lines of the traverse plot".

P. 156, Art 134, *from* line 13, make a new para with heading "134*a*. THE TRIANGULATION SYSTEM".

P. 157, line 13, *for* "camp" *read* "section".

P. 157, line 16, *for* "camp" *read* "section".

P. 157, Art. 137, line 3, *after* "scale" *insert* "Quadrilaterals should be split up into narrow rectangles by running subsidiary lines, called 'shikmi' more or less in the general direction that the field boundaries take and as much as possible along or near them. Blocks of very small fields and homesteads should be surveyed on double scale and plotted in a spare part of the sheet, as there would not be room for numbering them in the body of the map. Whole villages containing mostly small fields should be surveyed and plotted on the 32-inch scale. Whenever it is necessary to show on the map divisions that do not exist on the ground, either in waste or in lakes or in cultivation, they should be drawn in broken lines; temporary or indistinct field boundaries, and impermanent island cultivation, should be similarly shown. Existing permanent field and other distinguishable boundaries should be delineated by firm lines. No holes should be dug in the embankments of railways or canals; red flags are never to be planted on a railway line; all masonry pillars near the line, level-crossings, etc., are to be carefully fixed; the line with its banks and fences, is to be conventionally mapped".

P. 157, Art. 138, line 2, *after* "compounds" *insert* "homesteads".

P. 158, line 5, *after* "surveys" *insert* "The common margins of sheets of the same village need not be surveyed across. Extensions of cultivation outside of the village limits should not be surveyed as forming part of the village under survey".

P. 158, Art. 141, line 4, *for* "its" read "their".

P. 158, Art. 141, line 4, *for* "bed" *read* "beds".

P. 158, Art. 143, line 4, *after* "pencil" *add* "Conventional signs should be sparingly employed to permit of future details of cultivation, etc., being mapped without inconvenience. For specimens of conventional signs, *see* Map XLIV of the 'Calculating Tables' (5th edition)".

P. 158, Art. 144, line 10, *after* "line" *insert* "The total length of the independent test lines should not be less than 40 chains to every 320 acres".

P. 158, Art. 144, line 16, *for* "when" *read* "In the latter case".

P. 158, Art. 144, line 20, *for* "freshly" *read* "fresh"

P. 158, Art. 144, line [illegible], *after* "it" *insert* "There are two distinct sources of errors arising (1) from careless plotting of quadrilaterals, (2) from careless plotting of fields; the partaller should discriminate between these two kinds of error in closing work. The first is by far the more serious".

P. 159, line 3, *for* "on" *read* "by".

P. 159, line 4, *after* "latest" *add* "All test or partal lines should be recorded in the field-books, and projected, for easy comparison with the original survey, on tracing cloth or paper. All partals should be ruled in in broken lines, in blue, on the sheets".

P. 159, Art. 145, *after* line 4, *add* "Surveys should be accurate, otherwise they are worse than useless, for they are misleading. A survey if worth doing is worth doing well. It is well to impress this principle upon the staff. The best way to impress the principle is to punish heavily for any work which is obviously fudged. Every one is liable to make mistakes even if working honestly; and bad work due to fudging should be discriminated from that due to mistakes or even carelessness. For specimen plot of cadastral field-work, *see* Map LXXVII of the 'Calculating Tables' 5th edition)".

P. 159, Art 147, line 3, *for* "so also" *read* "as also the signatures".

P. 159, Art. 148, line 3, *after* "maps" *add* "Telegraph lines should be similarly shown and the positions of telegraph posts denoted on the sheets".

P. 159, Art. 151, line 5, *after* "level" *insert* "Stout steel chains are to be preferred to thin iron ones".

P. 159, *after* Art. 152, *insert* a new para. "152*a*. VILLAGE BOUNDARIES. The common boundaries of villages should be examined and passed by the Inspector before the sheets are sent in to the Camp Office; if they differ from each other, the Inspector should have the portions affected re-surveyed by the surveyors who did the original surveys".

P. 159, Art 153, line 5, *after* "oil-cloth" *insert* "(not wax-cloth)".

P. 159, Art. 15[illegible], line [illegible], *for* "stale bread-crumbs" *read* "the crumbs of stale bread"

P. 159, Art. 154, line 4, *for* "a" *read* "the"

P. 160, Art. 155, line 11, *after* "miles" *add* "For specimens of finished 16" and 2" sheets, *see* Maps LXXV and LXXVI of the 'Calculating Tables' (5th edition)"

P. 160, *after* Art. 155, *add* the following paras:

"156. INKING AND SHADING. Indian ink, which is used for the printing and inking up of maps, should be rubbed up fresh for use. The ink should be perfectly black, not brown, and the lines drawn on the sheets smooth, even, and unbroken, and not too fine. Light is supposed to come upon a map from the north-west corner, and the shadow sides of banks of rivers, tanks, villages, etc. should have a dark line for the sake of effect. In all such shading, the light is to be considered to fall at an angle of 45° from the north-west; that is, the *east* and *south* sides of any object in *elevation* and consequently above the surface of the ground, such as a house, and the *west* and *north* sides of any object in *depression* and consequently below the surface of the ground, such as a tank or a river, will be in shade, which is shown by the lines on those sides being drawn thick. For contouring, the rays of light are supposed to fall *vertically* upon the ground, and the degree of shade for expressing hills depends upon their slopes; the more these recede from the horizontal, the darker must be the shade.

157. MAPPING. (Fig. 507). When the maps are inked up and numbered, the boundaries passed, the scale and border lines drawn, and the printing done, they should be examined finally by the Inspector, and the necessary touching up done by a competent draughtsman. The numbering of the items in a sheet should be consecutive, from left to right, commencing at the north-west corner and ending at the south-east. Where there are more sheets than one the numbers should follow the course of the arrow as illustrated in Fig. 507. The shading of rivers, streams, tanks and houses should be carefully done, field boundaries should be uniformly drawn with a drawing pen only, the lettering and figuring should be of the proper size, and the transliteration rules should be strictly observed. Unless the original maps are of first-class quality in all respects the lithographed copies will in all probability be equally defective with the originals

Town Surveys.

158. TOWN SURVEYS EXPLAINED. The survey of important towns, inclusive of Municipalities, railway towns and others is now of some frequence, and the system of operations may thus be usefully described briefly. Surveys are generally done on the scale of 64 inches to the mile, though at times they are executed on other large scales such as 40 feet, 50 feet or 100 feet to the inch.

159. SYSTEM OF SURVEY. In each of the towns a main circuit should be run round the entire area for survey. Minor circuits should be formed inside the main circuit corresponding with the Ward divisions, and inside each of these the usual sub-traverses should be taken along roads or elsewhere, as found convenient, for the detail survey to follow. Care should be taken to fix stations on the berms of roads and not on frequented roads or paths, and there should be stations at all cross-roads; traverse lines should be short and

stations as numerous as circumstances justify, at about 5 chains apart. All old stations met with in the course of work should be picked up and as many of them used for the traverse stations as possible. Where the original marks of former traverses still exist, it should be possible to start the new traverses from an old survey station the true position of which should be quite assured before the commencement of operations. Numerous sub-traverses should be taken so as to fully provide the detail survey with a sufficiency of starting and closing points which are most necessary in a large-scale survey where every link of measurement tells, and where buildings and such other features are not to be given fanciful shapes. One or more verificatory azimuths should be observed on the main circuit to test the accuracy of the angular work. Sectional sheets should alone be used for the skeleton plots on which the positions of the survey stations should be projected by means of the coordinate values. These sheets, for 64' surveys (1¼ chains = 1 inch) are printed in blue, on Imperial, hand made drawing paper, blue-laid, and are 30" × 22" in size, outside measurement, and contain 35 × 25 squares of ⅘th inch divisions, each of which represents 1 chain (Gunter's) in length. Each block should be independently surveyed and mapped and treated as a distinct village, one or more blocks may form a headman's charge. For a specimen of a 64' Town Survey see Map LXXVIII of the "Calculating Tables" (5th edition).

160. DEMARCATION. Demarcation should always precede survey. The pipes (12") should be laid down at the demarcated points as the demarcation is proceeded with. Preliminary demarcation may be done by means of wooden pegs or posts, but the final demarcation must be effected by means of cylindrical clay pipes of the standard type. If the pipes are not ready to put down before the survey, the demarcation should be done in the first instance with wooden posts white-washed at the top. It will thus be necessary to survey the boundaries of house-plots as demarcated by wooden posts which would be replaced by the prescribed pipes later on.

161. STAFF. The staff should consist of picked surveyors with some experience, if possible, of the particular class of work. In the absence of previous knowledge men should be specially trained locally for the work. While at work in the field the detail surveyors should be kept together as much as possible in suitable batches; by this means better supervision is possible.

162. MAPPING, COMPUTATION AND AREA WORK. The mapping, computations and area calculations should be similar to those described for 16' surveys (see Arts. 154—157). The area statements should be in duplicate. The areas of items should be worked to three places of decimals, in acres, and also to the nearest foot, in square feet, see Table XVI of the "Calculating Tables" (5th edition).

163. CHAINING. The chaining must throughout be carefully done; for the traverse lines double measurements should be made, with two chains independently—of 100 feet and 66 feet lengths. Stout steel chains should invariably be employed; they are more reliable than thin iron ones. A test-chain should be given to each Inspector and Surveyor and used frequently, otherwise errors in measurements are sure to occur. There should be, in addition, a set of standard bars at the head office with which the test chains should be compared from time to time, as occasions offer. The outer traverse lines should hug the boundaries of the town, and the inner ones those of the wards and blocks, and offsets should be short, ordinarily not longer than 50 links; all chain lines should run through open spaces as much as possible, and cutting through gardens and other obstructions should be avoided as far as possible.

164. PERMANENT SURVEY MARKS. All theodolite stations should be permanently marked by means of clay cylinders (18") or triangular stone prisms of the usual pattern. A handful of charcoal or broken red-brick should be buried in the ground below the pipe or stone to better mark the positions of the stations (see Art. 107).

165. PROTECTION OF PERMANENT SURVEY MARKS. For the proper preservation of these important theodolite stations, each permanent survey mark should be protected by the erection of a masonry pillar 18" × 18" × 18" round the mark. Should the mark fall on the centre of a road, in the line of traffic, the pillar should be flat at the top and flush with the surface of the ground; if on the berm of a road or elsewhere, out of the way of traffic, the pillar should be raised 6" above the surface of the ground by projections to the centre from the four upper corners of the masonry.

166. SURVEY OF DETAILS. Unless expressly otherwise provided for in this section, the survey of details as well as the traversing should be performed more or less on the lines laid down for 16-inch surveys with the necessary deviations for the difference in scale. The sides of buildings and the like should be measured with a steel chain or measuring-tape independently of their positions as obtained from offsets, to avoid distortion resulting from the latter method alone. All topographical features whatsoever are to be duly mapped. The blocks should be cut up into smaller and more suitable quadrilaterals so as to permit of short offsets to corners of houses, drains, wells, etc. Before commencing the plane-table survey care should be taken that the plots to be filled in are not too large and that the frame-work of the detail survey is accurately fixed by means of cross and tie-lines. The areas covered by eaves and overhangings of houses are not to be represented on the maps; the survey system is to map the *ground* portion and to omit what may be up in the sky and does not descend to the earth.

167. TRAVERSE TO PRECEDE DETAIL SURVEY. The traverse survey should invariably precede the detail survey so that skeleton plots may be worked on. The object of a theodolite survey is to determine accurately the larger lines of the area to be surveyed. Details are then filled in with the plane-table. If the process is reversed several difficulties occur. The procedure to be adopted in order to overcome them is described in Arts. [illegible] and [illegible], in respect to work on the triangulation system. As town surveys are not, as a rule, very urgent they should be postponed until the traverse survey can be done in advance of the detailed survey.

168. PARTALLING. The check lines, or 'partals', should be carefully laid out, so as to cut across work diagonally so far as this is practicable; in the more crowded localities the best cross-lines that circumstances permit of must be taken. Work that is out generally by more than 10 links should be treated as of the 2nd class, while that which is out by 20 links and over should be peremptorily rejected.

169. DRAWING OF SCALE ON MAP. When the final maps are ready scales (diagonal) showing both links and feet should be drawn in their margin length-wise and breadth-wise in the usual manner. The reason for drawing the double scale is that local assessments are sometimes made on the basis of measurements in feet and survey measurements are usually by links (Gunter's). The primary division of the links scale is 0·8 inch and of the feet scale 1·21 inches, representing 100 links and 100 feet (*i.e.*, 1 chain each, Gunter's and feet), respectively.

170. NUMBERING. There should be one serial set of survey numbers for each block commencing from the north-west and ending at the south-east. All items whatsoever should receive numbers, but the included waste, houses and the like of house-plots, should receive separate sub-numbers—1, 2, 3, etc., which should be smaller in size than the main survey numbers. In some surveys small italic letters are employed for sub-numbers and answer the purpose equally well.

171. KEY-MAP. A key-map, by reduction to the 8-inch, of the large-scale sheets should be prepared showing the relative situation of the block, traverse lines and stations, principal roads, creeks, and the more important features of the ground. This is useful as a guide for both traverse and cadastral work. It also serves as an index to the theodolite stations, and is useful for the subsequent preparation of the pipe-map. Ward numbers should be entered in blue and sheet numbers in red; the boundaries of wards should be

delineated by ribands of blue and of the sheet by red-ink lines. Simple scales, showing links and feet should be drawn on the map; the primary divisions would equal 1 inch and 1·51 inches, representing 1,000 links and 1,000 feet (= 10 chains each, Gunter's, and feet), respectively. A specimen key-map appears as Map LXXIX in the "Calculating Tables" (5th edition).

172. Lithos. The maps when inked up, numbered and finished should be lithographed. Blocks in the congested area of the residential sections are necessarily smaller than in the less crowded parts. In the case of the smaller blocks, 2, 3 or even 4 adjoining ones may be printed together on a single sheet each with its distinct serial number, to economize in the cost of printing and labour. Blocks in different wards or headmen's charges, should not be put on the same map.

173. Traverse Data. All traverse computations relating to main and minor circuits and block and sub-traverses should be properly stitched together in suitable volumes and indexed.

Fishery Surveys.

174. Fishery Surveys explained. The survey of fisheries is now often undertaken, and a short description of the methods will therefore prove serviceable. Special establishments are generally required for the demarcation and survey of fisheries. Fisheries, it has to be remembered, are large or small streams or lakes, with or without distributaries, in open country or in dense high grass or tree jungle and of both. The work of demarcation and survey may thus be heavy or light. The circumstances of each case would, therefore, have to be carefully considered in the framing of estimates, and no hard-and-fast rule can be laid down of one kind in the matter, for where two fisheries may with difficulty be mapped in a given period four fisheries of another kind may easily be completed in the like time. The extent of the fisheries, the nature of the country to be operated in, the distances to be travelled over, and other like factors would all have to be duly weighed in drawing up schemes for demarcation and survey. The number and general description of the fisheries for survey would thus have to be considered in any proposals in the matter.

175. Demarcation. The demarcation would have to be done by a special officer and all limits duly marked by posts well buried in the ground or blazed trees. Rough demarcation maps should be prepared as a guide to the surveyors. These should give the names of the lessees, sub-lessees, the number and approximate positions of the posts put down, names of the streams and their branches with subdivisions of fisheries, and the number, nature and approximate position of fixed obstructions, and the like. The customary boundaries, and not arbitrary ones, at the time of demarcation should be taken in demarcating the fisheries; these may be the high or low water limits or any natural divisions. Demarcation must invariably be done in advance of survey.

176. Survey. Surveys should be executed in the dry-weather only, all mapping work being done in the rains. The scale of the survey should be four inches to the mile, but where there is much detail to be mapped and the fisheries are intricate and full of branches and subdivisions it may be advisable to survey on the 8-inch scale and reduce subsequently by pantagraph to the 4-inch; or, the map may be kept on the original scale of the survey without reduction. The scale (simple) should be drawn on a spare part of the map, in the customary way; the primary divisions of the 4-inch and 8-inch scales to represent 10 chains (Gunter's) in length are ½ inch and 1 inch, respectively. The boundaries of cultivation, with the names of the cultivators and the divisions of their holdings, within two chains distance from the fishery limits should also be shown. The stations at which the prismatic compass was put up need not be marked in any permanent manner; if desired, wooden pegs may be used to denote them temporarily. Surveys should be made with the prismatic compass and chain (Gunter's), and the topography should be truthfully delineated, the widths of streams shown, and all fixed obstructions, limits of fisheries and their subdivisions properly marked on the map, and the names of the fisheries, with their branches (or feeders), and of the lessees noted down. All angular and linear measurements should be recorded in a field-book and the plotting done on sectional sheets after the day's work is done. The survey should show the boundaries of all streams and pools, the natural banks in the case of fisheries in which these are well defined, and in other cases the boundaries as demarcated. Surveyors should be careful to follow the line of demarcation previously laid down. What they are required to show is the area indicated by these lines and not the area actually under water at the time of survey. All permanent, natural or artificial objects within the boundaries of the fishery or adjoining them should be shown, but no attempt need be made to survey cultivation or to go into excessive detail. A sufficient area outside the boundary of each fishery should be included so as to show clearly its position relative to adjoining fisheries and villages. The office work in the recess would be light, and would consist of first reducing the maps, when required, and then inking them up and finishing them generally.

177. Test-chain. A spare chain, well tested with the standard steel bars, should be in the possession of each Inspector and Surveyor. Frequent comparison should be made between chains in use for survey work and the test-chain and errors promptly rectified.

178. Map Printing. When the maps are completed, with the inking in of boundaries and the various details, printing of the names, drawing of the scales, magnetic north point, border lines, and the like, they should be lithographed. For a specimen of a finished sheet, see Map LXXX of the "Calculating Tables" (5th edition).

Grant Surveys

179. Grant Surveys explained. Frequently special staffs have to be engaged for the survey of lands applied for in grant. In the preparation of proposals for grant surveys, details should be furnished of the number of cases to be dealt with the approximate area for survey and generally as to the situation of the lands applied for and whether the areas are compact blocks or scattered, and whether or not the surveys present special difficulties.

180. Demarcation. The demarcation of lands applied for as grants should be done beforehand by the applicants. Wooden posts should be required to be put up at all angles and bends of the demarcated land.

181. Test-chain. Each Inspector and surveyor should be provided with a spare chain which has been well tested with the standard steel bars. The chain in use for survey work should be frequently compared with the test-chain and any defects should be at once remedied.

182. Survey. The survey should be made by means of the plane-table and chain, on the scale of 16 inches to the mile, and should be plotted direct on the board on the usual sectional sheets used for 16″ cadastral surveys, or, where required, in a fresh copy of the published 16″ village map. The inking up of the plots, and general finishing up is subsequently done. The special survey establishment is usually employed for the open-season only; there is no work for them in the recess. The maps do not require to be printed".

P. 160, Art. 8, line 8, *after* "obtained" *insert* "by the method"

P. 103, Art. 10, line 6, *for* "the Revenue Survey of India" *read* "Revenue Surveys".

P. 175, 1st col., line 12 from bottom, *for* "$(X_c - X)$" *read* "$(X_c - X_B)$"

P. 175, 1st col., line 11 from bottom, *for* "$(X - X)$" *read* "$(X_c - X_B)$"

P. 182, Art. 11, line 1, *for* "Its use" *read* "Use of the Dumpy Level"

P. 181, Art. 10, line 1, *for* "Its description" *read* "Description of the Dumpy Level"

P. 183, Art. 15, line 1, *for* "your" *read* "the".

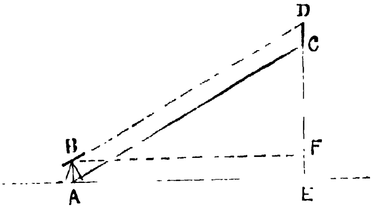
P. 186, Art. 29, line 10, *for* "object to" *read* "have no use for".

P. 190, Art. 42, line 2, *for* "interior" *read* "inferior".

P. 190, Art. 42, line 3, *for* "heavy" *read* "cumbrous".

P. 191, line 14, *for* "—tangent" *read* "sine".

P. 191, *after* line 16, *add* "*Example*: Required the height (CE) of a staff station (C) above the theodolite station (A); given, the sloping distance (AC) = 4,864 feet, height of theodolite (AB) = height of staff (CD) = 5 feet, and vertical angle (CAE = DBF) = + 4° 52′ (elevation).



log AC, 4,864 ... = 3·686 994
L sin CAE, 4° 52′ ... = 8·928 587

Sum = log. CE .. = 2·615 581
∴ CE = 412·65 feet.

P. 193, Art. 7, line 1, *after* "earth" *put* two asterisks (**).

P. 193, Art. 13, line 2, *for* "a place" *read* "a plane".

P. 193, Art. 17, *for* "78," *read* "478."

P. 193, at bottom, *insert* a third footnote: "The figure of the earth, geodetically considered, has reference to the ocean level, disregarding continents, and is approximately an ellipsoid of which the polar diameter is 7,899·6 miles and the equatorial diameter 7,926·6 miles."

P. 194, Art. 22, last line, *after* "referred" *insert* "(see Fig. 479)".

P. 196, Art. 39, line 10, *insert* a comma *after* 'altitude'.

P. 197, Art. 44, line 8, *against* "meridian" *place* two asterisks (**).

P. 197, at bottom, *insert* a fifth footnote: "** The sidereal noon, thus, is the moment at which the vernal equinox crosses the meridian of the place."

P. 199, line 6, from bottom, *after* "465" *add* "*Naked stars* are those stars which are visible to the naked eye to distinguish them from *telescopic stars* which can be seen only by the aid of the telescope. The time between two risings and settings of the same star is 23h. 56m. 4s. In the course of a year while the sun rises 365 times, a star rises 366 times. A sidereal day is the interval between two successive passages of a star over the meridian, and is 3m. 56s. less than the solar day. A sidereal clock thus gains 3m. 56s. daily on a mean time clock, and shows the same time at the same position of the stars all the year round. The constellation of Aries marked the first sign of the zodiac 2,000 years ago, and the equinox then was just below *α* Arietis."

P. 199, footnote, line 6 from bottom, after previous addition, *add* "The 'astronomical unit' is the mean distance between the earth and the sun, or 92,885,000 miles, and is used for all sidereal measurements. For measurements of the sun, moon and planets, the unit is the mean radius of the earth or 3,958·704 miles. The parallax of a star is the angle subtended at the star by the radius of the earth's orbit round the sun, and that of the sun, moon and planets is the angle subtended at the sun, etc., by the semi-diameter of the earth. A celestial object with a parallax of 1″ of arc is at a distance from the earth of 206,265 times the unit of measure; hence, to obtain the distance of a celestial object multiply the fraction 1″ divided by the parallax in seconds into 206,265 into the unit. Thus, required the distance from the earth of (i) a star and (ii) a planet whose parallax is (i) 0·35″ and (ii) 1·46″. Star = $\frac{1}{0{\cdot}35} \times 206{,}265 \times 92{,}885{,}000 = 51{,}037{,}130{,}500{,}000$ miles. Planet = $\frac{1}{1{\cdot}46} \times 206{,}265 \times 3{,}958{\cdot}704 = 559{,}288{,}113$ miles. By the term 'magnitude' is meant the ratio between the light of any star and that of another exactly one magnitude fainter. This value is 2·512 (of which the log. is 0·4). Thus, a star of the 1st magnitude is 2·512 times the brightness of a star of the 2nd magnitude; a star of the 2nd 2·512 times brighter than one of the 3rd, and so on. This makes a star of the 1st magnitude just 100 times brighter than one of the 6th. The 'radian,' or 'angular unit,' is the angle at the centre of a circle subtended by an arc equal in length to the radius."

P. 201, first footnote, line 5 from bottom, *after* "E. or W." *add* "The inner group of the major planets consists of Mercury, Venus, the Earth and Mars, and the outer group of Jupiter, Saturn, Uranus and Neptune; the asteroids, or minor planets, are situated between the groups. A superior planet when in opposition rises at sunset and sets at sunrise; an inferior planet can never be in opposition. When the elongation of an inferior planet is east from the sun it is seen in the western sky after sunset; when west it is seen in the eastern sky before sunrise. Mercury or Venus cannot thus be visible in the east in the evening, or in the west in the morning. The synodic revolution of an inferior planet is its revolution from one inferior conjunction to another. When Mercury is at its greatest elongation $\frac{\text{east}}{\text{west}}$ of the sun, it $\frac{\text{sets}}{\text{rises}}$ about $1\frac{1}{4}$ to $1\frac{1}{2}$ hours $\frac{\text{after}}{\text{before}}$ the sun; it is best placed for observation when at its eastern or western elongations shortly before sunset or after sunrise, respectively. Venus is at its greatest brightness about half-way between inferior conjunction and greatest elongation; it then $\frac{\text{sets}}{\text{rises}}$ about 2 hours $\frac{\text{after}}{\text{before}}$ the sun, if $\frac{\text{east}}{\text{west}}$ of it. The names assigned to Venus by the ancients were Hesperus when an evening star, and Phosphorus when a morning star."

P. 201, first footnote, *add* the following clause: "The existence of a new major planet, situated beyond Neptune, has been announced on the authority of photographic evidence by Professor Pickering. If the results obtained at Harvard are confirmed, Professor Pickering's discovery will be regarded as a memorable achievement for the year 1909."

P. 202, line 8, *after* "moon" *insert* 2 asterisks (**).

P. 202, footnotes, line 2 from bottom, *for* "mutation" *read* "nutation".

P. 202, line 9, from bottom, *after* "Hunter's moon" *add* "For the $\frac{\text{northern}}{\text{southern}}$ hemisphere the harvest moon and the hunter's moon occur when the moon is full nearest to $\frac{\text{September 22–23}}{\text{March 21–22}}$, at the autumnal equinox, when they respectively precede or follow that date. *Halos* are circles of light round the moon or sun, and are produced by the refraction of light from clouds of ice spicules at great elevations. The common belief that halos round the moon prognosticate wet weather is regarded as a fallacy by scientists; it is unlikely that there can be any connection between them and the 'weather', considering that the latter is usually at a comparatively low elevation (perhaps a few miles, as against the moon's distance of some 240,000 miles), and therefore quite independent of the clouds of crystals; it is also certain that fine weather follows halos quite as often as foul. At times, however, the coincidence of rain with the appearance of halos may be remarkable, and would tend to support the theory in question. It has also been asserted that an amount of rain may be subject to a slight variation depending on the age of the moon. But there is no reason to suppose that such changes arise from any other cause than the accidental vicissitudes to which the weather is at all times subject. There is perhaps some authority for the opinion that the rays of the full moon clear away clouds, but even this is improbable, since the reflected heat from the moon is infinitesimal."

P. 202, footnote, at bottom, *add* "Darwin's theory of tidal evolution, as relating to the moon, was that the moon was originally part of a semi-solid, or viscous earth: by the tidal pull of the sun it became separated from the earth, and in ages long past revolved close to its surface. Tidal action between these two bodies caused the moon to move gradually further away. According to the theory, the earth-moon was a semi-liquid, viscous body, becoming extremely flattened at the poles through its rapid rotation, assuming successively perhaps various elongated oval forms till the circular or globe-like shape was reached. Darwin has shown that this course of evolution is possible according to the laws of mathematical physics, and that it is still probably going on—the ultimate stage being that the earth and moon will keep the same forces to each other, and the month and day be the same length. But the notion that we can point out the spot on the earth where the moon was torn out is pure speculation. The evolution of the moon (if Darwin's theory is correct) took place when the earth was in a totally different condition from what it is at present. It must have been a semi-fluid mass, and probably there was no distinction between land and water; so it would be idle to speculate as to which ocean, whether the south pacific, south atlantic or the arabian sea, was the moon's place of origin."

P. 203, line 2, from bottom, *after* "partial" *place* two asterisks (**).

P. 205, at bottom, *add* a fourth footnote: "In *central* eclipses of the sun, if the apparent size of the moon exceeds that of the sun, the eclipse is said to be *total*; but if the sun is the larger, the latter is called *annular*."

P. 206, line 12, *for* semicolon *after* "A.M." *put* comma.

P. 219, Art. 69, line 37, *for* "90°" *read* "89°".

P. 220, line 4, *for* "13h. 55m. 35s." *read* "13h. 54m. 35s.".

P. 220, line 5, *for* "1h. 55m. 35s." *read* "1h. 54m. 35s.".

P. 220, line 22, *for* "1h. 55m. 35s." *read* "1h. 54m. 35s.".

P. 220, first footnote, line 2, *for* "-" *read* "=".

P. 225, line 12—13, 1st col., *for* "I." *read* "$\overset{I}{R}$".

P. 225, line 16—17, 1st col., *for* "R" *read* "$\overset{R}{I}$".

P. 225, line 19, *after* "Polaris" *insert* "W".

P. 225, lower form, col. 11, draw head "^" on top of meridian line.

P. 230, line 13 from bottom, col. 6, *for* "—0·0146" *read* "—1·0146".

P. 234, last line, *for* "educt" *read* "deduct".

P. 241, first footnote, add the following clause: "The latitude of a place, within a belt of 10° to 30° north of the equator, may also be found, with fair accuracy, by waiting at night till the two stars, *Sirius* (α Canis, Majoris: R A. = 6h. 41m., Dec. = 16° 35′ S.) and *Canopus* (α Argûs: R A. = 6h. 22m., Dec. = 52° 39′ S.), are in the same vertical plane. At that instant *Polaris* (α Ursæ Minoris: R A. = 1h. 26m., Dec. = 88° 49′ N.) is at its greatest elongation, W. and its altitude, *i.e.*, the angle of elevation, referred to the horizon, observed then with a transit theodolite will give the approximate latitude of the place. The phenomenon occurs at about 7 p.m. on the 29th March, at about 8 p.m. on the 14th March, and at about 9 p.m. on the 27th February, approximately. The instrument would have to be in adjustment for the purpose of the observation, and the two stars would require to be truly on the central vertical wire of the diaphragm of the telescope when moved in a great circle vertically through the southern section of the arc. A similar result may be obtained, for a region of 10° to 60° north of the equator, when the two stars, *Polaris* (α Ursæ Minoris: R.A. = 1h. 26m., Dec. = 88° 49′ N.) and *Alioth* (ε Ursæ Majoris: R.A. = 12h. 50m., Dec. = 56° 27′ N.) are in the same horizontal plane, *i.e.*, when they have a like altitude as referred to the horizon, at the time that both are at their respective elongations, E. or W., on opposite sides of the celestial pole. This can be ascertained by swinging the telescope of the theodolite, when in adjustment, from side to side in azimuth till both the stars are exactly on the horizontal wire of the diaphragm of the telescope. The phenomena take place, approximately, at 7 p.m. about the 29th March, at 8 p.m. about the 14th March, and at 9 p.m. about the 27th February, with *Polaris* and *Alioth* at their respective western and eastern elongations, and at 7 p.m. about the 28th September, at 8 p.m. about the 13th September, and at 9 p.m. about the 28th August, with *Polaris* and *Alioth* at their respective eastern and western elongations. Both the processes referred to for finding the approximate latitude are somewhat tedious, and the operations may occupy a considerable part of an evening."

P. 243, line 12, *for* "0·8-657" *read* "0·8657".

P. 249, second footnote, line 5, *after* "enlarged" *insert* "The Royal Observatory of Greenwich was founded in A.D. 1675."

P. 259, line 5, from bottom, *after* "limb" *insert* "and [1]".

P. 263, line 4, *for* "por-" *read* "posi-".

P. 263, footnote, last line, *add* "The initiation of the Zone Time System, which was adopted for use for the first time on the Railways of America in 1883, was due to Mr. W. F. Allen".

P. 265, *after* line 52 *insert* in col. 1 "Maymyo, Phalan H. S.".

P. 265, *after* line 52 *insert* in col. 2 "Mandalay (B)".

P. 265, *after* line 52 *insert* in col. 3 "22° 01′ 06″ N.".

P. 265, *after* line 52 *insert* in col. 4 "96° 29′ 24″ E.".

P. 265, *after* line 52 *insert* in col. 5 "6h. 25m. 58s. E."

P. 265, *after* line 52 *insert* in col. 6 "3,780".

P. 265, footnote, line 8, *for* "2° 35″ 36″ 25" *read* "2° 35′ 36″·25".

P. 265, footnote, line 18, *for* "80° 14′ 54·30″" *read* "80° 14′ 51·30″".

(ii) Illustrations.

P. ii, Reference Table, column 3, line 40, *for* "277" *read* "377"

P. ii, Reference Table, col. 6, line 43, *for* "X, 24" *read* "X, 23, 24"

P. ii, Reference Table, col. 8, line 40, *after* "XI, 11" *add* "XII, 157".

P. ii, Reference Table, col. 6, line 54, *delete* "68".

P. ii, Reference Table, col. 6, line 58, *after* "VIII" *insert* "68".

P. 32, Fig. 480, *below* "*" *insert* "s".

P. 36, Fig. 508, *for* "16 = 1 Mile" *read* "16″ = 1 Mile".

P. 37, Fig. 516, *for* "Autumnat" *read* "Autumnal".

(iii) Contents.

P. xiii, Chap. XIII, line 8, *after* "(16)" *insert* "Definition of terms used in observing (16*a*)"

P. xiii, Chap. XIII, line 11, *delete* "using"

P. xiii, Chap. XIII, line 12, *for* "Lifting the instrument" ***read*** "Placing the theodolite"

P. xiii, Chap. XIII, line 19, *before* "Levelling" *insert* "Horizontal".

P. xiv, line 5, *for* "manner of observing" *read* "Traversing".

P. xiv, line 17, *after* "(114)" *insert* "Apportionment of work among traverse-surveyors (114*a*). Jungle blocks (114*b*)"

P. xiv, line 18, *after* "(123)" *insert* "Measurements over bad ground (123*a*)"

P. xiv, line 21, *after* "(131)" *insert* "Receipts for survey marks (131*a*)"

P. xiv, line 23, *after* "(134)" *insert* "The triangulation system (134*a*)"

P. xiv, line 20, *after* "(152)" *insert* "Village boundaries (152*a*)".

P. xiv, Chap. XV, line 5, *for* "Its use" *read* "Use of the Dumpy Level".

P. xiv, Chap. XV, line 5, *for* "Its description" *read* "Description of the Dumpy Level".

P. xiv, line 30 *after* "(155)" *add* "Inking and shading (156). Mapping (157). *Town Surveys*: Town surveys explained (158). System of survey (159). Demarcation (160). Staff (161). Mapping, computation and area work (162). Chaining (163). Permanent survey marks (164). Protection of permanent survey marks (165). Survey of details (166). Traverse to precede detail survey (167). Partalling (168). Drawing of scale on map (169). Numbering (170). Key-map (171). Lithos (172). Traverse data (173). *Fishery Surveys*: Fishery surveys explained (174). Demarcation (175). Survey (176). Test-chain (177). Map printing (178). *Grant Surveys*: Grant surveys explained (179). Demarcation (180). Test-chain (181). Survey (182).

(iv) Index.

P. i, *after* line 5 from bottom, *insert* "Area-work. Mapping, computation and (Town) XIII, 162 ... 160".

P. ii, *after* line 33, *insert* "Boundaries. Village (Cadl.) XIII, 152*a* ... 159"

P. ii, line 33, *for* "(Cadl.)" *read* "(Trav.)".

P. ii, *after* line 10 from bottom, *insert* "Chaining (Town) XIII, 163 ... 160"

P. iii, *after* line 5 from bottom, *insert* "Computation and area work. Mapping (Town) XIII, 162 ... 160"

P. iv, *after* line 20, *insert* "Data. Traverse (Town) XIII, 173 ... 160"

P. iv, *after* line 30, *insert* "Demarcation (Fish.) XIII, 175 ... 160"

P. iv, *after* line 30, *insert* "Demarcation (Grant) XIII, 180 ... 160"

P. iv, *after* line 30, *insert* "Demarcation (Town) XIII, 160 ... 160"

P. iv, *after* line 39, *insert* "Details. Survey of (Town) XIII, 166 ... 160"

P. iv, *after* line 39, *insert* "Detail survey. Traverse to precede (Town) XIII, 167 ... 160".

P. iv, *after* line 39, *insert* "Drawing of scale on map (Town) XIII, 169 ... 160"

P. vi, *after* line 4, *insert* "Fishery surveys explained (Fish.) XIII, 174 ... 160".

P. vi, *after* line 4, *insert* "Fishery Surveys (Theod.) XIII 174–178 ... 160"

P. vi, *after* line 27, *insert* "Grant surveys explained (Grant) XIII, 179 ... 160"

P. vi, *after* line 27, *insert* "Grant Survey (Theod.) XIII, 179–182 ... 160"

P. vi, *after* line 5 from bottom, *insert* "Inking and Shading (Cadl.) XIII, 156 ... 160".

P. vii, *after* line 14, *insert* "Jungle blocks. (Trav.) XIII, 114*b* ... 150".

P. vii, *after* line 15, *insert* "Key-map (Town) XIII, 171 ... 160"

P. vii, line 55, *before* "(Theod.)" *insert* "Horizontal".

P. viii, *after* line 4, *insert* "Lithos (Town) XIII, 172 ... 160".

P. viii, *after* line 45, *insert* "Map. Drawing of Scale on (Town) XIII, 169 ... 160"

P. viii, *after* line 45, *insert* "Map printing (Fish.) XIII, 178 ... 160"

P. viii, *after* line 45, *insert* "Mapping (Cadl.) XIII, 157 ... 160"

P. viii, *after* line 45, *insert* "Mapping, computation and area work (Town) XIII, 162 ... 160"

P. viii, *after* line 48, *insert* "Marks. Permanent survey (Town) XIII, 164 ... 160"

P. viii, *after* line 48, *insert* "Marks. Protection of permanent survey (Town) XIII, 165 ... 160"

P. viii, *after* line 48, *insert* "Marks. Receipts for survey (Trav.) XIII, 131*a* ... 150"

P. viii, *after* line 63, *insert* "Measurements over bad ground (Trav.) XIII, 123*a*, ... 154"

P. ix, *after* line 20, *insert* "Numbering (Town) XIII, 170 ... 160"

P. ix, *after* line 31, *insert* "Observing. Definition of terms used in (Theod.) XIII 16*a* ... 117".

P. ix, *after* line 11 from bottom, *insert* "Partalling (Town) XIII, 168 ... 160".

P. x, *after* line 9, *insert* "Permanent survey marks Protection of (Theod.) XIII, 165 ... 160"

P. x, *after* line 9, *insert* "Permanent survey marks (Town) XIII, 164 ... 160"

P. xi, *after* line 12, *insert* "Printing. Map (Fish.) XIII, 178 ... 160"

P. xi, *after* line 32, *insert* "Protection of permanent survey marks (Town) XIII, 165 ... 160"

P. xii, *after* line 10, *insert* "Scale on map. Drawing of (Town) XIII, 169 ... 160"

P. xii, *after* line 31, *insert* "Shading. Inking and (Cadl.) XIII, 156 ... 160"

P. xii, *after* line 48, *insert* "Surveys explained (Grant), XIII, 179 ... 160".

P. xii, *after* line 48, *insert* "Survey (Grant) XIII, 182 ... 160"

P. xii, *after*, line 48, *insert* "Survey marks. Protection of (Town) XIII, 165 ... 160"

P. xii, *after* line 48, *insert* "Survey of details (Town) XIII, 166 ... 160"

P. xii, *after* line 48, *insert* "Survey. Traverse to precede (Town) XIII, 167 ... 160".

P. xii, *after* line 6 from bottom, *insert* "Surveys Fishery (Theod.) XIII, 174—178 ... 160".

P. xii, *after* line 6 from bottom, *insert* "Surveys. Grant (Theod.) XIII, 179-182 ... 160"

P. xii, *after* line 6 from bottom, *insert* "Surveys. Town (Theod.) XIII, 158—173 ... 160"

P. xii, *after* line 6 from bottom, *insert* "Surveys explained. Town (Town) XIII, 158—160"

P. xii, *after* line 6 from bottom, *insert* "Survey marks. Permanent (Town) XIII, 164 ... 160

P. xii, *after* line 6 from bottom, *insert* "Survey System of (Town) XIII, 159 ... 160"

P. xii, *after* line 5 from bottom, *insert* 'System of survey (Town) XIII, 159 ... 160".

P. xiii, *after* line 12, *insert* "Staff (Town) XIII, 161 ... 160"

P. xiii, *after* line 48, *insert* "Survey (Fish.) XIII, 176 ... 160"

P. xiii, *after* line 48, *insert* "Surveys explained. Fishery (Fish.) XIII, 174 ... 160"

P. xiii, *after* line 6 from bottom, *insert* "Surveys. Fishery XIII, 174—178 ... 160. Surveys. Grant XIII, 179—182 ... 160. Surveys. Town XIII, 158—173 ... 160".

P. xiv, *after* line 14, *insert* "Test-chain (Fish.) XIII, 177 ... 160"

P. xiv, *after* line 14, *insert* "Test-chain (Grant) XIII, 181 ... 160".

P. xiv, line 22, *for* "manner of observing" *read* "Traversing".

P. xiv, line 24, *for* "Lifting" *read* "Placing"

P. xiv, *after* line 47, *insert* "Town Surveys (Theod.) XIII, 158—173 ... 160"

P. xiv, *after* line 47, *insert* "Town surveys explained (Town) XIII, 158 ... 160"

P. xiv, *after* line 62, *insert* "Traverse surveyors. Apportionment of work among (Trav.) XIII, 114*a* ... 150".

P. xiv, *after* line 31 from bottom, *insert* "Traverse data (Town) XIII, 173 ... 160"

P. xiv, *after* line 23 from bottom, *insert* "Traverse to precede detail survey (Town) XIII, 167 ... 160"

P. xiv, *after* line 4 from bottom, *insert* "Triangulation system. The (Cadl.) XIII, 134*a* ... 150".

P. xiv, line 4 from bottom, *for* "(Cadl.)" *read* "(Trav.)"

P. xv, line 19 from bottom, *for* "(Cadl.)" *read* "(Trav.)".

P. xv, *after* line 19 from bottom, *insert* "Village boundaries (Cadl.) XIII, 152*a* ... 159"

ERRATA ET CORRIGENDA

(i) Text.

P. 3, Art. 40, line 4, *for* "co-secant" *read* "its co-secant".

P. 5, Art. 72, line 1, *for* "It in" *read* "In".

P. 8, Art. 107, line 1, *for* "side" *read* "sides".

P. 9, Art 119, line 6, *for* "contain" *read* "contains".

P. 20, Art. 8, line 1, *for* "B" *read* "C".

P. 54, Art. 47, line 34, *for* "clains" *read* "chains".

P. 69, Art. 57, line 3, *for* "s e" *read* "see".

P. 80, Art. 7, line 18, *after* "length" *insert* "(assumed)".

P. 84, Art. 19, line 1, *for* "522" *read* "122".

P. 87, Art 36, line 5, *for* "Figs." *read* "Fig.".

P. 104, 3rd footnote, lines 1 and 2, *for* "direct-tion" *read* "direc-tion".

P. 117, Art. 19, line 3, *for* "29" *read* "19".

P. 129, 2nd footnote, line 3, *for* "be" *read* "b".

P. 131, Art. 74, line 13, *after* "1·154272" *insert* ";".

P. 133, para. 76, line 4, column 8, *for* "Values" *read* "Value".

P. 139, Art. 82, line 11, *for* "Article" *read* "Articles".

P. 141, Art. 84, line 11, *for* "18 34 42 38" *read* "18$^{h.}$ 34$^{m.}$ 42·38$^{s.}$".

P. 141, footnote, line 22, *for* "Conut" *read* "Count".

P. 146, Art. 94, line 1, *for* "448" *read* "488".

P. 156, Art. 134, line 1, *for* "Filed-surveyors" *read* "Field-surveyors"

P. 156, 2nd footnote, line 5, *for* "15′ 1° or 4° lat." *read* "15′, 1° or 4° latitude".

P. 157, Art. 137, line 6, *for* "do" *read* "dot".

P. 163, footnote, line 11, *for* "eastings" *read* "east-ings".

P. 163, footnote, line 15, *for* "Adopted" *read* "Adapted".

P. 167, line 19, *for* "Art. 13" *read* "Art. 14."

P. 169, Art. 17, line 9, *for* "Ab^1" *read* "Ab′".

P. 177, Art. 22, line 40, left column, *for* "1·938792" *read* "$\bar{1}$·938792".

P. 177, Art. 22, line 41, left column, *for* "1·511776" *read* "$\bar{1}$·511776".

P. 177, Art 22, line 43, left column, *for* "3′ 21·2″" *read* "3′ 23·2″".

P. 177, Art. 22, line 43, right column, *for* 3′ 21·2″" *read* "3′ 23·2″".

P. 196, para. 38, line 4, *for* "E" *read* "O′".

P. 196, para. 39, line 2, *for* "from" *read* "and".

P. 199, 4th footnote, line 14, *for* "Vega" *read* "Sulaphat".

P. 206, line 13, *for* semicolon after "P.M." *put* comma

P. 206, line 41, *for* "9·47" *read* "9-47".

P. 206, fig. 1, *for* "E" *read* "E$_1$".

P. 207, line 11, *for* "91° 32′ 13″" *read* "91° 31′ 13″".

P. 208, line 23, *for* the 1st "owing" *read* "due".

P. 212, line 32, *for* "Sideral" *read* "Sidereal".

P. 232, line 17, *for* "8,928 553" *read* "8·928 553".

P. 239, Example 3, line 23, *for* "v" *put* "γ".

P. 240, 2nd footnote, line 1, *for* "meredian" *read* "meridian".

P. 248, line 40, *for* "cot. *i z*" *read* "cot. *z*".

P. 250, 2nd footnote, line 1, *after* "XVIII" close the bracket

P. 259, footnote, *for* "Adopted" *read* "Adapted".

P. 260, 3rd footnote, line 3, *for* "os (sigma)" *read* "σ (sigma)".

P. 260, 3rd footnote, line 3, *insert* a comma *after* "(Phi)".

P. 260, 4th footnote, line 7, *insert* "the" *before* "heavens".

P. 261, right col. line 35, *for* "Pisces" *read* "Piscis".

P. 261, right col., line 47, *for* "Sculptoris" *read* "Scalptoris".

P. 261, left col, line 42, *for* "Pisces" *read* "Piscis".

P. 264, line 43, column 5, *for* "5 06 41E." *read* "5 53 20E".

P. 265, 1st footnote line 17, *for* "1,910" *read* "1910".

P. 265, 2nd footnote, line 1, *for* "deducted" *read* "deduced".

P. 266, col. 3, line 16, *for* "24° 00′ 37″" *read* "14° 00′ 37″".

P. 266, footnote, line 2, *for* "05" *read* "5".

P. 267, line 40, *for* "at the" *read* "at that".

P. 267, line 7, *remove* comma *after* "assumed".

P. 270, line 4, *for* "date the" *read* "the date".

P. 270, line 11, *for* "or which" *read* "on which".

P. 270, line 51, *for* "mumbers" *read* "numbers".

P. 272, line 6, *insert* "the" *before* "Right".

P. 273, line 9, *for* "q′" *read* "q,".

P. 273, line 39, *for* "of the" *read* "of an".

(ii) Illustrations.

P. 5, Fig. 172, lower figures of subdivision, *for* "10" *read* "1"; *for* "5" *read* "·5".

P. 33, Fig. 489, R.A. 15$^{h.}$ 30$^{m.}$ Dec. 5° N., *for* "SERPENS" *read* "SERPEN".

P. 33, Fig. 489, right-hand top corner, *insert* "489", the number of the figure.

P. 33, Fig. 489, from R.A. 15$^{h.}$ 50^{m}, Dec 24° N. to R.A. 15^{h} 15$^{m.}$, Dec. 24° N, *draw* a dotted divisional line between the constellations "CORONA BOREALIS" and "SERPENTARIUS".

P. 33, Fig. 489, from R.A. 14^{h} 55$^{m.}$, Dec. 3° N. to R.A. 14^{h} 55$^{m.}$, Dec. 3° S, *draw* a dotted divisional line between the constellations "SERPENTARIUS" and "VIRGO".

P. 33, Fig. 488, R.A. 4$^{h.}$ 30^{m}, Dec 38° S., *for* "SCULPTORIS" *read* "SCALPTORIS".

P. 36, Fig. 500, *for* "dem" *read* "dem".

☞ It is particularly requested that notice of any errors discovered in the large edition (3rd) of the "Aid to Land Surveying" may be promptly sent to the Author.

AID TO LAND-SURVEYING.

CHAPTER I.

Practical Geometry.

FIRST PRINCIPLES.

1. GEOMETRY.—*Geometry* is the science that treats of the properties of magnitude in general. It is distinguished into *theoretical* and *practical*. The subjects which it considers are extent of distance, extent of surface, and extent of capacity or solid content. It is the foundation of Mensuration, Surveying, and other practical branches of Mathematics.

DEFINITIONS.

2. SUPERFICIES (Figs. 1—2).—A *Superficies*, or *surface*, has length and breadth, but no thickness. It may be flat (or plane) (*see* Fig. 1) or curved (*see* Fig. 2).

3. LINE (Figs. 3—4).—A *line* has length only, without breadth or thickness, and is the boundary of a superficies. It may be straight (right) (*see* Fig. 3) or curved (*see* Fig. 4).

4. POINT (Figs. 5—6).—A *point* is one of the extremities of a line; it marks position, but has neither parts nor dimensions.

5. PLANE (Fig. 7).—A *plane* is a perfectly flat superficies or even surface, and is such that if any two points in it be joined by a straight line, the whole of the straight line will lie in the superficies.

6. PLANE FIGURES (Fig. 8) Planes bounded by either straight or curved lines, called their *perimeter*, are termed *plane figures*.

7. PARALLEL LINES (Fig. 9).—*Parallel lines* are lines in the same plane, everywhere at the same distance from each other, and which, though infinitely produced, would never meet.

8. HORIZONTAL LINES (Fig. 10).—*Horizontal lines* are such as are parallel to the horizon* or level.

9. VERTICAL LINES (Fig. 11).—*Vertical lines* are such as are parallel to the position at rest assumed by a plumb line freely suspended in a still atmosphere. They are perpendicular to horizontal lines.

10. INCLINED LINES (Figs. 12—13).—*Inclined lines* are such as are neither horizontal nor vertical, but occupy intermediate positions (*see* Fig. 12). Two or more lines which converge or run together, and which would meet at a point if produced far enough, are said to be inclined to one another (*see* Fig. 13).

11. ANGLE (Figs. 14—16).—An *angle* is the inclination or divergence of two lines, which meet or cut each other at a point (*see* Figs. 14—15). One angle is said to be greater or less than another, according as the inclination or opening of the lines forming the one angle is greater or less than that of the lines forming the other (*see* Fig. 16).

12. PERPENDICULAR (Fig. 17).—One right line is said to be *perpendicular* to another, when it meets or cuts it and makes the angles on both sides equal. Thus, CD is perpendicular to AB.

13. RIGHT ANGLE (Fig. 18).—A *right angle* is formed by two right lines that are perpendicular to each other, as CDB.

OBSERVATION.

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* **For definition of the term *horizon* (*see* Chap. XVI, Art. 13.)**

14. OBTUSE ANGLE (Fig. 19).—An *obtuse angle* is greater than a right angle, as at A.

15. ACUTE ANGLE (Fig. 20).—An *acute angle* is less than a right angle, as at B.

16. OBLIQUE ANGLE (Figs. 19—20).—An *oblique angle* is any angle other than a right angle, and is either obtuse (*see* Fig. 19) or acute (*see* Fig. 20), as A or B.

17. TRIANGLE (Figs. 21—26).—A *triangle* is a plane figure of three sides and three angles: it is called *equilateral*, when the sides are all equal (*see* Fig. 21); *isosceles*, when two sides only are equal (*see* Fig. 22); *scalene*, when the sides are all unequal (*see* Fig. 23); *right angled*, when one angle is a right angle (*see* Fig. 24); *obtuse-angled*, when one angle is obtuse (*see* Fig. 25); *acute-angled*, when all the angles are acute (*see* Fig. 26); and *oblique-angled*, when it is either obtuse-angled or acute-angled (*see* Figs. 25—26).

18. HYPOTENUSE, BASE, AND PERPENDICULAR (Fig. 27).—In a right-angled triangle, the side which is opposite to the right angle is called the *hypotenuse*, as *c*; and of the other two sides, one is termed the *base*, as *a*; and the other the *perpendicular*, as *b*.

19. QUADRILATERAL (Figs. 28—33).—A *quadrilateral* is a plane figure of four sides and four angles (*see* Figs. 28—33); when the opposite sides are parallel it is called a *parallelogram* (*see* Figs. 28—29, 32—33); if its angles be right angles and its length exceeds its breadth, it is a *rectangle* (*see* Fig. 28); if its angles be right angles and all its sides be equal, it is a *square* (*see* Fig. 29); if only two of its sides be parallel, it is a *trapezoid* (*see* Fig. 30); if none of its sides be parallel, it is a *trapezium* (*see* Fig. 31); if the sides be equal and the opposite angles equal but not right angles, it is a *rhombus* (*see* Fig. 32); if only the opposite sides and angles, which are not right angles, be equal, it is a *rhomboid* (*see* Fig. 33).

20. DIAGONAL (Fig. 34).—A *diagonal* is a straight line joining two opposite angles of a quadrilateral, as AB.

21. POLYGONS (Figs. 35—36).—*Polygons* are plane figures of more than four sides, termed *regular* when the sides and angles are all equal. All regular polygons may be inscribed in (*see* Fig. 35) or circumscribed by (*see* Fig. 36) circles.

22. NAMES OF POLYGONS (Figs. 37—40).—A *pentagon* is a polygon of five sides (*see* Fig. 37); a *hexagon*, one of six (*see* Fig. 38); a *heptagon*, one of seven (*see* Fig. 39); an *octagon*, one of eight (*see* Fig. 40), etc.

23. CIRCLE (Fig. 41).—A *circle*, as ACF, is a plane figure bounded by a continuous curved line, called the *circumference* or *periphery*, which is everywhere equally distant from a point within called the *centre*, as D.

24. DIAMETER (Fig. 42).—The *diameter* of a circle is a straight line passing through its centre, and terminated both ways by the circumference, as BE. The diameters of a circle are all equal.

25. RADIUS (Fig. 43).—The *radius* of a circle is a straight line drawn from the centre to the circumference, and is equal to half the diameter, as DF. The radii of a circle are all equal.

26. ARC (Fig. 44).—An *arc* of a circle is any part of the circumference, as AB.

27. CHORD (Fig. 45).—A *chord* of a circle is a straight line joining the extremities of an arc, as AC.

28. SEGMENT (Fig. 46).—A *segment* of a circle is any part contained between an arc and its chord, as ABCA.

29. SECTOR (Fig. 47).—A *sector* is any part of a circle contained between two radii and their connecting arc, as ABCDA.

30. SEMICIRCLE (Fig. 48).—A *semicircle* is half a circle, or the segment cut off by any diameter, as EAB.

31. QUADRANT (Fig. 49).—A *quadrant* is the quarter of a circle, or a sector having the two radii forming its sides perpendicular to each other, as DFED.

32. Tangent (Fig. 50).—A *tangent* is a right line touching the circumference of a circle in one point only and cutting off no part of it, if it be produced, as HBJ.

33. Concentric Circles (Fig. 51).—*Concentric circles* are those that have one common centre.

34. Division of Circumference of Circle.—The circumference of every circle is supposed to be divided off into 360 equal parts, called *degrees*; each degree is subdivided into 60 equal parts, called *minutes*; and each minute into 60 equal parts, called *seconds*.

35. Division of Quadrant of Circle.—A *quadrant* of a circle will therefore contain 90 degrees, and a semicircle 180 degrees.

36. Symbols of Degrees.—Degrees are expressed thus °; minutes, thus ′; seconds, thus ″.

37. Measure of an Angle (Fig. 52).—The *measure* of an angle is an arc of a circle, whose centre is the angular point, and is said to be of so many *degrees*, *minutes*, and *seconds*, as are contained in its measuring arc, as CF.

38. Complement (Fig. 53).—The *complement* of an arc or angle, is what it exceeds or wants, of a right angle, or 90 degrees. Thus, of the arc BC, the complement is CF; and of the angle BDC, the complement is CDF.

39. Supplement (Fig. 54).—The *supplement* of an arc or angle, is what it wants of two right angles, or 180 degrees. Thus, of the arc BC, the supplement is CE, and of the angle BDC, the supplement is CDE.

40. Several Parts of a Quadrant named (Fig. 55).—Let DHB be a *quadrant* of a circle described by the radius CB. Then HB is an *arc* of it, and DH its *complement*; HL or FC the *sine*; FH or CL its *co-sine*; BK its *tangent*; DI its *co-tangent*; CK its *secant*; CI *co-secant*; and LB its *versed sine*.

41. Problem and Theorem.—A *problem* is a proposition in which some operation or construction is required; a *theorem* is a proposition to be proved by a chain of reasoning.

42. Geometrical Construction.—*Geometrical construction* signifies the series of operations performed with mathematical instruments, in determining the points and lines involved in the solution of a geometrical problem.

43. Lines of Construction.—*Lines of construction* are the lines used in solving problems.

44. Given Point or Line.—A *given point* or a *given line*, means a point or line of fixed position or length.

45. The term "Describe an Arc."—*Describe an arc*, signifies that an arc of a circle is to be drawn by the dividers, in pencil or ink.

46. Setting off a Distance.—To *set off* or *lay off a distance* or *length*, is to mark on the drawing a given distance by the dividers, the compasses, or the pricker; and, with the compasses, it is done most conveniently by describing a short arc.

47. The term "Lines Cut."—Lines *cut* each other when they intersect or cross.

48. Setting off an Angle.—To *set off* or *lay out an angle*, is to draw a line making a given angle with another.

49. Joining two Points.—To *join any two points*, is to draw a straight line between them.

50. Producing a Line.—To *produce a line*, is to lengthen a given line as far as may be necessary.

SIGNS.

51. Equality.—The sign = denotes the quantities between which it stands are equal.

52. Addition.—The sign + denotes the quantity it precedes is to be added.

53. Subtraction.—The sign − denotes the quantity which it precedes is to be subtracted.

54. Multiplication.—The sign × denotes the quantities between which it is are to be multiplied into each other.

55. DIVISION.—The sign ÷ stands for division.
56. GREATER THAN.—The sign > stands for greater than.
57. LESS THAN.—The sign < stands for less than.
58. DIFFERENCE BETWEEN. The sign ∽ stands for difference between.
59. PROPORTION.—To denote that four quantities, A, B, C, D, are proportional, they are usually written thus, A : B : : C : D, and read thus, A is to B as is C to D; but when three quantities, A, B, C, are proportional, the middl quantity is repeated, and they are written A : B : : B : C.

RULES FOR GEOMETRICAL DRAWING.

60. RULE I.—Never draw a single line that is not absolutely necessary: therefore do not commence operations hastily, without well considering the proceeding, and while drawing with a pencil, use one marked F or H, cut to a fine point, and press very lightly upon it; the pencil lines need be only just visible, and by attending to this the paper is kept cleaner, and the constructions are made more accurate.

61. RULE II.—Arcs of circles, or right lines by which an important point is to be found, should never intersect each other very obliquely, or at an angle of less than 15 or 20 degrees; and if this cannot be avoided, some other proceeding should be had recourse to, to define the point more precisely.

62. RULE III.—When one arc of a circle is described, and a point in it is to be determined by the intersection of another arc, this latter need not be drawn at all, but only the point marked off on the first, as it is always desirable to avoid the drawing of unnecessary lines. The same observation applies to a point to be determined on one straight line by the intersection of another.

63. RULE IV.—A right line should never be obtained by the prolongation of a very short one, unless some point in that prolongation is first found by some other means, especially in any essential part of a problem.

64. RULE V.—The larger the scale on which any problem, or part of one, is constructed, the less liable is the result to error: hence all angles should be set off, and points determined, by means of the largest circles which circumstances will admit of being described.

65. RULE VI.—All lines perpendicular or parallel to another, should be drawn long enough at once, to obviate the necessity of producing them.

66. RULE VII.—Whenever it is practicable, lines should be drawn *from* a given point and *not* to it; and if the line is to pass through two points, before drawing it the pencil should be moved along the rule, so as to ascertain whether the line will, when drawn, pass through them both.

67. RULE VIII.—Avoid setting off equal lengths on a given straight line by continual repetition of one such length, but mark off, on the line, a convenient multiple of the given length and subdivide it, *i.e.*, work from the whole to parts, not from parts to the whole; this is a great principle in surveying as well as plan drawing, and is especially to be observed in the construction of scales.

GEOMETRICAL THEOREMS.

68. THEOREM I (Fig. 56). If a right line falls on another, as AB or EB does on CD, it either makes with it two right angles, or two angles equal to two right angles, that is CBA + ABD = CBE + EBD = two right angles.

69. THEOREM II (Fig. 57). If one right line crosses another, as AC does BD, the opposite angles, made by those lines, will be equal to each other, that is, AEB to CED, and BEC to AED.

70. THEOREM III (Fig. 58).—If a right line crosses two parallel lines, as GH does AB and CD, then,

1st.—The sum of the interior angles on the same side will be equal to two right angles, that is, AEF + CFE = two right angles, and BEF + DFE = two right angles.

2nd.—The alternate angles will be equal, that is, AEF = EFD, and BEF = CFE.

3rd.—The exterior angle will be equal to the interior and opposite one on the same side, that is, AEG = CFE, and BEG = DFE.

71. THEOREM IV (Fig. 59).—If in any triangle ABC, one of its sides, as BC, is produced towards D, it will make the external angle ACD equal to the two internal opposite angles taken together; *viz.*, to B and A.

72. THEOREM V (Fig. 60).—If in any triangle ABC, all the three angles taken together are equal to two right angles, *viz.*, A + B + C = two right angles.

73. THEOREM VI (Fig. 61). If in any two triangles, ABC, DEF, there be two sides, AB, AC, in the one, severally equal to two sides, DE, DF, in the other, and the angle A contained between the two sides in the one, equal to the angle D in the other; then the remaining angles of the one will be severally equal to those of the other, *viz.*, B = E and C = F; and the base of the one, BC, will be equal to EF, that of the other.

74. THEOREM VII (Fig. 62). The angle BCD, at the centre of a circle ABDE, is double the angle BAD at the circumference on the same base.

75. THEOREM VIII (Fig. 63). If from the centre C of a circle ABE, there be let fall the perpendicular CD on the chord AB, it will bisect it in the point D.

76. THEOREM IX (Fig. 64). If from the centre of a circle ABE there be drawn a perpendicular CD on the chord AB, and produced till it meets the circle in F, that line, CF, will bisect the arc AB in the point F.

77. THEOREM X (Fig. 65). –If two equal and parallel lines AB, CD, be joined by two other lines AD, BC, those lines will be also equal and parallel.

78. THEOREM XI (Fig. 66).—All parallelograms on the same or equal bases and between the same parallels are equal to one another, that is, if BD = GH, and the lines BH and AF are parallel, then the parallelogram ABDC = BDFE = EFGH.

79. THEOREM XII (Fig. 67). –In every right-angled triangle ABC, the square of the hypotenuse or longest side, BC, or BCMH, is equal to the sum of the squares made on the other two sides, AB and AC, that is, to ABDE and ACGF.

80. THEOREM XIII (Fig. 68). In all circles the chord of 60 degrees is always equal in length to the radius. Thus in the circle AEBD, if the arc AEB be an arc of 60 degrees, and the chord AB be drawn; then AB = AC = CB.

81. THEOREM XIV (Fig. 69). –If in two triangles, ABC, abc, all the angles of one be each respectively equal to all the angles of the other, that is, A = a, B = b, C = c: then the sides opposite to the equal angles will be proportional, *viz.*:—

AB : ab : : AC : ac;
AB : ab : : BC : bc;
and AC : ac : : BC : bc.

82. THEOREM XV (Fig. 70).—If from a point A without a circle DBCE there be drawn two lines ADE, ABC, each of them cutting the circle in two points; the product of one whole line into its external part, *viz.*, AC × AB, will be equal to that of the other line into its external part, *viz.*, AE × AD.

83. THEOREM XVI (Fig. 71).—Triangles ABC, BCD, and parallelograms ABCF, BDEC having the same altitude, have the same proportion between themselves as their bases, AB and BD; that is, AB : BD : : ABC : BDC or : : ABCF : BDEC.

84. THEOREM XVII (Fig. 72).—Triangles ABC, DEF, standing upon equal bases, AB and DE, are to each other as their altitudes CG and FH; that is, ABC : DEF : : CG : FH.

85. THEOREM XVIII (Fig. 73).—If a right line BE, be drawn parallel to one side of a triangle ACD, it will cut the two other sides proportionally, *viz.*, AB : BC : : AD : ED.

86. THEOREM XIX (Fig. 74).—If two triangles ABC, ADE, have one angle BAC in the one, equal to one angle DAE in the other, and the sides about the equal angles proportional, that is, AB : AD : : AC : AE, then the triangles will be mutually equiangular, that is, B = D and C = E.

87. THEOREM XX (Fig. 75).—Equiangular triangles ABC, DEF, are to one another as the squares ABKI and DEML of their homologous sides, that is, ABC : DEF : : AK : DM.

GEOMETRICAL PROBLEMS.

88. PROBLEM I.—*To bisect, or divide into two equal parts, a given straight line or arc AB.* (Fig. 76.)

From A and B, as centres, with any radius greater than half the line, describe two arcs, intersecting each other in C and D; draw the line CD, and it will bisect the given line or arc AB in E or F.

89. PROBLEM II.—*To erect a perpendicular from a given point C in a given right line AB.* (Figs. 77—78.)

(*a*) When the point is near the middle of the line. (*See* Fig. 78.)

On each side of the point C take equal distances CD, CE; from D and E, with any radius greater than DC, describe two arcs intersecting each other in F; draw the line CF, and it will be the perpendicular required.

(*b*) When the point C is at or near the end of the line. (*See* Fig 77.)

From any point D over the line, with the radius DC, describe an arc, cutting the line AB in E and C; through E and D draw the right line EDF, cutting the arc in F; join FC, and it will be the perpendicular required.

90. PROBLEM III.—*From a given point C over a given right line AB, to let fall a perpendicular on it.* (Figs. 79—80.)

(*a*) When the point is nearly over the middle of the line. (*See* Fig. 79).

From the point C, with any convenient radius, describe an arc cutting AB in G and D; from G and D with a radius longer than the half of GD, describe two arcs crossing each other in E; draw the line CE, cutting the given line in F, and CF will be the perpendicular required.

(*b*) When the point is nearly opposite the end of the line. (*See* Fig. 80.)

From any two points D, E, at some distance apart in the given line, and with radii DC, EC, respectively, describe arcs cutting at C and F; join CF cutting AB in G; CG will be the perpendicular required.

91. PROBLEM IV.—*To draw a straight line parallel to a given straight line AB.* (Figs. 81—82.)

(*a*) When the parallel line is to be at a given distance E. (*See* Fig. 81.)

At any two points as C and D in the line AB, erect perpendiculars (by Prob. II, Art. 89), on which lay off CF and DG, each equal to E. Through F and G draw the line FG, and it will be parallel to AB.

(*b*) When the parallel line is to pass through a given point D. (*See* Fig. 82.)

From the point D let fall a perpendicular DC on AB (by Prob. III, Art. 90); at any point E, as distant from C as possible, erect a perpendicular (by Prob. II, Art. 89); and set off EG equal to CD. Through G and D draw the straight line GD; it will be parallel to AB.

92. PROBLEM V.—*To divide a given straight line AB into any required number of equal parts.* (Fig. 83.)

Draw the line AC, making an acute angle with AB; set off on AC, with any convenient distance, the required number of equal parts A1, 1 2, 2 3, 3 4, 4 5; join 5B, and draw parallels to it to meet the line AB from the other points of division in AC (by Prob. IV, Art. 91). These parallels divide AB as required.

93. Problem VI.—*To divide into any required number of equal parts a given straight line AB, which is too short to admit of distinct divisions.* (Fig. 84.)

At A erect a perpendicular AC (by Prob. II, Art. 89), and at any equal distances A4, 43, 32, 21, 1C, in AC, draw as many straight lines parallel to AB (by Prob. IV, Art. 91) as there are parts required—say, in this case, five. Join CB, cutting each of these parallels in 5; then 1—5 will be $\frac{1}{5}$, 2—5 will be $\frac{2}{5}$, 3—5 will be $\frac{3}{5}$, and 4—5 will be $\frac{4}{5}$ of AB.

94. Problem VII.—*To divide a given right line AH into any number of parts exactly proportional to the parts of another right line.* (Fig. 85.)

Let AB be the line divided into any number of parts, AC, CD, DE, EF, FG, and GB; and let AH be the line to be divided; draw HB, and parallel to the same draw (by Prob. IV, Art. 91) the lines GJ, FK, EL, DM, etc.; then the line AH will be divided exactly proportional to the line AB.

95. Problem VIII.—*At a given point A to make an angle equal to a given angle HBI.* (Fig. 86.)

From the given point A draw any line AJ, and from A and B as centres, with any and the same radius, describe arcs FE, DC: from F as a centre, with radius equal to the chord of the arc DC, describe an arc cutting FE in G; through G draw the line AGK, and JAK will be the angle required.

96. Problem IX.—*To bisect, or divide into two equal parts, a given angle ABC.* (Fig. 87.)

From B as a centre, with any radius, describe the arc DE; then from the points D and E, with any radius longer than the half of DE, describe arcs intersecting at F; draw the line BF, and it will bisect the angle ABC.

97. Problem X.—*To bisect the inclination of two lines of which the intersection would fall without the paper.* (Fig. 88.)

Upon the given lines CB, CH, draw at any points perpendiculars EF, HG (by Prob. II, Art. 89), of equal length; and through F and G draw parallels to the respective lines (by Prob. IV, Art. 91), cutting at S. Bisect the angle FSG so formed by the line SD (by Prob. IX, Art. 96); and this line equally divides the inclination of the given lines.

98. Problem XI.—*To trisect, or divide into three equal parts, a right angle ABC.* (Fig. 89.)

From the point B with any radius, describe the arc DE, cutting BA, BC, in D and E. From the points D and E, with the same radius BD or BE, cross the arc in M and N respectively. Then, draw the lines BM, BN, and they will trisect the angle as required.

99. Problem XII.—*To make a triangle with three given lines AB, AC and BC provided the sum of any two of them be greater than the third.* (Fig. 90.)

On any of the given lines, as AB, from A as a centre, with a radius equal to AC, describe an arc; then from B, as a centre, with a radius equal to BC, describe another arc cutting the former in the point C; draw AC, BC; and ABC will be the triangle required.

100. Problem XIII.—*Upon a given right line AB to describe an equilateral triangle.* (Fig. 91.)

From the points A and B, with radius equal to AB, describe arcs cutting each other at C. Draw AC and BC, and ABC will be the triangle required.

101. Problem XIV.—*To make an equilateral triangle to a given height AB.* (Fig. 92.)

Bisect AB in C. With B and C as centres and radius BC describe arcs intersecting at D and E; join AD and AE and produce them. At the point B draw FBG perpendicular to AB (by Prob. II, Art. 89), meeting AD and AE produced in F and G; and AFG will be the equilateral triangle required.

102. Problem XV.—*Upon a given base AB to describe an isosceles triangle, the other sides of which shall be equal to the line C.* (Fig. 93.)

From the points A and B as centres, and the line C as radius, describe arcs intersecting in E; join AE, BE, and ABE will be the isosceles triangle required.

103. PROBLEM XVI.—*To make a triangle equal to a given quadrilateral figure ABCD.* (Fig. 94.)

Draw the diagonal AC, and parallel to it DE (by Prob. IV, Art. 91), meeting BA produced in E; then draw CE, and ECB will be the triangle required.

104. PROBLEM XVII.—*To make a triangle equal to a given five-sided figure ABCDE.* (Fig. 95.)

Draw DA and DB, and also EH and CF parallel to them (by Prob. IV, Art. 91), meeting AB produced in H and F; then draw DH, DF, and the triangle HDF will be the one required.

105. PROBLEM XVIII.—*To construct a square upon a given line AB.* (Fig. 96.)

From the point B erect the perpendicular BC (by Prob. II, Art. 89), equal to AB; and from A and C, with the radius AB, describe arcs intersecting at D; draw AD and CD, and the square ABCD will be completed.

106. PROBLEM XIX.—*To construct a rectangular parallelogram, the length and breadth of which shall be equal to two given lines AMB and CNB.* (Fig. 97.)

At B erect the perpendicular BC equal to the breadth N. With A as centre and radius equal to BC, describe an arc; with C as centre and radius equal to M, describe another arc cutting the former in D; join AD and CD, and the required figure ABCD will be completed.

107. PROBLEM XX. *To make a rhombus whose side shall be equal to a given line W.* (Fig. 98.)

Draw the line AD equal to W; from D as centre and DA as radius describe the arc G; from A as centre and the same radius describe an arc intersecting the former at B; and from B as centre and the same radius describe another arc intersecting the first in F. Draw AB, BF, and FD, and the rhombus ADFB will be completed.

108. PROBLEM XXI.—*To make a rhomboid the longer and shorter sides of which shall be equal to the lines M and N respectively.* (Fig. 99.)

Draw the line AB equal to M; then from A set off AD, making any angle DAB that may be required, equal to N. With D as centre and radius equal to M describe the arc *e*, and with B as centre and radius equal to N describe an arc cutting the former in C; draw AD, DC, and CB, and the required rhomboid ABCD will be completed.

109. PROBLEM XXII.—*To make a square equal to a given rectangle AB CD.** (Fig. 100.)

Produce one of the sides AB till BE equals BC; bisect AE in O (by Prob. I, Art. 88), on which as centre with radius OA, describe a semicircle, and produce BC to meet it in F. On BF describe the square BFGH (by Prob. XVIII, Art. 105), and it will be equal to the rectangle ABCD, as required.

110. PROBLEM XXIII. *To form a square equal in area to a given triangle ABC.* (Fig. 388).

On the longest side produced set off CD, equal to half the perpendicular height BE, and at C erect the perpendicular CF; then on AD describe a semicircle cutting CF in G, and on CG, form the square CGHI, which will be equal in area to the given triangle.

111. PROBLEM XXIV.—*To find the centre of a given circle.* (Fig. 101).

Draw any chord AB, and bisect it (by Prob. I, Art. 88) with the perpendicular CD (by Prob. II, Art. 89), which also bisect in O (by Prob. I, Art. 88) for the centre of the circle.

112. PROBLEM XXV.—*To find the centre of a circle of which an arc only is given.* (Fig. 102.)

Draw any two chords AB, BC, and bisect them by perpendiculars DO, EO (by Probs. I, II, Arts. 88–89); the point O, in which they meet, will be the centre of the circle of which ABC is an arc.

OBSERVATION.

[*] To find a mean proportional, is equivalent to finding a square = a rectangle.

113. PROBLEM XXVI.—*To draw a tangent to a given circle that shall pass through a given point A.* (Figs. 103—105).

(*a*) When the point A is in the circumference (*see* Fig. 103).

Draw the radius AO and the line DC perpendicular to it at the point A (by Prob. II, Art. 89); DAC will be the tangent required.

(*b*) When the point A is in the circumference, but the centre of the circle is inaccessible. (*See* Fig. 104.)

Take any two equal distances AB, BC, upon the circumference from the given point A. Draw the chord AC, and with A as centre and radius AB, describe an arc DBE cutting the chord AC in E; make BD equal to BE, and draw AD, which will be the tangent required.

(*c*) When the point A is without the circle. (*See* Fig. 105.)

Draw the line AB from the given point A to B, the centre of the circle, and bisect AB in C (by Prob. I, Art. 88). From C as centre, with radius CA or CB, describe a circle cutting the given circle in D and E; draw AD, AE, and they will both be tangents to the given circle.

114. PROBLEM XXVII.—*To describe the circumference of a circle through three given points A, B, and C.* (Fig. 106.)

Draw the lines AB and BC, and bisect them with two others drawn perpendicular to them (by Probs. I—II, Arts. 88—89), producing the same till they meet in the point D. With D as a centre and radius DA, DB, or DC, describe a circle which will pass through the points as required.

115. PROBLEM XXVIII.—*To find a third proportional to two given right lines, AB and FC.* (Fig. 107.)

At either extremity of the given line AB, draw AE, making any angle with it, from which cut off AG, equal to the other line FC, and join BG. Produce AB and make BD equal to FC, and from D draw DE parallel to BG (by Prob. IV, Art. 91); GE will be the third proportional required; that is, AB : FC : : FC : GE.

116. PROBLEM XXIX.—*To find a fourth proportional to three given lines, P, Q, and R.* (Fig. 108.)

Draw two lines AD and AE, making any angle; make AB and BD equal to the given lines P and Q, and AC equal to the line R; join BC, and through D, draw DE parallel to BC, and the line CE will be the fourth proportional required; that is, P : Q : : R : CE.

117. PROBLEM XXX.—*To construct a scale of chords.* (Figs. 111, 185.)

Transfer the divisions on the arc of a quadrant to its chord. Thus, suppose ACB is a quadrant and the right line BA the chord of its arc. Let this arc be divided into 90 equal parts or degrees; then, if one foot of a pair of compasses be kept on the point B, and arcs successively described with the other from each of the 90 divisions on the arc to meet BA, those arcs will divide it into a line of chords. For a larger scale of chords, *see* Fig. 185.

118. PROBLEM XXXI.—*To make an angle of any proposed number of degrees.* (Figs. 109—111.)

Take the first 60 degrees from a scale of chords (*see* Fig. 111) as a radius, and from A as a centre (*see* Fig. 109), describe an arc BC. Next take the proposed number of degrees from the scale of chords, and set them off from B to C; then draw the lines AD, AE, and the required angle will be made. If the angle is to contain more than 90 degrees, it must be taken at two operations (*see* Fig. 110).

119. PROBLEM XXXII.—*To find the number of degrees contained in an angle BAC.* (Figs. 109—111.)

From A for a centre (*see* Fig. 109), and with a chord of 60 degrees for a radius (*see* Fig. 111), describe the arc BC. Next take the distance BC, and apply it to the scale of chords, which will show the number of degrees. If the angle contain more than 90 degrees, it must be taken at two operations (*see* Fig. 110).

CHAPTER II.

Mensuration of Surfaces.

DEFINITIONS.

1. MENSURATION. —*Mensuration* is the art of measuring figures of every form, whether superficial or solid, regular or irregular.

2. MEASURES. —*Measures* are of three kinds: Lineal, Square, and Solid.

3. ORDINARY LINEAL MEASURES. —The ordinary lineal measures are Inches, Feet, Yards, Chains, and Miles. The other measures are the squares or cubes of these.

4. AREA. —*Area* means the superficial content of any plane geometrical figure, estimated by the squares of some known lineal measure, as inches, links, chains, etc.

5. AREA, HOW RECKONED. (Fig. 202.) —The area of any plane figure is the measure of the space contained within its lines or bounds. This area or content is estimated by the number of little squares that may be contained in the figure; the side of those squares being an inch, a foot, a yard, an acre, or any other fixed quantity. And hence the area or content is said to be so many square inches, or square feet, or square yards, etc. Thus, if the figure to be measured be the rectangle ABCD, and the little square E, whose side is one inch, be the measuring unit: then, whatever number of those little squares may be made to fit into the figure, so many square inches it is said to contain, which in the present case is 12.

6. ALTITUDE. (Fig. 207.) —The *altitude* of any figure is the straight line drawn from its vertex perpendicular to its base, as AB.

7. PERIMETER. (Fig. 60.) —*Perimeter* is the outer boundary of a figure, or the sum of all its sides, as ABC.

8. LUNE. (Fig. 205.) —A *lune* is a figure bounded by two arcs of circles, as ABCDA.

9. ELLIPSE. (Fig. 203). —An *ellipse* is an oval figure, the section of a cone formed by a plane cutting both sides of it obliquely, as ABC.

10. CURVILINEAR. (Fig. 118.) —A *curvilinear* figure is a figure bounded by curve lines, as ABC.

11. ORDINATE. (Fig. 118.) —An *ordinate*, as CD, is a straight line drawn from any point in a curve perpendicularly to another straight line called the *abscissa*, as AB. The abscissæ and ordinates together are called the *co-ordinates* of the point.

LENGTHS.

12. LINEAL MEASURE. —The lineal measures commonly used are as follows, namely :—

12 inches = 1 foot.
3 feet = 1 yard.
22 yards, or 66 feet = 1 chain (Gunter's) of 100 links.
10 chains = 1 furlong.
8 furlongs, or 80 chains = 1 mile.

The following is a table of lineal measure*:—

In.	Lk.	Ft.	Yd.	Ch.	Fur.	Ml.
7·92 =	1					
12 =	1·515 =	1				
36 =	4·545 =	3 =	1			
792 =	100 =	66 =	22 =	1		
7,920 =	1,000 =	660 =	220 =	10 =	1	
63,360 =	8,000 =	5,280 =	1,760 =	80 =	8 =	1

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* For a complete table of lineal measure, see Table LX of the "Calculating Tables" (5th edition).

The English cubit = 19·8 inches, or $\frac{1}{40}$ of a chain. The Burmese cubit = 19·05 inches.

In India and Burma the duodecimal system of measuring prevails, whereof the foot of 12 inches is the basis. The yard, which is equal to 3 feet, is now made by enactment the standard of lineal measurement for this country.

A *geographical* or *nautical* mile = 6,082·66 feet.

Some of the other known, but little used, small measures are: a *point* = $\frac{1}{72}$ inch; a *line* = 6 points = $\frac{1}{12}$ inch; a *palm* = 3 inches; a *hand* = 4 inches; a *span* = 9 inches.

† Sections 2 and 4 of Act II of 1889 of the Legislative Council of the Governor-General of India run as follows: "The imperial standard yard for the United Kingdom shall be the legal standard measure of length in British India and be called the *standard yard*. One-third of the standard yard shall be called a *standard foot*, and one-thirty-sixth part of such a yard shall be called a *standard inch*."

TRIANGLES.

13. *The sides of a right-angled triangle being given, to find the hypotenuse.*

Rule: Extract the square root of the sum of the squares of the given sides (*see* Chap. I, Art. 79).

Example: One side of a right-angled triangle is 3 feet, and the other side is 4 feet; find the hypotenuse.

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ feet}.$$

14. *The hypotenuse and one side of a right-angled triangle being given, to find the other side.*

Rule: Extract the square root of the difference of the squares of the given sides (*see* Chap. I, Art. 79).

Example: The hypotenuse of a right-angled triangle is 13 inches, and one side is 5 inches; find the other side.

$$\text{Required side} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ inches}.$$

15. *In an equilateral triangle one side being given, and in an isosceles triangle the base and one of the equal sides being given, to find the altitude.*

Rule: Extract the square root of the difference of the squares of the given side and half the base.

Example: Required the altitude of an isosceles triangle one of whose equal sides is 10 and base 6.

$$\text{Altitude} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8.$$

16. *Having given the base and perpendicular of a right-angled triangle, to find the perpendicular let fall on the hypotenuse from the right angle; and also the segments into which this perpendicular divides the hypotenuse.*

Rule: First find the hypotenuse, then divide the square of the greater side by the hypotenuse for the greater segment, and the square of the lesser side by the hypotenuse for the lesser segment. Next, extract the square root of the product of these two segments for the perpendicular required.

Example: The base and pependicular of a right-angled triangle are respectively 9 and 12; required the segments into which the hypotenuse is divided by a perpendicular let fall on it from the right angle; also, required this perpendicular.

$$\text{First, hypotenuse} = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15;$$

$$\text{then, greater segment} = \frac{12^2}{15} = \frac{144}{15} = 9{\cdot}6,$$

$$\text{and lesser segment} = \frac{9^2}{15} = \frac{81}{15} = 5{\cdot}4;$$

$$\text{also, perpendicular} = \sqrt{9{\cdot}6 \times 5{\cdot}4} = \sqrt{51{\cdot}84} = 7{\cdot}2.$$

CIRCLES.

17. *To find the circumference of a circle, the diameter being given.*

Rule: Multiply the diameter by 22, and divide the product by 7.*

Example: The diameter of a circle is 21; what is the circumference?

$$\text{Circumference} = \frac{21 \times 22}{7} = 3 \times 22 = 66.$$

18. *To find the diameter of a circle, the circumference being given.*

Rule: Multiply the circumference by 7, and divide the product by 22.†

Example: The circumference of a circle is 198; what is the diameter?

$$\text{Diameter} = \frac{198 \times 7}{22} = 9 \times 7 = 63.$$

CHORDS.

19. *Any two of the following dimensions being given, to find the others* (*see* Arts. 13 and 14). (Fig. 204.)

CA, the radius.

AD, the chord of the whole arc;

OBSERVATIONS.

* More correctly: Multiply the diameter by 3·141593.

† More correctly: Multiply the circumference by 0·318310.

AB, the chord of half the arc;
BE, the height of the arc, also called "versed sine;"
EC, the central distance.[*]

1. *Given BE, AB; required BF.*

Formula: $BF = \frac{AB^2}{BE}$.[†]

Example: The height of an arc is 2 feet, and the chord of half the arc is 6 feet; find the diameter of the circle.

$$\text{Diameter} = \frac{6^2}{2} = \frac{36}{2} = 18 \text{ feet.}$$

Given AB, BF; required BE.

Formula: $BE = \frac{AB^2}{BF}$.[‡]

Example: The chord of half the arc is 10 yards, and the diameter of the circle 75 yards; find the height of the arc.

$$\text{Height of arc} = \frac{10^2}{75} = \frac{100}{75} = 1\tfrac{1}{3} \text{ yards.}$$

3. *Given BE, BF; required AB.*

Formula: $AB = \sqrt{BE \cdot BF}$.[§]

Example: The height of an arc is 3 feet 9 inches, and the diameter of the circle is 33 feet 9 inches; required the chord of half the arc.

$$\text{Chord of half the arc} = \sqrt{3 \text{ feet } 9 \text{ inches} \times 33 \text{ feet } 9 \text{ inches}} = \sqrt{3\tfrac{3}{4} \times 33\tfrac{3}{4}} = \sqrt{\tfrac{2025}{16}} = \tfrac{45}{4} = 11\tfrac{1}{4} = 11 \text{ feet } 3 \text{ inches.}$$

4. *Given AD, BE; required BF.*

Formula: $BF = \frac{AD^2}{4BE} + BE$.[‖]

Example: The chord of an arc is 40 furlongs, and the height of the arc is 8 furlongs; required the diameter of the circle.

$$\text{Diameter} = \frac{40^2}{4 \times 8} + 8 = \frac{1600}{32} + 8 = 50 + 8 = 58 \text{ furlongs.}$$

5. *Given BE, BF; required AD.*

Formula: $AD = 2\sqrt{BE \times (BF - BE)}$.[¶]

Example: The height of an arc is 6 miles, and the diameter of the circle 156 miles; what is the chord of the arc?

$$\text{Chord of the arc} = 2\sqrt{6 \times (156 - 6)} = 2\sqrt{6 \times 150} = 2\sqrt{900} = 2 \times 30 = 60 \text{ miles.}$$

6. *Given AD, BF; required BE.*

Formula: $BE = BF - \sqrt{BF^2 - AD^2}$.[**]

OBSERVATIONS.

[*] Geometry provides us with the two following important results which are much employed in the solution of these problems, namely: $FB \cdot BE = AB^2$; and $FE \times EB = AE \times ED$.

[†] *Reason*: Since $FB \cdot BE = AB^2$, $\therefore BF = \frac{AB^2}{BE}$.

[‡] *Reason*: Since $FB \cdot BE = AB^2$, $\therefore = BE = \frac{AB^2}{BF}$.

[§] *Reason*: Since $FB \cdot BE = AB^2$, $\therefore = AB = \sqrt{BE \cdot BF}$.

[‖] *Reason*: Since $FE \cdot EB = AE \cdot ED$, $\therefore FE = \frac{AE \cdot ED}{EB} = \frac{AE^2}{EB} = \frac{AD^2}{4BE}$; but $BF = FE + BE = \frac{AD^2}{4BE} + BE$.

[¶] *Reason*: Since $FE \cdot EB = AE \cdot ED = AE^2$, $\therefore AE = \sqrt{FE \cdot BE} = \sqrt{(BF - BE)\,BE}$, and $2AE$ or $AD = 2\sqrt{(BF - BE)\,BE}$.

[**] *Reason*: Since $AC^2 = AE^2 + EC^2$, $\therefore EC = \sqrt{AC^2 - AE^2}$, but $BE = BC - EC = BC - \sqrt{AC^2 - AE^2} = \frac{BF}{2} - \sqrt{\frac{BF^2}{4} - \frac{AD^2}{4}} = \frac{BF - \sqrt{BF^2 - AD^2}}{2}$.

Example: The chord of an arc is 4 chains, and the diameter of the circle 5 chains; required the height of the arc.

$$\text{Height of arc} = \frac{5 - \sqrt{5^2 - 4^2}}{2} = \frac{5 - \sqrt{25 - 16}}{2} = \frac{5 - \sqrt{9}}{2} = \frac{5 - 3}{2} = \frac{2}{2} = 1 \text{ chain.}$$

7. *Given AD, BF; required AB.*

Formula: $\mathrm{AB} = \sqrt{\dfrac{\mathrm{BF}^2 - \mathrm{BF}\sqrt{\mathrm{BF}^2 - \mathrm{AD}^2}}{2}}$.*

Example: The chord of an arc is 40 links, and the diameter of the circle is 50 links; required the chord of half the arc.

$$\left.\begin{array}{l}\text{Chord of half}\\ \text{the arc.}\end{array}\right\} = \sqrt{\frac{50^2 - 50\sqrt{50^2 - 40^2}}{2}} = \sqrt{\frac{2{,}500 - 50\sqrt{2{,}500 - 1{,}600}}{2}}$$

$$= \sqrt{\frac{2{,}500 - 50\sqrt{900}}{2}} = \sqrt{\frac{2{,}500 - 50 \times 30}{2}} = \sqrt{\frac{2{,}500 - 1{,}500}{2}}$$

$$= \sqrt{\frac{1{,}000}{2}} = \sqrt{500} = 22{\cdot}36 \text{ links.}$$

8. *Given AB, BF; required AD.*

Formula: $\mathrm{AD} = \dfrac{2\mathrm{AB}}{\mathrm{BF}}\sqrt{\mathrm{BF}^2 - \mathrm{AB}^2}$.†

Example: The chord of half the arc is 7·2 poles, and the diameter of the circle 48 poles; find the chord of the arc.

$$\text{Chord of the arc} = \frac{2 \times 7{\cdot}2}{48}\sqrt{48^2 - 7{\cdot}2^2} = 0{\cdot}3 \times \sqrt{2{,}304 - 51{\cdot}84} = 0{\cdot}3 \times \sqrt{2{,}252{\cdot}16} = 0{\cdot}3 \times 47{\cdot}46 = 14{\cdot}24 \text{ poles.}$$

9. *Given AD, AB; required BF.*

Formula: $\mathrm{BF} = \dfrac{2\mathrm{AB}^2}{\sqrt{4\mathrm{AB}^2 - \mathrm{AD}^2}}$.‡

Example: The chord of an arc and the chord of half the arc are 36 and 19·5 fathoms respectively; find the diameter of the circle.

$$\text{Diameter} = \frac{2 \times 19{\cdot}5^2}{\sqrt{4 \times 19{\cdot}5^2 - 36^2}} = \frac{2 \times 380{\cdot}25}{\sqrt{4 \times 380{\cdot}25 - 1{,}296}} = \frac{760{\cdot}5}{\sqrt{1{,}521 - 1{,}296}}$$

$$= \frac{760{\cdot}5}{\sqrt{225}} = \frac{760{\cdot}5}{15} = 50{\cdot}7 \text{ fathoms.}$$

20. *Table of the Radii of inscribed and circumscribed circles, and of the sides of Polygons.*

S = Side of Polygon.
R = Radius of circumscribed circle.
r = Radius of inscribed circle.

OBSERVATIONS.

* *Reason*: Since $\mathrm{AC}^2 = \mathrm{AE}^2 + \mathrm{EC}^2$, $\therefore \mathrm{EC} = \sqrt{\mathrm{AC}^2 - \mathrm{AE}^2}$, but $\mathrm{BE} = \mathrm{BC} - \mathrm{EC} = \mathrm{BC} - \sqrt{\mathrm{AC}^2 - \mathrm{AE}^2}$; again since $\mathrm{AB}^2 = \mathrm{AE}^2 + \mathrm{EB}^2$, $\therefore \mathrm{AB} = \sqrt{\mathrm{AE}^2 + \mathrm{EB}^2} = \sqrt{\mathrm{AE}^2 + (\mathrm{BC} - \sqrt{\mathrm{AC}^2 - \mathrm{AE}^2})^2}$; $\sqrt{\mathrm{AE}^2 + \mathrm{BC}^2 + \mathrm{AC}^2 - \mathrm{AE}^2 - 2\mathrm{BC}\sqrt{\mathrm{AC}^2 - \mathrm{AE}^2}} = \sqrt{2\mathrm{BC}^2 - 2\mathrm{BC}\sqrt{\mathrm{AC}^2 - \mathrm{AE}^2}}$ $= \sqrt{\dfrac{\mathrm{BF}^2}{2} - \mathrm{BF}\sqrt{\dfrac{\mathrm{BF}^2}{4} - \dfrac{\mathrm{AD}^2}{4}}} = \sqrt{\dfrac{\mathrm{BF}^2 - \mathrm{BF}\sqrt{\mathrm{BF}^2 - \mathrm{AD}^2}}{2}}$.

† *Reason*: Since $\mathrm{FB} \cdot \mathrm{BE} = \mathrm{AB}^2$, $\therefore \mathrm{BE} = \dfrac{\mathrm{AB}^2}{\mathrm{FB}}$; and since $\mathrm{AB}^2 = \mathrm{AE}^2 + \mathrm{EB}^2$, $\therefore \mathrm{AE} = \sqrt{\mathrm{AB}^2 - \mathrm{BE}^2}$ $= \sqrt{\mathrm{AB}^2 - \dfrac{\mathrm{AB}^4}{\mathrm{BF}^2}} = \sqrt{\dfrac{\mathrm{AB}^2(\mathrm{BF}^2 - \mathrm{AB}^2)}{\mathrm{BF}^2}} = \dfrac{\mathrm{AB}}{\mathrm{BF}}\sqrt{\mathrm{BF}^2 - \mathrm{AB}^2}$; hence 2AE or AD $= \dfrac{2\mathrm{AB}}{\mathrm{BF}}\sqrt{\mathrm{BF}^2 - \mathrm{AB}^2}$.

‡ *Reason*: Since $\mathrm{AB}^2 = \mathrm{AE}^2 + \mathrm{EB}^2$, $\therefore \mathrm{BE} = \sqrt{\mathrm{AB}^2 - \mathrm{AE}^2}$; again since $\mathrm{FB} \cdot \mathrm{BE} = \mathrm{AB}^2$, $\therefore \mathrm{BF} = \dfrac{\mathrm{AB}^2}{\mathrm{BE}} = \dfrac{\mathrm{AB}^2}{\sqrt{\mathrm{AB}^2 - \mathrm{AE}^2}} = \dfrac{\mathrm{AB}^2}{\sqrt{\mathrm{AB}^2 - \dfrac{\mathrm{AD}^2}{4}}} = \dfrac{\mathrm{AB}^2}{\frac{1}{2}\sqrt{4\mathrm{AB}^2 - \mathrm{AD}^2}} = \dfrac{2\mathrm{AB}^2}{\sqrt{4\mathrm{AB}^2 - \mathrm{AD}^2}}$.

Name.	Number of sides.	Interior angle.	S = R ×	S = *r* ×	R = S ×	*r* = S ×
Trigon	3	60	1·7320	3·4641	0·5774	0·2887
Tetragon	4	90	1·4142	2·0000	0·7071	0·5000
Pentagon	5	108	1·1755	1·4536	0·8507	0·6882
Hexagon	6	120	1·0000	1·1547	1·0000	0·8660
Heptagon	7	128 4/7	0·8678	0·9631	1·1524	1·0383
Octogon	8	135	0·7653	0·8284	1·3066	1·2071
Nonagon	9	140	0·6840	0·7280	1·4619	1·3737
Decagon	10	144	0·6180	0·6498	1·6180	1·5388
Undecagon	11	147 3/11	0·5634	0·5873	1·7747	1·7028
Duodecagon	12	150	0·5176	0·5359	1·9319	1·8660

POLYGONS.

21. *To find the side of a regular polygon, the radius of the circumscribing or inscribed circle being given* ; *and* vice versâ.

Rule : Multiply the given quantity by the tabular radius or side (*see* Art. 20), as the case requires.

Example : Find the side of a pentagon, (*a*) when the radius of the circumscribing circle is 8, (*b*) when the radius of the inscribed circle is 3 ; also, when the side of a trigon is 10, find the radii of the circumscribing and inscribed circles,

$$S = 8 \times 1·1755 = 9·4040, (a),$$
$$S = 3 \times 1·4536 = 4·3608, (b),$$
$$R = 10 \times 0·5774 = 5·7740,$$
$$r = 10 \times 0·2887 = 2·8870.$$

ARCS.

22. *To find the length of an arc of a circle, when the number of degrees in the angle subtended by it at the centre of the circle is given.*

Rule : As 360 degrees is to the number of degrees in the angle, so is the circumference of the circle to the length of the arc required.

Example : The circumference of a circle is 24 inches, and the angle subtended by the arc at the centre is 54 degrees ; find the length of the arc.

As $360° : 54° :: 24$ inches : length of arc in inches, ∴ length of arc

$$= \frac{54 \times 24}{360} = \frac{18}{5} = 3·6 \text{ inches.}$$

23. *To find the number of degrees in the angle subtended by the arc at the centre of a circle, when the length of the arc is given.*

Rule : As the circumference of the circle is to the length of the arc, so is 360 degrees to the required number of degrees in the angle.

Example : The circumference of a circle is 250 feet, and the length of the arc 40 feet ; find the number of degrees contained in the angle subtended by the arc at the centre.

As 250 feet : 40 feet : : 360° : the required number of degrees in the angle,

$$\therefore \text{required angle} = \frac{40 \times 360}{250} = \frac{288}{5} = 57°36'.$$

24. *To find the length of an arc of a circle, the chord of the whole arc and the chord of half the arc being given.*[*] (Fig. 204.)

Rule : From eight times the chord of half the arc subtract the chord of the whole arc, and divide the remainder by 3 ; the quotient will be the length of the arc *nearly*.

OBSERVATION.

[*] *To find the diameter of a circle, when there are given* (*i*) *the chord of half the arc and the height of the segment,* (*ii*) *the chord of the whole arc and the height of the segment, and* (*iii*) *the chord of the whole arc and the chord of half the arc.*

Rule : For (*i*) divide the square of the chord of half the arc by the height ; (*ii*) to the square of the whole chord divided by four times the height, add the height ; and (*iii*) divide twice the square of the chord of half the arc by the square root of the difference of the square of the whole chord taken from four times the square of the chord of half the arc.

Example: The chord of the whole arc, or AD, is 32 inches, and the chord of half the arc, or AB, 20 inches; find the length of the arc.

$$\text{Length of arc} = \frac{8 \times 20 - 32}{3} = \frac{160 - 32}{3} = \frac{128}{3} = 42\tfrac{2}{3} \text{ inches.}$$

ELLIPSES.

25. *To find the circumference of an ellipse, the axes being given.*

Rule: Multiply half the sum of the axes by $\frac{22}{7}$.*

Example: What is the circumference of an ellipse whose axes are 8 and 12?

$$\text{Circumference} = \frac{8 + 12}{2} \times \frac{22}{7} = 10 \times \frac{22}{7} = \frac{220}{7} = 31{\cdot}43.$$

AREAS.

26. SQUARE MEASURE.—The square measures commonly used are as follows, namely:—

144 square inches ... ... = 1 square foot.
9 square feet ... ... = 1 square yard.
484 square yards, or 4,356 square feet ... = 1 square chain (Gunter's).
10 square chains, or 4,840 square yards... = 1 acre.
640 acres ... ... = 1 square mile.

The following is a table of square measure†:

Sq. In.	Sq. Lk.	Sq. Ft.	Sq. Yd.	Sq. Ch.	Ac.	Sq. Ml.
62·726 =	1					
144 =	2·296 =	1				
1,296 =	20·661 =	9 =	1			
627,264 =	10,000 =	4,356 =	484 =	1		
6,272,640 =	100,000 =	43,560 =	4,840 =	10 =	1	
4,014,489,600 =	64,000,000 =	27,878,400 =	3,097,600 =	6,400 =	640 =	1.

TRIANGLES.

27. *To find the area of a triangle, the base and perpendicular height being given.*‡ (Fig. 155.)

Rule: Multiply the base by the perpendicular height, and divide the product by 2.

Example: The base AC of a triangle BAC (*see* Fig. 155) is 9, and the perpendicular height BD 7; find its area.

$$\text{Area} = \frac{9 \times 7}{2} = \frac{63}{2} = 31{\cdot}5.$$

28. *To find the area of a triangle, the three sides being given.* (Fig. 60.)

Rule: From half the sum of the three sides subtract each side separately; next multiply together the three remainders and the half sum, and extract the square root of the product.

Example: The sides of a triangle AB, BC, and CA (*see* Fig. 60) are 26, 28, and 30 respectively; find the area.

$$\text{Half sum of sides} = \frac{26 + 28 + 30}{2} = \frac{84}{2} = 42;$$

$$\begin{aligned}\text{area} &= \sqrt{42 \times (42 - 26) \times (42 - 28) \times (42 - 30)} \\ &= \sqrt{42 \times 16 \times 14 \times 12} = \sqrt{112{,}896} = 336.\end{aligned}$$

OBSERVATIONS.

[*] More correctly: 3·141593.

[†] For a complete table of square measure, *see* Table LXI of the "Calculating Tables" (5th edition).

The *acre* is the accepted unit of square measure everywhere, being a most convenient particle of land measure for area calculations, and has been fixed by statute to consist of 160 square perches. It is equal to a square whose side is 120½ English cubits nearly, or about 83½ Burmese paces (the ordinary pace being 30 inches in length); it also equals 316·23 links of a Gunter's chain, or 208·71 feet, or about 70 English paces of 36 inches. Two and a half acres would be equal to a square plot within 5 chain sides, and 100 Burmese paces square (or, in Burma, the space of ground that would be covered by a basket of paddy-seed sown broadcast) would equal about 1½ acres.

[‡] *To find the base, the area and perpendicular height being given.* *Rule*: Divide the area by half the perpendicular height.

To find the perpendicular height, the area and base being given. *Rule*: Divide the area by half the

QUADRILATERALS.

29. *To find the area of a square, the side or diagonal being given.*[*] (Fig. 96.)

Rule: Multiply the side by itself, or take half the square of the diagonal.

Example: Find the area of a square whose side AB is 6 or diagonal AC, $= 6\sqrt{2}$. (*See* Fig. 96.)

$$\text{Area} = 6 \times 6 = 36;$$

$$\text{or area} = \frac{6\sqrt{2} \times 6\sqrt{2}}{2} = \frac{36 \times 2}{2} = 36.$$

30. *To find the area of a rectangle, the sides being given.*[†] (Fig. 97.)

Rule: Multiply the length by the breadth.

Example: The length of a rectangle ABCD (*see* Fig. 97) is 18 and the breadth 8; find its area.

$$\text{Area} = 18 \times 8 = 144.$$

31. *To find the area of a parallelogram, the base and perpendicular height being given.*[‡] (Fig. 208.)

Rule: Multiply the base by the perpendicular height.

Example: Find the area of a parallelogram ABCD (*see* Fig. 208) whose length BC is 88, and its altitude AE, 9.

$$\text{Area} = 88 \times 9 = 792.$$

32. *To find the area of a trapezoid, the parallel sides and the perpendicular distance between them being given.*[§] (Fig. 209.)

Rule: Multiply half the sum of the parallel sides by the perpendicular distance.

Example: What is the area of the trapezoid ABCD (*see* Fig. 209), the parallel sides AD, BC, being 16 and 18, and the perpendicular distance DE, being 7?

$$\text{Area} = \frac{16 + 18}{2} \times 7 = \frac{34}{2} \times 7 = 17 \times 7 = 119.$$

33. *To find the area of a trapezium, the diagonal and the perpendiculars on it from the opposite angles being given.*[||] (Fig. 211.)

Rule: Multiply the diagonal by half the sum of the perpendiculars.

Example: The diagonal BD of a trapezium ABCD (*see* Fig. 211) is 64, and the perpendiculars AE, CF, are 14 and 6; find its area.

$$\text{Area} = 64 \times \frac{14 + 6}{2} = 640.$$

OBSERVATIONS.

[*] *To find the side or diagonal, the area being given* *Rule*: For the side, extract the square root of the area; for the diagonal, extract the square root of double the area.

[†] *To find the length or breadth, the area being given. Rule*: Divide the area by the given side.

To find the diagonal, the length and breadth being given. Rule: Extract the square root of the sum of the squares of the given sides (*see* Art. 13).

[‡] *To find the base, the area and perpendicular being given. Rule*: Divide the area by the perpendicular.

To find the perpendicular, the area and base being given. Rule: Divide the area by the base.

[§] *To find the perpendicular distance, the area and parallel sides being given. Rule*: Divide double the area by the sum of the parallel sides.

To find the sum of the parallel sides, the area and perpendicular distance being given. Rule: Divide double the area by the perpendicular distance.

[||] *To find the diagonal, the area and perpendiculars being given. Rule*: Divide double the area by the sum of the perpendiculars.

To find the sum of the perpendiculars, the area and diagonal being given. Rule: Divide double the area by the diagonal.

POLYGONS.

34. *Table of the Areas of Regular Polygons.*

S = side of Polygon.

NAME.	No. of sides.	Interior angle.	AREA = S² ×
Trigon	3	60°	0·4330
Tetragon	4	90	1·0000
Pentagon	5	108	1·7205
Hexagon	6	120	2·5981
Heptagon	7	128 4/7	3·6339
Octagon	8	135	4·8284
Nonagon	9	140	6·1818
Decagon	10	144	7·6942
Undecagon	11	147 3/11	9·3656
Duodecagon	12	150	11·1962

35. *To find the area of a regular polygon, a side being given.** (Fig. 36.)

Rule: Multiply the square of the side by the tabular area (from Art. 34).

Example: The side ABC (*see* Fig. 36) of a hexagon is 7; what is the area?

$$\text{Area} = 7^2 \times 2{\cdot}5981 = 127{\cdot}3069.$$

36. *To find the area of any irregular polygon.* (Fig. 212.)

Rule: Divide it into convenient triangles and quadrilaterals, and take the sum of their several areas.

Example: Find the area of the irregular polygon ABCDE (*see* Fig. 212) which has AB = 3, BC = 8, CD = 6, DE = 9, EA = 5, BE = 7, and BD = 13.

Area of triangle ABE = 6·50 (by Art. 28).
Do. BDE = 29·95 do.
Do. BCD = 16·69 do.
∴ Area of polygon = 6·50 + 29·95 + 16·69 = 53·14.

CIRCLES.

37. *To find the area of a circle, the diameter being given.*† (Fig. 48.)

Rule: Multiply the square of the diameter by $\frac{11}{14}$.‡

Example: Find the area of a circle ABCE (*see* Fig. 48) whose diameter EB is 8.

$$\text{Area} = 8^2 \times \frac{11}{14} = \frac{352}{7} = 50{\cdot}29.$$

38. *To find the diameter of a circle, the area being given.*§ (Fig. 48.)

Rule: Multiply the area by $\frac{14}{11}$‖ and extract the square root of the product.

Example: The area of a circle ABCE (*see* Fig. 48) is 3,927; find its diameter EB.

$$\text{Diameter} = \sqrt{3{,}927 \times \tfrac{14}{11}} = \sqrt{4{,}998} = 70{\cdot}7.$$

OBSERVATIONS.

* *To find the side, the area being given.* *Rule*: Divide the given area by the tabular area (from Art. 34), and extract the square root of the quotient.

† *To find the area, the circumference being given.* *Rule*: Multiply the square of the circumference by $\frac{7}{88}$. (*a*)

To find the area, the diameter and circumference being given. *Rule*: Multiply half the circumference by half the diameter.

‡ More correctly: 0·785398.

§ *To find the circumference, the area being given.* *Rule*: Multiply the area by $\frac{88}{7}$ (*b*), and extract the square root of the product.

‖ More correctly: 1·273240.

(*a*) More correctly: 0·079577. | (*b*) More correctly: 12·566371.

SECTORS AND SEGMENTS.

39. *To find the area of a sector of a circle, the length of the arc and radius being given.** (Fig. 216.)

Rule: Multiply half the length of the arc by the radius.

Example: The length of the arc ABC of a circle (*see* Fig. 216) is 25, and the radius AD, 35; required the area of the sector ABCD.

$$\text{Area} = \frac{25}{2} \times 35 = 437{\cdot}5.$$

40. *To find the area of the segment of a circle, the area of the sector and the area of the triangle formed by the chord of the segment and the radii of the circle, being given.*† (Fig. 204.)

Rule: Subtract the area of the triangle from the area of the sector.

Example: Find the area of the segment ABC of a circle (*see* Fig. 204), having given the chord AC = 12, and the height BE = 3.

$$\text{Now } AB = \sqrt{AE^2 + EB^2} = \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 6{\cdot}7082 \ (\textit{see } \text{Art. } 13),$$

$$\text{and } BF = \frac{AE^2}{BE} + BE = \frac{6^2}{3} + 3 = \frac{36}{3} + 3 = 15 \ (\textit{see } \text{Art. } 19),$$

$$\text{and length of arc } ABC = \frac{8 \times AB - AC}{3} = \frac{8 \times 6{\cdot}7082 - 12}{3} = \frac{53{\cdot}6656 - 12}{3} = \frac{41{\cdot}6656}{3} = 13{\cdot}8885 \ (\textit{see } \text{Art. } 24),$$

$$\therefore \text{area of sector } ABCD = \frac{\text{length of arc}}{2} \times AD = \frac{13{\cdot}8885}{2} \times \frac{15}{2} = \frac{208{\cdot}3275}{4} = 52{\cdot}082 \ (\textit{see } \text{Art. } 39);$$

$$\text{again } DE = DB - EB = 7{\cdot}5 - 3 = 4{\cdot}5,$$

$$\therefore \text{area of triangle } ADC = \frac{AC \times DE}{2} = \frac{12 \times 4{\cdot}5}{2} = 27 \ (\textit{see } \text{Art. } 27);$$

$$\therefore \text{area of segment} = \text{area of sector} - \text{area of triangle} = 52{\cdot}082 - 27 = 25{\cdot}082.$$

LUNES.

41. *To find the area of a lune the areas of the two segments upon the common chord forming the lune being given.* (Fig. 215.)

Rule: Subtract the area of the lesser segment from that of the greater.

Example: The length of chord AC = 40, the height ED = 4, and the height EB = 12; what is the area of the lune ABCDA (*see* Fig. 215)?

$$\text{Area of segment } ABCEA = \frac{2}{3}(BE \times AC) + \frac{BE^3}{2AC} = \frac{2}{3}(12 \times 40) + \frac{12^3}{2 \times 40} = \frac{2}{3} \times 480 + \frac{1{,}728}{80} = \frac{1{,}708}{5} = 341{\cdot}6$$ (*see* Note to Art. 40),

$$\text{and area of segment } ADCEA = \frac{2}{3}(DE \times AC) \times \frac{DE^3}{2AC} = \frac{2}{3}(4 \times 40) + \frac{4^3}{2 \times 40} = \frac{2}{3} \times 160 + \frac{64}{80} = 107{\cdot}47$$ (*see* Note to Art. 40),

$$\therefore \text{area of lune } ABCDA = \text{area of } ABCEA - \text{area of } ADCEA = 341{\cdot}6 - 107{\cdot}47 = 234{\cdot}13.$$

OBSERVATIONS.

* *To find the area, the number of degrees in the arc of the sector and the area of the circle being given.*

Rule: Multiply the degrees in the arc by the area of the circle, and divide the product by 360 degrees

To find the area of the circle, the area of the sector and the number of degrees in its arc being given.

Rule: Multiply the area of the sector by 360 degrees, and divide the product by the degrees in the arc.

† *To find the area, the height of the segment and the chord of the arc being given.* *Rule*: To two-thirds of the product of the height into the chord add the cube of the height divided by twice the chord.

ELLIPSES.

42. *To find the area of an ellipse, the transverse and conjugate diameters being given.* (Fig. 117.)

Rule: Multiply the product of the two diameters by $\frac{11}{14}$.*

Example: Find the area of an ellipse (*see* Fig. 117) whose transverse diameter AB is 7, and conjugate diameter CD, 17.

$$\text{Area} = 7 \times 17 \times \frac{11}{14} = \frac{187}{2} = 93{\cdot}5.$$

CURVILINEALS.

43. *To find the area of any curvilineal figure, the lengths of the ordinates and common distance being given.* (Fig. 206.)

Rule: Add together the first ordinate, the last ordinate, twice the sum of all the other odd ordinates, and four times the sum of all the other even ordinates; and multiply the result by one-third of the common distance between two adjacent ordinates.

Example: Find the area of the figure ABDC (*see* Fig. 206), having the ordinates of the following lengths: AB = 4, cf = 6, gh = 8, kl = 10, mn = 13, op = 12, qr = 9, st = 7, CD = 5; and the common distance between them 6.

$$\text{Area} = \left\{ (4+5) + 2\,(8+13+9) + 4\,(6+10+12+7) \right\} \times \frac{6}{3}$$

$$= (9+60+140) \times 2 = 209 \times 2 = 418.$$

CHAPTER III.

Plane Trigonometry.

DEFINITIONS.

1. PLANE TRIGONOMETRY.—*Plane Trigonometry* treats of the relations and calculations of the sides and angles of plane triangles.

2. PARTS OF A TRIANGLE.—The *parts of a triangle* are its three sides and its three angles; and when any three of these parts are given, except these be the three angles, the others can be found by calculation.

3. TRIGONOMETRICAL RATIOS.—The *trigonometrical ratios* of an angle are certain relations which exist between certain lines of a right-angled triangle connected with the angle, upon which principles the calculations are founded. They are the *sine*, *co-sine*, *tangent*, *co-tangent*, *secant*, *co-secant*, *versed sine*,† and *co-versed sine*† of that angle.

4. COMPLEMENT.—The *complement* of an angle is its difference from a right angle or 90°.

5. SUPPLEMENT.—The *supplement* of an angle is its difference from two right angles or 180°.

TRIGONOMETRICAL RATIOS.

6. RATIOS OF ANGLES A AND C. (Fig. 154). Let ABC be any right-angled triangle having the right angle at B, of which CB or a is the perpendicular, AB or c the base, and AC or b the hypotenuse, and BAC or A, ACB or C,

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OBSERVATIONS.

* More correctly: 0·785398.

† The terms *versed sine* and *co-versed sine* are scarcely ever used; hence no allusion is made to them hereafter.

CBA or B, its three angles respectively.[*] Then the following are the trigonometrical ratios of the angle A:—

$\frac{a}{b}$, or $\frac{\textit{perpendicular}}{\textit{hypotenuse}}$, is called the *sine* of the angle A;
$\frac{c}{b}$, ,, $\frac{\textit{base}}{\textit{hypotenuse}}$, ,, ,, *co-sine* ,, ,,;
$\frac{a}{c}$, ,, $\frac{\textit{perpendicular}}{\textit{base}}$, ,, ,, *tangent* ,, ,,;
$\frac{c}{a}$, ,, $\frac{\textit{base}}{\textit{perpendicular}}$, ,, ,, *co-tangent* ,, ,,;
$\frac{b}{c}$, ,, $\frac{\textit{hypotenuse}}{\textit{base}}$, ,, ,, *secant* ,, ,,;
$\frac{b}{a}$, ,, $\frac{\textit{hypotenuse}}{\textit{perpendicular}}$, ,, ,, *co-secant* ,, ,,;

Also, the trigonometrical ratios of the angle C will be as follows:—

$\frac{c}{b}$, or $\frac{\textit{perpendicular}}{\textit{hypotenuse}}$, is called the sine of the angle C;
$\frac{a}{b}$, ,, $\frac{\textit{base}}{\textit{hypotenuse}}$, ,, ,, *co-sine* ,, ,,;
$\frac{c}{a}$, ,, $\frac{\textit{perpendicular}}{\textit{base}}$, ,, ,, *tangent* ,, ,,;
$\frac{a}{c}$, ,, $\frac{\textit{base}}{\textit{perpendicular}}$, ,, ,, *co-tangent* ,, ,,;
$\frac{b}{a}$, ,, $\frac{\textit{hypotenuse}}{\textit{base}}$, ,, ,, *secant* ,, ,,;
$\frac{b}{c}$, ,, $\frac{\textit{hypotenuse}}{\textit{perpendicular}}$, ,, ,, *co-secant* ,, ,,;

7. DEFINITIONS ABBREVIATED.—The above definitions are usually abbreviated and written thus:—

$$\begin{array}{llll} \textit{sin.}\ A=\frac{a}{b}, & \textit{cos.}\ A=\frac{c}{b}, & \textit{sin.}\ C=\frac{c}{b}, & \textit{cos.}\ C=\frac{a}{b}, \\ \textit{tan.}\ A=\frac{a}{c}, & \textit{cot.}\ A=\frac{c}{a}, & \textit{tan.}\ C=\frac{c}{a}, & \textit{cot.}\ C=\frac{a}{c}, \\ \textit{sec.}\ A=\frac{b}{c}, & \textit{cosec.}\ A=\frac{b}{a}. & \textit{sec.}\ C=\frac{b}{a}, & \textit{cosec.}\ C=\frac{b}{c}. \end{array}$$

8. ANGLES A AND B IN TERMS OF EACH OTHER.—From (Art. 7) it will be seen that—

$$\begin{array}{ll} \textit{sin.}\ A=\frac{a}{b}=\ \textit{cos.}\ C; & \textit{cos.}\ A=\frac{c}{b}=\textit{sin.}\ C; \\ \textit{tan.}\ A=\frac{a}{c}=\ \textit{cot.}\ C; & \textit{cot.}\ A=\frac{c}{a}=\textit{tan.}\ C; \\ \textit{sec.}\ A=\frac{b}{c}=\textit{cosec.}\ C; & \textit{cosec.}\ A=\frac{b}{a}=\textit{sec.}\ C; \end{array}$$

and *vice versâ*; also the angle A is the complement of the angle C, and *vice versâ*. Hence:

the sine of an angle =the cosine of its complement, or sin. A=cos. (90°—A),
,, tangent ,, = ,, co-tangent ,, ,, tan. A=cot. (90°—A),
,, secant ,, = ,, co-secant ,, ,, sec. A=cosec.(90°—A),
,, co-sine ,, = ,, sine ,, ,, cos. A=sin. (90°—A),
,, co-tangent ,, = ,, tangent ,, ,, cot. A=tan. (90°—A),
,, co-secant ,, = ,, secant ,, ,,cosec.A=sec. (90°—A).

9. RATIOS CHANGE WITH ANGLE. (Fig. 152.) So long as the angle remains unchanged, the trigonometrical ratios continue the same; for instance,

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OBSERVATION.

[*] To express an angle of any right-angled triangle, in terms of its sides, the side *adjacent* to the given angle is assumed as the *base*, and the side *opposite* to the given angle as the *perpendicular*.

let BAC be any angle; from any two points P, P′, let drop PQ, P′Q′ perpendiculars on AB. Then, by similar triangles, $\frac{PQ}{AQ} = \frac{P'Q'}{AQ'}$; that is, the *sine* of the angle A is the same whether it be formed from the triangle APQ or from the triangle AP′Q′. The same result holds for the other trigonometrical ratios.

10. VALUES OF RATIOS. To obtain a trigonometrical value of each of the trigonometrical ratios, proceed thus, from the definitions given in Art. 7:—

$$\sin. A \times \text{cosec.}\ A = \frac{a}{b} \times \frac{b}{a} = 1,\ \therefore \sin. A = \frac{1}{\text{cosec.}\ A},\quad \text{cosec.}\ A = \frac{1}{\sin. A};$$

$$\cos. A \times \sec. A = \frac{c}{b} \times \frac{b}{c} = 1,\ \therefore \cos. A = \frac{1}{\sec. A},\quad \sec. A = \frac{1}{\cos. A};$$

$$\tan. A \times \cot. A = \frac{a}{c} \times \frac{c}{a} = 1,\ \therefore \tan. A = \frac{1}{\cot. A},\quad \cot. A = \frac{1}{\tan. A};$$

$$\text{Also } \tan. A = \frac{a}{c} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{\sin. A}{\cos A};\quad \cot. A = \frac{c}{a} = \frac{\frac{c}{b}}{\frac{a}{b}} = \frac{\cos. A}{\sin. A}.$$

CHANGES IN THE RATIOS AS THE ANGLE CHANGES.

11. ANGLE IN EACH OF THE FOUR QUADRANTS. (Fig. 157.) With a centre A and any radius describe a circle BCB′C′; through A draw the diameters BB′, CC′, at right angles to each other, and consequently dividing the circle into four quadrants, BAC being the *first quadrant*, CAB′ the *second quadrant*, B′AC′ the *third quadrant*, and C′AB the *fourth quadrant*. In each of the four quadrants draw any radius AP; then the angle BAP′ is said to be in the first quadrant, the angle BAP″ in the second quadrant, the angle BAP‴ in the third quadrant, and the angle BAP⁗ in the fourth quadrant.

12. VARIATION OF VALUES OF RATIOS. (Fig. 491.) In order to trace the variation of the values of the trigonometrical ratios in the four quadrants, draw two straight lines BB′, and CC′, cutting each other at right angles in A; let a straight line turn round the point A from right to left, commencing at and closing on the line AB; and let it come into any position AP in each of the four quadrants; draw PM perpendicular to BB′. Let the angle BAP be denoted by A; then the trigonometrical ratios of A in each of the four quadrants are thus defined:—

$$\sin. A = \frac{PM}{AP},\qquad \tan. A = \frac{PM}{AM},\qquad \sec. A = \frac{AP}{AM},$$

$$\cos. A = \frac{AM}{AP},\qquad \cot. A = \frac{AM}{PM},\qquad \text{cosec.}A = \frac{AP}{PM}.$$

It will thus be seen that the relations established in (Art. 10) between the trigonometrical ratios of angles not exceeding a right angle, hold universally between the trigonometrical ratios of angles of any magnitude.

13. *Angles of the first quadrant.* (Fig 492.)
Now, as in (Art. 12),

$$\sin. \quad (0^\circ + A) = \sin.BAP = \frac{PM}{AP} = \sin. \quad A,$$

$$\cos. \quad (0^\circ + A) = \cos \quad ,, = \frac{AM}{AP} = \cos. \quad A,$$

$$\tan. \quad (0^\circ + A) = \tan \quad ,, = \frac{PM}{AM} = \tan. \quad A,$$

$$\cot. \quad (0^\circ + A) = \cot. \quad ,, = \frac{AM}{PM} = \cot. \quad A,$$

$$\sec. \quad (0^\circ + A) = \sec. \quad ,, = \frac{AP}{AM} = \sec. \quad A,$$

$$\text{cosec.}(0^\circ + A) = \text{cosec.} \quad ,, = \frac{AP}{PM} = \text{cosec.}\ A.$$

14. *Angles of the second quadrant.* (Fig. 493.) Let the angle CAP be called A; draw PM′ parallel to BB′; then the angle BAP = 90° + A.

Now, as in (Art. 12),

$$\text{sin.}\quad (90^\circ+A) = \text{sin. BAP} = \frac{PM'}{AP} = \frac{AM'}{AP} = \text{cos. } A,$$

$$\text{cos.}\quad (90^\circ+A) = \text{cos.}\quad ,, \quad = \frac{AM}{AP} = \frac{PM'}{AP} = \text{sin. } A,$$

$$\text{tan.}\quad (90^\circ+A) = \text{tan.}\quad ,, \quad = \frac{PM}{AM} = \frac{AM'}{PM'} = \text{cot. } A,$$

$$\text{cot.}\quad (90^\circ+A) = \text{cot.}\quad ,, \quad = \frac{AM}{PM} = \frac{PM'}{AM'} = \text{tan. } A,$$

$$\text{sec.}\quad (90^\circ+A) = \text{sec.}\quad ,, \quad = \frac{AP}{AM} = \frac{AP}{PM'} = \text{cosec. } A,$$

$$\text{cosec.}\ (90^\circ+A) = \text{cosec.}\ ,, \quad = \frac{AP}{PM} = \frac{AP}{AM'} = \text{sec. } A.$$

15. *Angles of the third quadrant.* (Fig. 494.) Let the angle B′AP be called A; then the angle BAP = 180° + A.

Now, as in (Art. 12),

$$\text{sin.}\quad (180^\circ+A) = \text{sin. } B'AP = \frac{PM}{AP} = \text{sin. } A,$$

$$\text{cos.}\quad (180^\circ+A) = \text{cos.}\quad ,, \quad = \frac{AM}{AP} = \text{cos. } A,$$

$$\text{tan.}\quad (180^\circ+A) = \text{tan.}\quad ,, \quad = \frac{PM}{AM} = \text{tan. } A,$$

$$\text{cot.}\quad (180^\circ+A) = \text{cot.}\quad ,, \quad = \frac{AM}{PM} = \text{cot. } A,$$

$$\text{sec.}\quad (180^\circ+A) = \text{sec.}\quad ,, \quad = \frac{AP}{AM} = \text{sec. } A,$$

$$\text{cosec.}(180^\circ+A) = \text{cosec.}\ ,, \quad = \frac{AP}{PM} = \text{cosec. } A.$$

16. *Angles of the fourth quadrant.* (Fig. 495.) Let the angle C′AP be called A; draw PM′ parallel to BB′; then the angle BAP=270°+A.

Now, as in (Art. 12),

$$\text{sin.}\quad (270^\circ+A) = \text{sin. BAP} = \frac{PM}{AP} = \frac{AM'}{AP} = \text{cos. } A,$$

$$\text{cos.}\quad (270^\circ+A) = \text{cos.}\quad ,, \quad = \frac{AM}{AP} = \frac{PM'}{AP} = \text{sin. } A,$$

$$\text{tan.}\quad (270^\circ+A) = \text{tan.}\quad ,, \quad = \frac{PM}{AM} = \frac{AM'}{PM'} = \text{cot. } A,$$

$$\text{cot.}\quad (270^\circ+A) = \text{cot.}\quad ,, \quad = \frac{AM}{PM} = \frac{PM'}{AM'} = \text{tan. } A,$$

$$\text{sec.}\quad (270^\circ+A) = \text{sec.}\quad ,, \quad = \frac{AP}{AM} = \frac{AP}{PM'} = \text{cosec. } A,$$

$$\text{cosec.}\ (270^\circ+A) = \text{cosec.}\ ,, \quad = \frac{AP}{PM} = \frac{AP}{AM'} = \text{sec. } A.$$

PROPERTIES OF TRIANGLES.

17. PROBLEM I. *In any triangle the sides are proportional to the sines of the opposite angles.* (Fig. 155.) Let ABC be a triangle, having the three angles A, B, C; calling the sides opposite to these angles respectively a, b, and c. Draw BD or p perpendicular to AC. Then from the right-angled triangles ABD and

CBD, we have by (Art. 6) sin. $A = \frac{p}{c}$, and sin. $C = \frac{p}{a}$; and dividing the former by the latter, we have

$$\frac{\sin. A}{\sin. C} = \frac{\frac{p}{c}}{\frac{p}{a}} = \frac{a}{c},$$

similarly, $\frac{\sin. A}{\sin. B} = \frac{a}{b}$, and $\frac{\sin. B}{\sin. C} = \frac{b}{c}$;

that is, the sides are proportional to the sines of the opposite angles. This rule holds equally good for obtuse-angled triangles, right-angled triangles, or acute-angled triangles.

The above results may be written symmetrically thus: $\frac{\sin. A}{a} = \frac{\sin. B}{b} = \frac{\sin. C}{c}$.

18. PROBLEM II. *In any triangle of which two sides and the angle contained between them are known, the sum of the two known sides is to their difference as the tangent of half the sum of the unknown angles to the tangent of half their difference.* (Fig. 156.) Let ABC be any triangle; then by (Art. 17) $\frac{a}{b} = \frac{\sin. A}{\sin. B}$, and algebraically, $\frac{a+b}{a-b} = \frac{\sin. A + \sin. B}{\sin. A - \sin. B} = \frac{\tan. \frac{1}{2}(A+B)}{\tan. \frac{1}{2}(A-B)}$.*

SOLUTION OF RIGHT-ANGLED TRIANGLES.

19. *Four cases of Right-Angled Triangles.* (Fig. 154.) Let ABC be a right-angled triangle, having the right angle at B; then one of its acute angles being known, the other can be found by subtracting it from 90°. In the solution of a right-angled triangle the following four cases occur, where two of the elements are given and the others can be calculated out.—

1st case.—Given the hypotenuse and an acute angle;
2nd case.—Given a side and an acute angle;
3rd case.—Given the hypotenuse and a side;
4th case.—Given the two sides.

20. *Solution of the four cases.* (Fig. 154.) Solve the four cases above given, as follows:—

1st case.—Given b and A.

$C = 90° - A$;

By (Art. 7), $\frac{a}{b} = \sin. A, \therefore a = b. \sin. A$;

and $\frac{c}{b} = \sin. C, \therefore c = b. \sin. C$.

2nd case.— Given c and C.

$A = 90° - A$;

By (Art. 7), $\frac{c}{b} = \sin. C, \therefore b = \frac{c}{\sin. C}$;

and $\frac{c}{a} = \tan. C, \therefore a = \frac{c}{\tan. C}$.

3rd case.—Given b and c.

By (Art. 7), $\sin. C = \frac{c}{b}$; $A = 90° - C$;

and $\frac{a}{b} = \cos. A, \therefore a = b. \cos. A$.

OBSERVATION.

* This result is obtained by certain rules of trigonometry with which the student is not expected to be familiar. The above Chapter is designedly very elementary.

4th case.—Given a and c.

$$\text{By (Art. 7), tan. } C = \frac{c}{a}\text{ ; } A = 90^\circ - C\text{ ;}$$

$$\text{and } \frac{c}{b} = \text{sin. } C,\ \therefore b = \frac{c}{\text{sin. } C}.$$

21. *1st case.* (Fig. 154.) *Example*: Given hypotenuse b = 482, and angle A = 57° 13′; find the rest.

$$\begin{aligned}
&C = 90^\circ - A = 90^\circ - 57^\circ\ 13' = 32^\circ 47';\\
&\text{By (Art. 20), } a = b.\ \text{sin. } A,\\
&\therefore \text{by logarithms,* log. } a = \text{log. } b + \text{log. sin. } A = \text{log. } b + \text{L. sin. } A - 10\\
&\qquad = \text{log. } 482 + \text{L. sin. } 57^\circ\ 13' - 10\\
&\qquad = 2\cdot683047 + 9\cdot924654 - 10\\
&\qquad = 2\cdot607701 = \text{log. } 405\cdot2,\\
&\therefore a = 405\cdot2\ ;\\
&\text{and } c = b.\ \text{sin } C,\\
&\therefore \text{by logs., log. } c = \text{log. } b + \text{log. sin. } C. = \text{log. } b + \text{L. sin. } C - 10\\
&\qquad = \text{log. } 482 + \text{L. sin. } 32^\circ\ 47' - 10\\
&\qquad = 2\cdot683047 + 9\cdot733569 - 10\\
&\qquad = 2\cdot416616 = \text{log. } 261\cdot0,\\
&\therefore c = 261\cdot0.
\end{aligned}$$

22. *2nd case.* (Fig. 154.) *Example*: Given side c = 49, and angle C = 13° 51′; find the rest.

$$\begin{aligned}
&A = 90^\circ - C = 90^\circ - 13^\circ\ 51' = 76^\circ\ 9';\\
&\text{By (Art. 20), } b = \frac{c}{\text{sin. } C}.\\
&\therefore \text{by logs., log. } b = \text{log. } c - \text{log. sin. } C = \text{log. } c - \text{L. sin. } C + 10\\
&\qquad = \text{log. } 49 - \text{L. sin. } 13^\circ\ 51' + 10\\
&\qquad = 1\cdot690196 - 9\cdot379089 + 10\\
&\qquad = 2\cdot311107 = \text{log. } 204\cdot7,\\
&\therefore b = 204\cdot7\ ;\\
&\text{and } a = \frac{c}{\text{tan. } C}.\\
&\therefore \text{by logs., log. } a = \text{log. } c - \text{log. tan } C. = \text{log. } c - \text{L. tan. } C + 10\\
&\qquad = \text{log. } 49 - \text{L. tan. } 13^\circ 51' + 10\\
&\qquad = 1\cdot690196 - 9\cdot391903 + 10\\
&\qquad = 2\cdot298293 = \text{log. } 198\cdot8,\\
&\therefore a = 198\cdot8.
\end{aligned}$$

23. *3rd case.* (Fig. 154.) *Example*: Given hypotenuse b = 526·7, side c = 328·2, and angle B = 90°; find the rest.

$$\begin{aligned}
&\text{By (Art. 20), sin. } C = \frac{c}{b},\\
&\therefore \text{by logs., log. sin. } C = \text{log. } c - \text{log. } b,\\
&\therefore \text{L. sin. } C = 10 + \text{log. } c - \text{log. } b\\
&\qquad = 10 + \text{log. } 328\cdot2 - \text{log. } 526\cdot7\\
&\qquad = 10 + 2\cdot516139 - 2\cdot721563\\
&\qquad = 9\cdot794576 = \text{L. sin. } 38^\circ\ 32'\ 41'',\\
&\therefore C = 38^\circ\ 32'\ 41'';\\
&A = 90^\circ - C = 90^\circ - 38^\circ\ 32'\ 41'' = 51^\circ\ 27'\ 19'';\\
&\text{and } a = b.\ \text{cos. } C,\\
&\therefore \text{by logs., log. } a = \text{log. } b + \text{log. cos. } C = \text{log. } b + \text{L. cos. } C - 10\\
&\qquad = \text{log. } 526\cdot7 + \text{L. cos. } 38^\circ\ 32'\ 41'' - 10\\
&\qquad = 2\cdot721563 + 9\cdot893274 - 10\\
&\qquad = 2\cdot614837 = \text{log } 411\cdot9,\\
&\therefore a = 411\cdot9.
\end{aligned}$$

OBSERVATION.

* For the use of logarithms, *see* Chapter IV.

24. *4th case.* (Fig. 154.) *Example*: Given side a = 5,268·4, side c = 3,950·1, and angle B = 90°; find the rest.

$$
\begin{aligned}
&\text{By (Art. 20), } \tan.\ C = \frac{c}{a},\\
&\therefore \text{ by logs., } \log.\ \tan.\ C = \log.\ c - \log.\ a,\\
&\qquad \therefore\ L.\ \tan.\ C = 10 + \log.\ c - \log.\ a\\
&\qquad\qquad = 10 + \log.\ 3{,}950{\cdot}1 - \log.\ 5{,}268{\cdot}4\\
&\qquad\qquad = 10 + 3{\cdot}596608 - 3{\cdot}721679\\
&\qquad\qquad = 9{\cdot}874929 = \tan.\ 36^\circ\ 51'\ 41'',\\
&\qquad \therefore\ C = 36^\circ\ 51'\ 41'';\\
&A = 90^\circ - C = 90^\circ - 36^\circ\ 51'\ 41'' = 53^\circ\ 08'\ 19'';\\
&\text{and } b = \frac{c}{\sin.\ C},\\
&\therefore \text{ by logs., } \log.\ b = \log.\ c - \log.\ \sin.\ C = \log.\ c - L.\ \sin.\ C. + 10\\
&\qquad = \log.\ 3{,}950{\cdot}1 - L.\ \sin.\ 36^\circ\ 51'\ 41'' + 10\\
&\qquad = 3{\cdot}596608 - 9{\cdot}778065 + 10\\
&\qquad = 3{\cdot}818543 = \log.\ 6{,}584{\cdot}8,\\
&\qquad \therefore\ b = 6{,}584{\cdot}8.
\end{aligned}
$$

SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

25. *Four cases of Oblique-Angled Triangles.* (Fig. 156.) Let ABC be an oblique-angled triangle, of which if two angles be known, the third can be found by substracting their sum from 180°. In the solution of an oblique-angled triangle, the following four cases occur, where three of the elements are given and the others calculated out :—

1st case.—Given two angles and a side;

2nd case.—Given two sides and the angle opposite to one of them;

3rd case.—Given two sides and the included angle;

4th case.—Given the three sides.

26. *Solution of the four cases.* (Fig. 156.) Solve the four cases above given as follows :—

1st case.—Given A, C, and b.

Rule: As the unknown side is to the known side, so is the sine of the angle opposite to the unknown side to the sine of the angle opposite the known side. (*See* Art. 17.)

$$
\begin{aligned}
&B = 180^\circ - (A + C);\\
&\text{By (Art. 17), } \frac{a}{b} = \frac{\sin.\ A}{\sin.\ B},\ \therefore\ a = b.\frac{\sin.\ A}{\sin.\ B};\\
&\qquad \text{and } \frac{c}{b} = \frac{\sin.\ C}{\sin.\ B},\ \therefore\ c = b.\frac{\sin.\ C}{\sin.\ B}.
\end{aligned}
$$

2nd case.—Given a, b, and B.

Rule: As the side opposite the unknown angle is to the side opposite the known angle, so is the sine of the unknown angle to the sine of the known angle. (*See* Art. 17.)

$$
\begin{aligned}
&\text{By (Art. 17), } \frac{a}{b} = \frac{\sin.\ A}{\sin.\ B},\ \therefore\ \sin.\ A = \frac{a}{b}.\sin.\ B;\\
&\qquad C = 180^\circ - (A + B);\\
&\qquad \text{and } \frac{c}{b} = \frac{\sin.\ C}{\sin.\ B},\ \therefore\ c = b.\frac{\sin.\ C}{\sin.\ B}.
\end{aligned}
$$

3rd case.—Given a, c, and B.

Rule: As the sum of the two known sides is to their difference, so is the tangent of half the sum of the unknown angles to the tangent of half their difference. Half the difference of the two angles thus found, added to half their sum, gives the greater; and taken from half their sum, gives the less. (*See* Art. 18.)

$$
\begin{aligned}
&\text{Now } A + B + C = 180^\circ,\ \therefore\ A + C = 180^\circ - B;\\
&\text{and by (Art. 18), } \frac{a+c}{a-c} = \frac{\tan.\ \frac{1}{2}(A+C)}{\tan.\ \frac{1}{2}(A-C)},\\
&\therefore\ \tan.\ \tfrac{1}{2}(A - C) = \frac{a-c}{a+c}\ \tan.\ \tfrac{1}{2}(A + C), \text{ whence } \tfrac{1}{2}(A - C) \text{ is found.}
\end{aligned}
$$

Again $\frac{1}{2}(A + C) + \frac{1}{2}(A - C)$ = greater angle (A or C),
and $\frac{1}{2}(A + C) - \frac{1}{2}(A - C)$ = lesser angle (A or C).

4th case.—Given a, b, and c.

Rule: The tangent of half the unknown angle is equal to the square root of the following fraction, whose numerator is the product of the differences of each of its adjacent sides from half the sum of the three sides, and whose denominator is the product of half the sum of the three sides with the difference of its opposite side from half the sum of the three sides.

* $\tan. \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, whence A is found;

$\tan. \frac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$, ,, B ,, ,, ;

$\tan. \frac{1}{2} C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$, ,, C ,, ,, .

27. *1st case.* (Fig. 156.) *Example*: Given angle A = 65° 27′, angle C = 48° 31′, and side b = 293; find the rest.

B = 180° − (A + C) = 180° − (65° 27′ + 48° 31′) = 180° − 113° 58′ = 66° 02′.

By (Art. 26), $a = b. \frac{\sin. A}{\sin. B}$,

∴ by logs., log. a = log. b + log. sin. A − log. sin. B
= log. b + L. sin. A − 10 − L. sin. B + 10
= log. 293 + L. sin. 65° 27′ − L. sin. 66° 02′
= 2·466868 + 9·958850 − 9·960843
= 2·464875 = log. 291·7,
∴ a = 291·7;

and $c = b. \frac{\sin. C}{\sin. B}$,

∴ by logs., log. c = log. b + log. sin. C − log. sin. B
= log. b + L. sin. C − 10 − L. sin. B + 10
= log. 293 + L. sin. 48° 31′ − L. sin. 66° 02′
= 2·466868 + 9·874568 − 9·960843
= 2·380593 = log. 240·2
∴ c = 240·2.

28. *2nd case.* (Fig. 156.) *Example*: Given side a = 247, side b = 515, and angle B = 123° 36′; find the rest.

By (Art. 26), $\sin. A = \frac{a}{b}. \sin. B$,

∴ by logs., log. sin. A = log. a + log. sin. B − log. b
∴ L. sin. A = 10 + log. a + L. sin. B − 10 − log. b
= log. 247 + L. sin. 123° 36′ − log. 515
= 2·392697 + 9·920604 − 2·711807
= 9·601494 = L. sin. 23° 32′ 44″,
∴ A = 23° 32′ 44″;
C = 180° − (A + B) = 110° − (23° 32′ 44″ + 123° 36′)
= 180° − 147° 08′ 44″ = 32° 51′ 16″;

and $c = b. \frac{\sin. C}{\sin. B}$,

∴ by logs., log. c = log. b + log. sin. C − log. sin. B
= log. b + L. sin. C − 10 − L. sin. B + 10
= log. 515 + L. sin. 32° 51′ 16″ − L. sin. 123° 36′
= 2·711807 + 9·734405 − 9·920604
= 2·525608 = log. 335·4,
∴ c = 335·4.

29. *3rd case.* (Fig. 156.) *Example*: Given side a = 423·8, side c = 350·1, and angle B = 49° 03′ 42″; find the rest.

OBSERVATION.

* These results are arrived at by the application of the more advanced rules of trigonometry.

By (Art. 26), $A + C = 180^\circ - B = 180^\circ - 49^\circ\,03'\,42'' = 130^\circ\,56'\,18''$, and

$$\tan.\ \tfrac{1}{2}(A - C) = \frac{a - c}{a + c}.\ \tan.\ \tfrac{1}{2}(A + C),$$

$\therefore$ by logs., $\log.\ \tan.\ \tfrac{1}{2}(A - C)$
$= \log.\ (a - c) + \log.\ \tan.\ \tfrac{1}{2}(A + C) - \log.\ (a + c)$

$\therefore$ $L.\ \tan.\ \tfrac{1}{2}(A - C)$
$= 10 + \log.\ (a - c) + L.\ \tan.\ \tfrac{1}{2}(A + C) - 10 - \log.\ (a + c)$
$= \log.\ (423{\cdot}8 - 350{\cdot}1) + L.\ \tan.\ \tfrac{1}{2}(130^\circ\,56'\,18'') - \log.\ (423{\cdot}8 + 350{\cdot}1)$
$= \log.\ 73{\cdot}7 + L.\ \tan.\ 65^\circ\,28'\,09'' - \log.\ 773{\cdot}9$
$= 1{\cdot}867467 + 10{\cdot}340677 - 2{\cdot}888685$
$= 9{\cdot}319459 = L.\ \tan.\ 11^\circ\,47'\,12''$,

$\therefore \tfrac{1}{2}(A - C) = 11^\circ\,47'\,12''$.

Again, by (Art. 26), greater angle (or A) $= \tfrac{1}{2}(A + C) + \tfrac{1}{2}(A - C)$
$= 65^\circ\,28'\,09'' + 11^\circ\,47'\,12'' = 77^\circ\,15'\,21''$,
and lesser angle (or C) $= \tfrac{1}{2}(A + C) - \tfrac{1}{2}(A - C)$
$= 65^\circ\,28'\,09'' - 11^\circ\,47'\,12'' = 53^\circ\,40'\,57''$.

Next, by (Art. 26), $b = c.\ \dfrac{\sin.\ B}{\sin.\ C}$,

$\therefore$ by logs., $\log.\ b = \log.\ c + \log.\ \sin.\ B - \log.\ \sin.\ C$
$= \log.\ c + L.\ \sin.\ B - 10 - L.\ \sin.\ C + 10$
$= \log.\ 350{\cdot}1 + L.\ \sin.\ 49^\circ\,03'\,42'' - L.\ \sin.\ 53^\circ\,40'\,57''$
$= 2{\cdot}544192 + 9{\cdot}878186 - 9{\cdot}906199$
$= 2{\cdot}516179 = \log.\ 328{\cdot}2$,
$\therefore b = 328{\cdot}2$.

30. *4th case.* (Fig. 156.) *Example*: Given side $a = 222$, side $b = 318$, and side $c = 406$; find the rest.

Now s (or half the sum of the three sides) $= \dfrac{a + b + c}{2} = \dfrac{222 + 318 + 406}{2} = 473$,
and $s - a = 473 - 222 = 251$,
$s - b = 473 - 318 = 155$,
$s - c = 473 - 406 = 67$.

Then, by (Art. 26), $\tan.\ \tfrac{1}{2}A = \sqrt{\dfrac{(s - b)(s - c)}{s(s - a)}} = \sqrt{\dfrac{155 \times 67}{473 \times 251}}$,

$\therefore$ by logs., $\log.\ \tan.\ \tfrac{1}{2}A = \tfrac{1}{2}(\log.\ 155 + \log.\ 67 - \log.\ 473 - \log.\ 251)$,
and $L.\ \tan.\ \tfrac{1}{2}A = 10 + \tfrac{1}{2}(\log.\ 155 + \log.\ 67 - \log.\ 473 - \log.\ 251)$
$= 10 + \tfrac{1}{2}(2{\cdot}190332 + 1{\cdot}826075 - 2{\cdot}674861 - 2{\cdot}399674)$
$= 10 + \tfrac{1}{2} \times \bar{2}{\cdot}941872 = 10 + \bar{1}{\cdot}470936$
$= 9{\cdot}470936 = L.\ \tan.\ 16^\circ\,28'\,34''$,
$\therefore \tfrac{1}{2}A = 16^\circ\,28'\,34''$, and $A = 32^\circ\,57'\,08''$.

Again, $\tan.\ \tfrac{1}{2}B = \sqrt{\dfrac{(s - a)(s - c)}{s(s - b)}} = \sqrt{\dfrac{251 \times 67}{473 \times 155}}$,

$\therefore$ by logs., $\log.\ \tan.\ \tfrac{1}{2}B = \tfrac{1}{2}(\log.\ 251 + \log.\ 67 - \log.\ 473 - \log.\ 155)$,
and $L.\ \tan.\ \tfrac{1}{2}B = 10 + \tfrac{1}{2}(\log.\ 251 + \log.\ 67 - \log.\ 473 - \log.\ 155)$
$= 10 + \tfrac{1}{2}(2{\cdot}399674 + 1{\cdot}826075 - 2{\cdot}674861 - 2{\cdot}190332)$
$= 10 + \tfrac{1}{2} \times \bar{1}{\cdot}360556 = 10 + \bar{1}{\cdot}680278$
$= 9{\cdot}680278 = L.\ \tan.\ 25^\circ\,35'\,29''$,
$\therefore \tfrac{1}{2}B = 25^\circ\,35'\,29''$, and $B = 51^\circ\,10'\,58''$.

Again, $\tan.\ \tfrac{1}{2}C = \sqrt{\dfrac{(s - a)(s - b)}{s(s - c)}} = \sqrt{\dfrac{251 \times 155}{473 \times 67}}$,

$$\begin{aligned}
\therefore \text{ by logs., log. tan. } \tfrac{1}{2}\text{C} &= \tfrac{1}{2}\,(\text{log. } 251 + \text{log. } 155 - \text{log. } 473 - 67),\\
\text{and L. tan. } \tfrac{1}{2}\text{C} &= 10 + \tfrac{1}{2}\,(\text{log. } 251 + \text{log. } 155 - \text{log. } 473 - \text{log. } 67)\\
&= 10 + \tfrac{1}{2}\,(2{\cdot}399674 + 2{\cdot}190332 - 2{\cdot}674861 - 1{\cdot}826075)\\
&= 10 + \tfrac{1}{2} \times 0{\cdot}089070 = 10 + 0{\cdot}044535\\
&= 10{\cdot}044535 = \text{L. tan. } 47^\circ\, 55'\, 57'',\\
\therefore \tfrac{1}{2}\text{C} &= 47^\circ\, 55'\, 57'', \text{ and C} = 95^\circ\, 51'\, 54''.^{*}
\end{aligned}$$

AREAS OF TRIANGLES.

31. PROBLEM I. *To find the area of a triangle, having given two sides and the included angle.* (Fig. 155.)

Rule: Multiply half the product of the two sides by the sine of the included angle.

For, let ABC be any triangle, and BD the perpendicular from B on the base AC.

Then, by (Chap. II, Art. 27), the area = ½ AC. BD; and by (Art. 7), $\frac{\text{BD}}{\text{AB}}$ = sin. BAC,

∴ BD = AB. sin. BAC;

thus, the area = ½ AC. AB. sin. BAC or ½ b. c. sin. A.

32. *Example*: (Fig. 155.) Given side b = 28, side c = 36, and angle A = 63° 47′; find the area of the triangle.

$$\begin{aligned}
\text{By (Art. 31), Area} &= \tfrac{1}{2}\text{ b. c. sin. A} = \tfrac{1}{2} \times 28 \times 36 \times \sin 63^\circ\, 47'\\
&= 14 \times 36 \times \text{sin. } 63^\circ\, 47',\\
\therefore \text{ by logs., log. area} &= \text{log. } 14 + \text{log. } 36 + \text{log. sin. } 63^\circ\, 47'\\
&= \text{log. } 14 + \text{log. } 36 + \text{L. sin. } 63^\circ\, 47' - 10\\
&= 1{\cdot}146128 + 1{\cdot}556303 + 9{\cdot}952855 - 10\\
&= 2{\cdot}655286 = \text{log. } 452{\cdot}2,\\
\therefore \text{area} &= 452{\cdot}2.
\end{aligned}$$

HEIGHTS AND DISTANCES.

The following examples in Heights and Distances show the practical application to surveying, of the rules given for the solution of right-angled and oblique-angled triangles. (*See* Arts. 19—30.)

33. (1) *To find the height of an accessible object.* (Fig. 162.)

Let BC be an object accessible at its base, the height of which is required.

From its base measure a horizontal line AB of any convenient length, and at A observe the vertical angle BAC to the top of the object.

Then, by (Art. 20, 2nd case), BC = AB. tan. BAC.

Example: At a distance of 130 feet from the foot of a building, the angle of elevation of its top was found to be 61° 20′; required its height.

Ans. 237·78 feet.

34. (2) *To find the height of an inaccessible object.* (Fig. 158.)

Let CDE be an object inaccessible at its base, the height of which is required.

Measure a horizontal line AB in the direction of B, and at A and B observe the vertical angles DAC, DBC, to the top of the object.

Now, in the triangle ABC, angle ACB = DBC − BAC (*see* Chap. I, Art. 71), and, in the same triangle, by (Art. 25, 1st case), BC = AB. $\frac{\text{sin. BAC}}{\text{sin. ACB}}$; hence, in the triangle BCD, by (Art. 20, 1st case), DC = BC. sin. DBC.

OBSERVATIONS.

* To prove the accuracy of the results, sum up the three angles which should equal 180°; thus A + B + C = 32° 57′ 8″ + 51° 10′ 58″ + 95° 51′ 54″ = 180° 0′ 0″.

Also, the first two angles having been obtained, the third could have been deduced by substracting the sum of the two found angles from 180°; thus, C = 180° — (A + B) = 180° — (32° 57′ 08″ + 51° 10′ 58″) = 180° — 84° 08′ 06″ = 95° 51′ 54″.

Example: In order to find the height of a hill, a base line of 280 feet was measured in its direction, and the angles of elevation of the top of the hill at the extremities of the base were 28° and 49°; required its height.

Ans. 276·8 feet.

35. (3) *To find the height of an object situated on an inaccessible height.* (Fig. 159.)

Let ED be a hill inaccessible at its base, and CE an object, the height of which is required, situated on it.

Measure a horizontal line AB in the direction of the hill; at B observe the vertical angles DBC, DBE, to the top and bottom of the object, and at A observe the vertical angle DAC to the top of the object.

Now, in the triangle ABC, angle ACB = DBC — BAC (*see* Chap. I, Art. 71), and, in the same triangle, by (Art. 25, 1st case), $BC = AB.\frac{\sin. BAC}{\sin. ACB}$.

Again, in the triangle CBE, angle BEC = 90° + DBE, and angle CBE = DBC — DBE; hence, in the same triangle, by (Art. 25, 1st case), $CE = BC.\frac{\sin. CBE}{\sin. BEC}$.

Example: Find the height of a fort situated on the summit of an inaccessible hill, having given a base line AB in the direction of the hill = 390 feet, and the angles of elevation of the top and bottom of the fort at B the nearer extremity of the base = 62° 15′ and 47° 10′ respectively; and the angle of elevation of the top of the fort at A the farther extremity of the base = 36° 11′.

Ans. 200·56 feet.

36. (4) *To find the distance of an inaccessible object.* (Fig. 160.)

Let C be an object inaccessible from a point A, whence the distance AC is required.

Measure a horizontal line AB in any convenient direction, and from its extremities observe the horizontal angles CAB and CBA.

Now, angle C = 180° — (CAB + CBA),

then, by (Art. 25, 1st case), $AC = AB.\frac{\sin. CBA}{\sin. ACB}$.

Example: Find the breadth of a river, having given a base line AB along one bank = 63 chains 37 links, and the horizontal angles subtended by a tree on the opposite bank at A and B each end of the base = 83° 13′ and 34° 11′, respectively.

Ans. 40 chains 10 links.

37. (5) *To find the distance between two inaccessible objects.* (Fig. 426.)

Let M and N be two objects inaccessible from the position of the observer, whose distance asunder is required.

Measure a base AB; and at each of its extremities observe the horizontal angles between the other extremity and the two objects—namely, the angles BAN, BAM, ABM, and ABN.

Then, in the triangle ABN, angle ANB = 180° —(ABN + BAN);

∴ by (Art. 25, 1st case), $AN = AB.\frac{\sin. ABN}{\sin. ANB}$.

Again, in the triangle AMB, angle AMB = 180° — (BAM + ABM),

∴ by (Art. 25, 1st case), $AM = AB.\frac{\sin. ABM}{\sin. AMB}$.

Hence, in the triangle AMN, the two sides AM, AN are known, and the contained angle MAN = BAM — BAN;

∴ by (Art. 25, 3rd case), tan. ½ (AMN — ANM)

$$= \frac{AN - AM}{AN + AM} \times \tan. \tfrac{1}{2}(AMN + ANM);$$

whence the angles AMN and ANM are found.

Lastly, in the triangle AMN, by (Art. 25, 1st case), $MN = AM.\frac{\sin. MAN}{\sin. ANM}$.

Example: Find the distance between two houses M and N, on the opposite side of a river from that of the observer, and inaccessible therefrom, having given a base line AB = 5·26 chains; and at A and B, each of its extremities, the horizontal angles between the other extremity and the two houses 63° 58′, 104° 17′, and 55° 42′, 106° 12′, respectively.

Ans. 2,152·58 chains.

38. (6) *To find the distance between two accessible objects.* (Figs. 427, 428.)

Let A and B be two objects (*see* Fig. 427) accessible from C; their distance apart is required.

Measure the distances CA and CB, and observe the horizontal angle ACB;

Then, by (Art. 25, 3rd case), $\tan. \frac{1}{2}(B-A) = \frac{CA - CB}{CA + CB} \cdot \tan. \frac{1}{2}(B + A)$;

whence the angles B and A are found.

And, by (Art. 20, 1st case), $AB = CA. \frac{\sin. C}{\sin. B}$.

Example 1. Find the distance between two accessible trees, whose distances from the observer are 1,382 and 1,350 links respectively, and the angle between them 1° 38′. *Ans.* 50·4 links.

Example 2. From the top of a peak of the Himalayan mountains, the dip of the horizon * (*see* Fig. 428)) is 1° 58′ 10″. If the radius of the earth be 4,000 miles, what is the height of the peak, in miles? *Ans.* 2·364 miles.

CHAPTER IV.

Logarithms.

LOGARITHMS OF NUMBERS.†

1. LOGARITHMS EXPLAINED.—*Logarithms* ‡ are a series of artificial numbers adapted in a certain way to a series of natural numbers, to facilitate the process of numerical calculation. Thus, Multiplication of natural numbers can be performed by *addition* of their logarithms; Division by their *subtraction*; Involution, or raising of powers, by their *multiplication*; and Evolution, or extraction of roots, by their *division*.

The above expressed algebraically, where X and Y represent two natural numbers, is

$$\log. (X \times Y) = \log. X + \log. Y;$$

$$\log. \frac{X}{Y} = \log. X - \log. Y;$$

$$\log. X^n = n. \log. X;$$

$$\log. \sqrt[n]{X} = \log. X^{\frac{1}{n}} = \frac{1}{n}. \log. X.$$

2. INDEX AND MANTISSA.—The integral part of a logarithm is called its *index* or *characteristic*, and the decimal part is called its *mantissa*.

3. CALCULATION OF INDEX.—If the number whose logarithm is sought contains one or more integral figures, the index is always *one less* than the number

OBSERVATIONS.

---

* The *dip of the horizon* is the angle contained by two straight lines drawn from the observer's eye—the one to a point in the *visible* horizon, and the other parallel to the *sensible* horizon, the observer's eye being supposed to be elevated above the level of the sea. In (Fig. 428) angle AOB is the dip of the horizon, B*vb* the visible horizon, *mon* the sensible horizon, and AO*a* a horizontal plane at O. In practice, owing to the effect of refraction (*see* Chap. XVI, Art. 30), whereby rays from a point beyond B will be seen and reach A in a direction above BA, the angle of depression AOB is in reality diminished by $\frac{1}{13}$th of itself, and the visible distance *o*B increased also by $\frac{1}{13}$th of itself. Thus, if θ and x be taken to respectively represent the angle of depression and the visible distance, the reduced value of the one would be $\theta - \frac{1}{13}\theta$, and the increased value of the other $x + \frac{1}{13}x$. (*See also* Chap. XVI, Arts. 13 and 14.)

† For the logarithmic table, consult Table XXXI of the "Calculating Tables" (5th edition).

‡ The *logarithm* of a number is the exponent of the power to which another given number must be raised in order to produce the former number. Thus 1,000 = 10 × 10 × 10; that is, $10^3 = 1,000$, and the exponent 3 is the logarithm of 1,000.

of integral figures in the number, and is positive; thus, if the number contains one figure, the index of its logarithm is 0; if it contains two figures, the index is 1; if three figures, the index is 2; and so on.

If the number whose logarithm is sought is wholly a decimal, the index is always *one more* than the number of prefixed ciphers immediately after the decimal place, and is negative; thus, if there is no cipher after the decimal, the index is $\bar{1}$; if there is one prefixed cipher, the index is $\bar{2}$; if there are two prefixed ciphers, the index is $\bar{3}$; and so on.

It will be observed that the negative sign — is placed over the index. This is to show that the index alone is negative, since the mantissa is always positive.

4. MANTISSA, WHERE FOUND.—The mantissa of the logarithm is alone found in the table (*see* Table XXXI, "Calculating Tables," 5th edition); the index must therefore be supplied by the calculator himself, by the Rule in (Art. 3).

5. MANTISSA CONSTANT AS TO DECIMAL POINT.—The decimal part of a logarithm is the same for all numbers consisting of the same significant figures, wherever the decimal place is situated; the index alone changes according to the position of the decimal point.

Thus the decimal part of the logarithm of 357 being ·552668, the logarithm of 35,700 is 4·552668, the logarithm of 3·57 is 0·552668, and the logarithm of 0·00357 is $\bar{3}$·552668 which is equal to (— 3 + ·552668).

Examples on the method of supplying indices; and having given the indices to supply the decimal point of the numbers.

Number.			Logarithm.
5,206	...	... =	3·716504
520·6	...	... =	2·716504
52·06	...	... =	1·716504
5·206	...	... =	0·716504
·5206	...	... =	$\bar{1}$·716504
·05206	...	... =	$\bar{2}$·716504
·005206	...	... =	$\bar{3}$·716504
etc.	...	... =	etc.

6. *To find the logarithm of a given number.*

Case 1. When the number consists of 4 significant figures.

Rule: Look for it in the column marked No. (number), and opposite it, in the adjacent column marked Log. (logarithm), will be found the decimal part of the logarithm. The index must be supplied by the Rule in (Art. 3).

Example: Log. 5,487 = 3·739335.

Case 2. When the number consists of less than 4 significant figures.

Rule: Conceive ciphers to be annexed to it to make up 4 figures, and find its logarithm as above.

Example: Log. 74 or log. 74·00 = 1·869232.

Case 3. When the number consists of more than 4 significant figures.

Rule: Find the logarithm of the first 4 figures, as above; then look out for the fifth figure in the unit's place in the same space, and take out the corresponding number from the adjacent column marked Pr. Pt. (proportional part), and place it under the last figures of the preceding logarithm; in the same way look for the sixth and seventh figures, if there be so many, and place the corresponding proportional parts under the others, removing them one and two places respectively to the right. Add these numbers together, and the sum kept to the nearest sixth place will be the decimal part of the logarithm required.

Example: Find the logarithm of ·008060519.

Decimal part of log. of 8,060		=	·906335
Proportional part for	5	=	27
,, ,, ,,	1	=	5
,, ,, ,,	9	=	49

Hence the log. of ·008060519 = $\bar{3}$·906363.

7. *To find the number corresponding to a given logarithm.*[*]

Case 1. When not more than 4 significant figures are required.

Rule: Look for the logarithm, or if that is not exactly given, the nearest logarithm in the column marked Log., and the corresponding number in the adjacent column marked No. will be that which is required.

Example: Find the number corresponding to the logarithm 1·261037.

Nearest log. = ·261025;
Hence number corresponding to 1·261025 = 18·24.

Case 2. When more than 4 significant figures are required.

Rule: Find the next less logarithm and its corresponding number of 4 figures, as above; subtract this logarithm from the given one, and from the column marked Pr. Pt. take out the number next below this difference, and the corresponding figure in the unit's place of the column of numbers is a fifth figure to be annexed to the others. Subtract this proportional part from the difference above, and annex a cipher to it; the next number in the same column of proportional parts will give a sixth figure. Proceed in this manner as far as may be necessary, taking the last additional figure to the ***nearest*** unit.

Example: Find the number corresponding to the logarithm $\bar{3}$·906363.

$$\begin{array}{lcrcr} \text{Decimal part of given log.} & = & \cdot906363 & & \\ \text{Next less log.} & = & \cdot906335 & = & 8060 \\ & & & & \\ \text{Difference} & = & 28 & & \\ \text{Next lower pro. part} & = & 27 & = & 5 \\ \hline & & 10 & & \\ \text{,, ,, ,, ,,} & = & 5 & = & 1 \\ \hline & & 50 & & \\ \text{,, ,, ,, ,,} & = & 49 & = & 9 \end{array}$$

Hence the No. corresponding to the log. $\bar{3}$·906363 = ·008060519

8. *Calculation of negative indices.*—It is usual to make only the index of a logarithm negative, as the calculations can thus be made more conveniently. Calculations with negative indices are made according to the rules of algebraic addition and subtraction.

Case 1. To add two negative indices.

Rule: Take their sum and make it negative; thus, $\bar{1} + \bar{2} = \bar{3}$.

Example: Add $\bar{1}$·452761 and $\bar{2}$·274828.

$$\begin{array}{r} \bar{1}\cdot452761 \\ +\ \bar{2}\cdot274828 \\ \hline \bar{3}\cdot727589. \end{array}$$

Case 2. To add a positive and negative index.

Rule: Take their difference and give it the sign of the greater; thus, $6 + \bar{2} = 4$, $3 + \bar{5} = \bar{2}$.

Examples: Add 6·327410 and $\bar{2}$·411249, and 3·532488 and $\bar{5}$·842264.[†]

Case 3. To subtract a negative index.

Rule: Change its sign to plus, and then add it by the preceding rules for addition; thus, $\bar{2} - \bar{6} = \bar{2} + 6 = 4$, $\bar{5} - \bar{2} = \bar{5} + 2 = \bar{3}$,

$$3 - \bar{5} = 3 + 5 = 8,\ 6 - \bar{1} = 6 + 1 = 7.$$

OBSERVATIONS.

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[*] The number cannot be carried with accuracy to more places than the logarithm.

[†] In this example the sum in the first decimal place is 13, the 1 being carried to the indices which give $1 + 3 + \bar{5} = 4 + \bar{5} = \bar{1}$.

Examples: Subtract $\bar{6}{\cdot}129475$ from $\bar{2}{\cdot}457896$, $\bar{2}{\cdot}846320$ from $\bar{5}{\cdot}640728$, $\bar{5}{\cdot}241847$ from $3{\cdot}406842$, and $\bar{1}{\cdot}722436$ from $6{\cdot}348502$.

$$\begin{array}{rrrr} \bar{2}{\cdot}457896 & \bar{5}{\cdot}640728 & 3{\cdot}406842 & 6{\cdot}348502 \\ -6{\cdot}129475 & -\bar{2}{\cdot}846320 & -\bar{5}{\cdot}241847 & -\bar{1}{\cdot}722436 \\ \hline 4{\cdot}328421 & \bar{2}{\cdot}794408^{*} & 8{\cdot}164995 & 8{\cdot}626066^{\dagger} \end{array}$$

Case 4. To multiply a logarithm which has a negative index.

Rule: Multiply the decimal part by the ordinary rules; then multiply the negative index, which will give a negative product, and add the carriage, if any, to it (by the Rule of Case 2).

Thus $\bar{2} \times 5 = \overline{10}$, and if 2 of carriage is to be added, the result will be $\overline{10} + 2 = \bar{8}$.

Examples : Multiply $\bar{2}{\cdot}247981$ by 3, and $\bar{3}{\cdot}984075$ by 6.

$$\begin{array}{rr} \bar{2}{\cdot}247981 & \bar{3}{\cdot}984075 \\ 3 & 6 \\ \\ \bar{6}{\cdot}743943 & \overline{13}{\cdot}904450\ \ddagger \end{array}$$

Case 5. To divide a logarithm which has a negative index.

Rule: If the index is divisible by the divisor, write down the quotient with a negative sign, and divide the decimal part by the ordinary rules. But if the index is not divisible by the divisor, add such least negative number to it as will make it divisible, and prefix an equal positive integer to the decimal part of the logarithm; then divide the increased negative index and the other part of the logarithm separately, and the first quotient, taken negatively, will be the index to the decimal part of the second quotient.

Thus, $\bar{6} \div 2 = \bar{3}$. But $\overline{10} \div 3 = (\overline{10} + \bar{2} + 2) \div 3 = \overline{12} \div 3 + 2 \div 3 = \bar{4} + {\cdot}6 = \bar{4}{\cdot}6$.

Example: Divide $\bar{6}{\cdot}942568$ by 2, and $\overline{10}{\cdot}278504$ by 3.

$\bar{6}{\cdot}942568 \div 2 = \bar{3}{\cdot}471284$;

$\overline{10}{\cdot}278504 \div 3 = (\overline{12} + 2{\cdot}278504) \div 3 = \bar{4}{\cdot}759501$.

9. *To perform multiplication by logarithms.*

Rule: Add the logarithms of the factors, and the sum will be the logarithm of the product.

Examples: Multiply 3·24789 by 268·847; and multiply together ·0648, 26·41, and ·0019426.

$$\begin{array}{ll} \log.\ 3{\cdot}24789 = & 0{\cdot}511601 \\ \log.\ 268{\cdot}847 = & 2{\cdot}429505 \\ \hline & 2{\cdot}941106 = 873{\cdot}184, \text{ Product;} \end{array}$$

$$\begin{array}{ll} \log.\ {\cdot}0648 = & \bar{2}{\cdot}811575 \\ \log.\ 26{\cdot}41 = & 1{\cdot}421768 \\ \log.\ {\cdot}0019426 = & \bar{3}{\cdot}288383 \\ \hline & \bar{3}{\cdot}521726 = {\cdot}0033245, \text{ Product.} \end{array}$$

10. *To perform division by logarithms.*

Rule: From the logarithm of the dividend subtract that of the divisor, and the remainder will be the logarithm of the quotient.

Example: Divide 386·2 by 64·75, and ·0643 by 114·09.

$$\begin{array}{ll} \log.\ 386{\cdot}2 = & 2{\cdot}586812 \\ \log.\ 64{\cdot}75 = & 1{\cdot}811240 \\ \hline & 0{\cdot}775572 = 5{\cdot}96447, \text{ Quotient;} \end{array}$$

$$\begin{array}{ll} \log.\ {\cdot}0643 = & \bar{2}{\cdot}808211 \\ \log.\ 114{\cdot}09 = & 2{\cdot}057247 \\ \hline & \bar{4}{\cdot}750964 = {\cdot}00056359, \text{ Quotient.} \end{array}$$

OBSERVATIONS.

* In this example the difference in the first decimal place is 7 with 1 over, and is carried to the indices which give $1 + \bar{5} - \bar{2} = \bar{4} - 2 = \bar{4} + 2 = \bar{2}$.

† In this example the difference in the first decimal place is 6 with 1 over, and is carried to the indices which give $1 + 6 - \bar{1} = 7 - 1 = 7 + 1 = 8$.

‡ In this example $\bar{3} \times 6 + 5$ (the carriage) $= \overline{18} + 5 = \overline{13}$.

11. *To perform involution by logarithms.*

Rule: Multiply the logarithm of the given number by the index of the power, and the product will be the logarithm of the required power.

Examples: Find the cube of 2·4206, and the 287th power of 1·0058.

log. 2·4206 = 0·383923
3

1·151769 = 14·183, Power required;

log. 1·0058 = 0·002512
287

0·720944 = 5·2595, Power required.

12. *To perform evolution by logarithms.*

Rule: Divide the logarithm of the given number by the index of the root, and the quotient will be the logarithm of the required root.

Examples: Find the square root of 2,528·17, and the 83rd root of 429·063.

log. 2,528·17 = 3·402806, and 2)3·402806
1·701403 = 50·2809, Root required;

log. 429·063 = 2·632521, and 83) 2·632521
0·031717 = 1·07576, Root required.

13 ARITHMETICAL COMPLEMENT. The *Arithmetical Complement* of a logarithm is the number which remains after subtracting the logarithm from zero.

Thus the A. C. of 7·274326 = $\bar{8}$·725674; of 2·067843 = $\bar{3}$·932157.

Rule for obtaining the A. C. of a logarithm: Add 1 to the index and change its sign; and subtract all the figures of the decimal part from 9, except the right-hand significant one which subtract from 10.

Examples: Find the arithmetical complement of the logs. 1·784698, 0·185445, and $\bar{3}$·047862.

A. C. of 1.784698 = $\bar{2}$·215302;
" " 0.185445 = $\bar{1}$·814555;
" " $\bar{3}$.047862 = 2·952138.

14. *To subtract a logarithm is the same thing as adding its arithmetical complement.**

Examples: Of the following logs subtract 1·045283 from 2·527896, and $\bar{6}$·849953 from 1·000701.

2·527896
A. C. of 1·045283 = $\bar{2}$·954717

Remainder = 1·482613;

1·000701
A. C. of $\bar{6}$·849953 = 5·150047

Remainder = 6·150748.

LOGARITHMIC SINES AND TANGENTS.†

15. *Logarithmic Sines, Tangents, etc., explained.* The Logarithmic Sines, Tangents, etc., as arranged in the Table (*see* Table XXXII, "Calculating Tables," 5th edition), are the logarithms of the numerical values of the sines, co-sines, tangents, and co-tangents of all the arcs of a quadrant, calculated to a radius of 10,000,000,000. The logarithm of this radius is 10.

In the first horizontal line of each page, are written the degrees, whose sines and tangents are expressed on the top of the page, proceeding from 0° to 90° *forwards*; and the vertical column on the left is their column of minutes. In the last horizontal line of each page are written the degrees whose co-sines and co-tangents are expressed on the bottom of the page proceeding from 0° to 90° *backwards*; and the vertical column on their right is their column of minutes.

OBSERVATIONS.

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* Hence to perform division, add the A. C. of the log. of the divisor to the log. of the dividend, and the sum will be the log. of the quotient.

† For the logarithmic table, consult Table XXXII of the "Calculating Tables" (5th edition).

Since neither the sine nor co-sine of an angle is ever greater than unity, the logarithm of a sine or co-sine will never be a positive quantity. In order to avoid the occurrence of negative quantities in the Tables, it is found convenient to *add* 10 *to the logarithm of every trigonometrical ratio*, before registering it in the Tables; the logarithm so increased is called the *Tabular Logarithm*, and is usually denoted by the letter L. Thus—

$$\text{L. sin. A} = \text{log. sin. A} + 10,$$
$$\text{L. cos. A} = \text{log. cos. A} + 10,$$

and so on. Of course in calculation this addition to the real logarithms of the Trigonometrical Ratios must be allowed for.

Attention must be paid to the following rule: that as the angle *increases*, the sine and tangent also *increase*, but the co-sine and co-tangent *decrease*; hence, in the Tables, the Tabular differences are *additive* for the former case and *subtractive* for the latter. The signs + and — have accordingly been placed as a guide in the vertical columns of Differences; at the top for the sines and tangents, and at the bottom for the co-sines and co-tangents.

16. *To find the Logarithmic sine, tangent, co-sine or co-tangent of a given angle.*

Case 1. When the angle contains degrees and minutes only.

(*a*) For the sine or tangent. *Rule*: Look for the degrees at the *top* of the Table, and in the column marked Sine or Tangent, as the case may be, and opposite to the given number of minutes in the *left* hand column marked ′ (minutes), is the required Logarithm.

Examples: The L. sine of $18^\circ\ 45' = 9{\cdot}507099$;
The L. tangent of $18^\circ\ 45' = 9{\cdot}530781$.

(*b*) For the co-sine or co-tangent. *Rule*: Look for the degrees at the *bottom* of the Table, and in the column marked Co-sine or Co-tangent, as the case may be, and opposite to the given number of minutes in the *right* hand column marked ′ (minutes), is the required Logarithm.

Examples: The L. co-sine of $67^\circ\ 13' = 9{\cdot}587989$;
The L. co-tangent of $67^\circ\ 13' = 9{\cdot}623269$.

Case 2. When the angle contains degrees, minutes and seconds.

(*a*) For the sine or tangent. *Rule*: Find the Logarithm for the number of degrees and minutes as before; multiply the corresponding tabular difference for 1″ in the column marked Diff. (difference) between this Logarithm and the next *higher*, by the number of seconds given; and *add* the product to the Logarithm first found, retaining the sum to 6 places of decimals.

Example: To find the Logarithmic tangent of 27° 13′ 18″

$$\text{L. tan. } 27^\circ\ 13' = \qquad = 9{\cdot}711215$$
$$\text{Diff.} \times \text{Secs.} = 5{\cdot}18 \times 18 = \quad +\ 93{\cdot}24$$

$$\therefore \text{L. tan } 27^\circ\ 13'\ 18'' = 9{\cdot}711308$$

(*b*) For the co-sine or co-tangent. *Rule*: Find Logarithm of the number of degrees and minutes as before; multiply the corresponding tabular difference for 1″ in the column marked Diff., between this Logarithm and the next *lower*, by the number of seconds given; and *substract* the product from the Logarithm first found, preserving the difference to 6 places of decimals.

Example: To find the Logarithmic co-tangent of 73° 28′ 51″.

$$\text{L. cot.} \;\ldots\; \ldots \;\ldots = 9{\cdot}472532$$
$$\text{Diff.} \times \text{Secs.} = 7{\cdot}72 \times 51 \;\ldots = -\ \ 393{\cdot}72$$

$$\therefore \text{L. cot. } 73^\circ\ 28'\ 51'' \;\ldots = 9{\cdot}472138$$

17. ***To find the angle corresponding to a given Logarithmic sine tangent, co-sine or co-tangent.***

Case 1. When the Logarithm is found in the Tables exactly.

(*a*) For the sine or tangent. *Rule*: Look for the Logarithm in the column marked Sine or Tangent, as the case may be, when the degrees of the angle will be found at the *top* of the page, and the minutes in the *left* hand column marked ′, opposite to the given Logarithm.

Examples: The L. sine 9·423697 corresponds to 15° 23′;
" tangent 10·413561 " 68° 54′.

(*b*) For the co-sine or co-tangent. *Rule*: Look for the Logarithm in the column marked Co-sine or Co-tangent, as the case may be, when the degrees will be found at the *bottom* of the page, and the minutes in the *right* hand column marked ′, opposite to the given Logarithm.

Examples: The L. co-sine 9·544663 corresponds to 69° 29′;
" co-tangent 9·998958 " 3° 58′.

Case 2. When the Logarithm is not found in the Tables exactly.

(*a*) For the sine or tangent. *Rule*: Find the next *lower* Logarithm, and take the degrees and minutes corresponding to it as before; subtract this Logarithm from the given one; divide the remainder by the corresponding tabular difference for 1″ in the column marked Diff., between the Tabular Logarithm and the next *higher*, and the quotient will be the number of seconds required.

Examples: Find the angle corresponding to the Logarithmic sine 9·733849, and that corresponding to the Logarithmic tangent 10·498963.

Given. L sin. ... = 9·733849
Next lower Log. ... = 9·733765 = L. sin. 32° 48′

Tabular difference ... = 3·27)84·00(26″
654

1860
1962

∴ the required angle = 32° 48′ 26″;
Given L. tan. ... = 10·498963
Next lower Log. ... = 10·498641 = L. tan. 72° 24′

Tabular difference... = 7·31)322·00(44″
2924

2960
2924

∴ the required angle = 72° 24′ 44″.

(*b*) For the co-sine or co-tangent. *Rule*: Find the next *higher* Logarithm, and take the degrees and minutes corresponding to it as before; subtract the given Logarithm from this one; divide the remainder by the corresponding tabular difference for 1″ in the column marked Diff., between the Tabular Logarithm and the next *lower*, and the quotient will be the number of seconds required.

Examples: Find the angle corresponding to the Logarithmic co-sine 9·752834, and that corresponding to the Logarithmic co-tangent 10·238561.

Next higher Log. ... = 9·752944 = L. cos. 55° 31′
Given L. cos. ... = 9·752834

Tabular difference... = 3·07)110·00(36″
921

1790
1842

∴ the required angle = 55° 31′ 36″;

Next higher Log. ... = 10·238561 = L. cot. 30° 00′
Given L. cot. ... = 10·238415

Tabular difference ... = 4·86)146·00(30″
1458

20
00

∴ the required angle = 30° 00′ 30″.

18. *To find the Logarithmic sine, tangent, co-sine, or co-tangent of an angle greater than 90°.* (*See* Chap. III, Arts. 14 –16.)

(a) For the sine or tangent.

Rule : 1. When the angle is in the 2nd quadrant, subtract 90° from it, and find the Logarithmic co-sine or co-tangent, as the case may require, of the difference.

2. When the angle is in the 3rd quadrant, subtract 180° from it, and find the Logarithmic sine or tangent, as the case may require, of the difference.

3. When the angle is in the 4th quadrant, subtract 270° from it, and find the Logarithmic co-sine or co-tangent, as the case may require, of the difference.

Examples : Find the Logarithmic sine and tangent of 133° 4′ 5″, 230° 47′ 19″, and 315° 52′ 01″.

L. sin. 133°04′ 05″=L. cos. (133° 04′ 05″− 90°)=L. cos. 43°04′ 05″= 9·863646,
L. tan. 133 04 05 =L. cot. (133 04 05 − 90)=L. cot. 43 04 05 =10·029310 ;
L. sin. 230 47 19 =L. sin. (230 47 19 −180)=L. sin. 50 47 19 = 9·889201,
L. tan. 230 47 19 =L. tan. (230 47 19 −180)=L. tan. 50 47 19 =10·088358
L. sin. 315 52 01 =L. cos. (315 52 01 −270)=L. cos. 45 52 01 = 9·842813,
L. tan. 315 52 01 =L. cot. (315 52 01 −270)=L. cot. 45 52 01 = 9·986856

(b) For the co-sine or co-tangent.

Rule : 1. When the angle is in the 2nd quadrant, subtract 90° from it, and find the Logarithmic sine or tangent, as the case may require, of the difference.

2. When the angle is in the 3rd quadrant, subtract 180° from it, and find the Logarithmic co-sine or co-tangent, as the case may require, of the difference.

3. When the angle is in the 4th quadrant, subtract 270° from it, and find the Logarithmic sine or tangent, as the case may require, of the difference.

Examples : Find the Logarithmic co-sine and co-tangent of 91° 27′ 48″, 180° 2′ 22″, and 350° 15′ 57″.

L. cos. 91°27′ 48″=L. sin. (91°27′ 48″− 90°)=L. sin. 1°27′ 48″= 8·407169,
L. cot. 91 27 48 =L. tan. (91 27 48 − 90)=L. tan. 1 27 48 = 8·407310;
L. cos. 180 02 22 = L. cos. (180 02 22 −180)=L. cos. 0 02 22 =10·000000,
L. cot. 180 02 22 = L. cot. (180 02 22 −180)=L. cot. 0 02 22 =13·170677;
L. cos. 350 15 57 = L. sin. (350 15 57 −270)=L. sin. 80 15 57 = 9·993702,
L. cot. 350 15 L. tan. (350 15 57 −270)=L. tan. 80 15 57 =10·765616.

19. *To find the Logarithmic secant or co-secant of a given angle.*

(a) For the Logarithmic secant.

Rule : Obtain the complement of the Logarithmic co-sine of the given angle, to which add 10 for the Tabular Logarithm required.

(b) For the Logarithmic co-secant.

Rule : Obtain the complement of the Logarithmic sine of the given angle, to which add 10 for the Tabular Logarithm required.

Examples : Find the Logarithmic secant and co-secant of 27° 15′ 18″.

L. sec. 27° 15′ 18″=(10−L. cos. 27° 15′ 18″)+10=20−9·948891=10·051109 ;
L. cos. 27 15 18 =(10−L. sin. 27 15 18)+10=20−9·660820=10·339180.

20. *Rules for small angles.* The differences for the Logarithmic sines and tangents of small angles, up to 3°, are very irregular, and therefore the ordinary method of proportioning for seconds is subject to considerable error. It is seldom

such angles enter into calculations, but when they do so, they or their Logarithmic sines and tangents may be calculated from the subjoined rules.*

21. *To find the Logarithmic sine of a small angle.*

Rule : To the Logarithm of the angle reduced to seconds, add the constant Log. 4·685575, and from the sum subtract one-third of the arithmetical complement of the Logarithmic co-sine ; the remainder will be the Logarithmic sine of the given angle.

Example : Find the Logarithmic sine of 1° 17′ 23″·8.

1° 17′ 23″·8=4,643″·8, its log. ... = 3·666873
Constant Log. ... ... = 4·685575

8·352448
$\frac{1}{3}$ (10−L. cos. 1° 17′ 23″·8)=$\frac{1}{3}$ (10−9·999890) = 0·000037

∴ L. sin. 1° 17′ 23″·8 ... = 8·352411.

22. *To find the Logarithmic tangent of a small angle.*

Rule : Add together the logarithm of the angle reduced to seconds, the constant Log. 4·685575, and two-thirds of the arithmetical complement of the Logarithmic co-sine ; the sum will be the Logarithmic tangent of the given angle.

Example : Find the Logarithmic tangent of 1° 27′ 23″·8.

1° 17′ 23″·8 =4,643″·8, its log. = 3·666873
Constant Log. ... ... = 4·685575
$\frac{2}{3}$ (10−L. cos. 1° 17′ 23″·8)=$\frac{2}{3}$ (10−9·999890)= 0·000073

∴ L. tan. 1° 17′ 23″·8... ... = 8·352521

23. *To find a small angle, of which the Logarithmic sine is given.*

Rule : Add together the given Logarithmic sine, the constant Log. $\bar{5}$·314425, and one-third of the arithmetical complement of the Logarithmic co-sine ; the sum will be the logarithm of the number of ***seconds*** in the required angle.

Example : Find the angle corresponding to the Logarithmic sine 8·352411.

Given L. sin. ... ... = 8·352411
Constant Log. ... ... = $\bar{5}$·314425
$\frac{1}{3}$ (10−L. cos. 1° 17′ 23″·8)=$\frac{1}{3}$ (10−9·999890)= 0·000037

Log. 4,643″·8 ... ... = 3·666873
∴ Required angle = 4,643″·8 ... = 1° 17′ 23″·8

24. *To find a small angle, of which the Logarithmic tangent is given.*

Rule : To the given Logarithmic tangent add the constant Log. $\bar{5}$·314425, and from the sum subtract two-thirds of the arithmetical complement of the Logarithmic co-sine ; the remainder will be the logarithm of the number of ***seconds*** in the required angle.

Example : Find the angle corresponding to the Logarithmic tangent 8·352521.

Given L. tan. ... ... = 8·352521
Constant Log. ... ... = $\bar{5}$·314425

3·666946
$\frac{2}{3}$ (10−L. cos. 1° 17′ 23″·8) = $\frac{2}{3}$ (10−9·999890)= 0·000073

Log. 4,643″·8 ... ... = 3·666873
∴ Required angle=4,643″·8 ... = 1° 17′ 23″·8.

OBSERVATION.

* The numerical calculations of the sine, tangent, etc., of an angle by logarithms, are performed in the same way as those of natural numbers by logarithms. In so calculating the tabular 10 added to all logarithmic sines, tangents, etc. (*see* Article 16), must always be allowed for.

CHAPTER V.

Mathematical Drawing Instruments.

1. OBJECT OF THE CHAPTER. In this Chapter the object has been to describe only such instruments as are in ordinary use, and which will amply meet the wants of the Surveyor. For a knowledge of special instruments, various refinements, and contrivances, etc., that have been invented from time to time, reference must be made to special treatises, such as Simms', Stanley's, Heather's, etc.

2. CONTENTS OF BOX OF INSTRUMENTS. A complete box of mathematical drawing instruments usually contains one cf each of the following:—1. *Large compasses* with moveable (a) *plain point*, (b) *pencil point* (c) *ink point* (d) *lengthening bar*; 2. *Hair compasses*; 3. *Compass key*; 4. *Bow compasses*, namely, (a) *pencil*, (b) *ink*; 5. *Spring bows*, namely, (a) *dividers*, (b) *ink*; 6. *Drawing pen*; 7. *Double pen*; 8. *Dotting pen*; 9. *Pricker*; 10. *Proportional compasses*; 11. *Rectangular protractor*, 6″; 12. *Straight edge*; 13. *Parallel rule*; 14. *Marquois' scale*; 15. *Sector*.

3. LARGE COMPASSES. (Figs. 133, 136.) This instrument (*see* Fig. 133) consists of two legs moveable about a joint A, so that their extremities may be set at any distance from one another. The upper portion is formed of brass or electrum, and the legs of steel, ending in fine points. It is used to transfer and measure distances, and to describe arcs and circles; and is held erect in the right hand with one leg between the first and second fingers, the other leg between the third and fourth fingers, and with the thumb against both the legs close to the joint on the near side, as shown in (Fig. 136). To draw arcs or circles with pencil or ink, remove the plain point from the socket B into which it fits and replace it with the pencil point C, or the ink point D, as required. To strike larger circles and measure greater distances insert one end of the lengthening bar E into the socket B, and fit the socket at its other end with the plain point, pencil point or ink point, as necessary. The legs of some compasses, as also the pencil point and pen point, are furnished with a joint *a*, that they may be set nearly perpendicular to the paper.

4. HAIR COMPASSES. (Fig. 134.) The *hair compasses* are used where greater accuracy is required than can be obtained by the common compasses. In the hair compasses the upper part of one of the steel points is formed into a bent spring, which, being fastened at one extremity to the leg of the compasses almost close up to the joint, is held at the other end by a screw. A groove is formed in the shank, which receives the spring when screwed up tight; and by turning the screw backwards the steel point may be gradually allowed to be pulled backwards by the spring, and may again be gradually pulled forwards by the screw being turned forwards. In the figure, *a* represents these compasses when shut, and *b* represents them open, with the screw turned backwards, and the steel point *p*, in consequence, moved backwards by its spring *s*, from the position represented by the dotted lines, which it would have when screwed up tight.

5. COMPASS KEY. (Figs. 469, 134.) The *compass key* (*see* Fig. 469) has two points which fit into the two holes seen in the nut *n* of the joint (*see* Fig. 134); and by turning this nut the joint is made stiffer or easier, as desired.

6. BOW COMPASSES. (Fig. 135.) To describe small arcs or circles of less than an inch radius, a small pair of compasses, called *bow compasses*, with permanent pencil point (*see* Fig. 135, *a*) or with permanent ink point (*see* Fig. 135 *b*) are used. They are formed with a round head, which rolls with ease between the fingers.

7. SPRING BOWS. (Fig. 139.) These are bow compasses without the joint about which the legs move, being composed of a stout steel spring of which the two legs are attached by an adjusting screw, which brings the points to any required distance from each other and there keeps them immoveable (*see* Fig. 139); *a* represents spring bow dividers, which can measure small distances to almost a point; *b* represents a spring bow pen, which is well adapted to describe

arcs or circles of small radii with great exactness by means of the adjusting screw *s*.

8. DRAWING PEN.* (Fig. 140.) This instrument is used for drawing straight lines. It is formed of two evenly pointed blades of steel fixed to a handle; and they are so bent, that a sufficient cavity is left between them for the ink. The blades can be moved to or from each other by means of a mill-headed screw passing through them, so as to draw lines of any required fineness or thickness. In using the pen, hold it slightly inclined in the direction of the line to be drawn, taking care that the edges of both blades touch the paper.

9. DOUBLE PEN. (Fig. 141.) The *double* or *road pen* is used for drawing close parallel lines to represent canals, roads, etc., having an adjusting screw to set the points to any required distance.

10. DOTTING PEN. (Fig. 142.) The *dotting pen* is a drawing pen with a dotting wheel fitted on to the end, and is used to draw straight or curved dotted lines. It receives ink in the usual way, and has a little box scooped out on the top of the handle which contains several descriptions of wheels, some to make only dots, others dots and lines, etc., being used as wanted.

11. PRICKER. (Fig 143.) The *pricker* is a fine needle held in a pair of forceps fixed to a handle, and is used to mark the intersection of lines, or to set off divisions from the plotting scale and protractor; and may also be used to prick through a drawing upon an intended copy, or, the needle being reversed, the eye end forms a good tracing point.

12. PROPORTIONAL COMPASSES. (Fig. 137.) By means of *proportional compasses* drawings may be enlarged or reduced, so that all the lines of the copy, or the areas or solids represented by the several parts, shall bear any required proportion to the lines, areas, or solids of the original drawing. They also serve to inscribe regular polygons in circles, and to take the square roots and cube roots of numbers; but the chief use of the proportional compasses is to *reduce* a drawing in any given proportion. They consist of two flat brass or electrum blades, having steel points, and united by a nut C, carrying a slider, which can be moved along the whole length of the groove of the blades. On the face of each limb are marked 10 divisions, denominated " Lines," " Circles," " Planes " and " Solids." To set the instrument it must first be accurately closed, so that the two blades appear as one; the nut C being then unscrewed, the slider is moved until its zero line coincides with any required division upon any of the scales; tighten the screw, and the compasses are set.

To enlarge or reduce the lines of a drawing.—Set the zero line of the slider to one of the divisions, 1, 2, 3, 4, etc., on the scale of lines, and the points A, B, will open to the same, double, triple, four times, etc., the distances of the points D, E; and conversely.

To inscribe in a circle a regular polygon of any number of sides from 6 to 20.—Set the zero line of the slider to the given number on the scale of circles, open the points A and B to the length of any radius, then the points D and E will prick off a polygon of that number of sides in the circle described with this radius.

To enlarge or reduce the area of a drawing.—Set the zero line of the slider to any number on the scale of plans, and if the points D and E be opened out to the lengths of the several lines of a drawing, the points A and B will prick off a copy occupying so many times greater than the area as the number indicates; and conversely.

To take the square root of a number.—Set the zero lines of the slider to the given number upon the scale of plans, open the points A and B to take the number from any scale of equal parts, then the points D and E applied to the same scale of equal parts will give the square root of the number.

OBSERVATION.

* Always wipe off the ink from the blades of the bow compasses and drawing pen after use.

To enlarge or reduce the contents of solids.—Set the zero line of the slider to any number on the scale of solids, then the distance between the points A and B will give the side of a solid of so many times the content as the number indicates, of that of which a like side is given by the distance of the points D and E; and conversely.

To take the cube root of a number.—Set the zero line of the slider to the given number upon the scale of solids, and opening the points A, B, to take off the number upon any scale of equal parts, the points D, E, will then take off the cube root of the number from the same scale of equal parts.

13. RECTANGULAR PROTRACTOR. For a description and use of this, as well as the semi-circular protractor (*see* Chap. VI, Arts. 37–40). The former usually has, in addition to angular divisions, various simple and diagonal scales marked on both sides of it.

14. STRAIGHT EDGE. (Fig. 145.) The figure represents a *straight edge* and a *triangle*. The former is used to draw straight lines, and with the help of the latter parallel straight lines are drawn, perpendiculars erected on straight lines, etc. In using these it is of the utmost importance to test the accuracy of their edges and see if they are quite straight. The simplest method of doing this is to place the edges of both in contact and slide them along each other while held up between the eye and the light. If the light is excluded, the edges are good and fit for use.

15. PARALLEL RULES. (Fig. 138.) There are various kinds and sizes of parallel rules, being as much as 3 feet in length, used for the purpose of drawing parallel lines, but the most useful for short lines are those which are 6 inches long, made of ebony or ivory. The *common parallel rule* (*see* Fig. 138, *a*) consists of two straight rules, which are connected together, and always maintained in a parallel position by two equal and parallel bars, which move freely on their centres. The *double parallel rule* (*see* Fig. 138, *b*), is constructed on the same principle as the above, but with this advantage that, in using it, the moveable rule can always be so placed that its ends may be exactly over, or even with, the ends of the fixed rule, whereas in the former they are always shifting away from the ends of the first rule. The *cross-barred parallel rule* (*see* Fig. 138, *c*) consists of two straight rules, joined by two brass bars, which cross each other, and turn on their intersection as on a centre; one end of each bar moves on a centre, the other slides in a groove, as the rules recede from each other.

To draw a line parallel to a given right line by either of the foregoing instruments.—Set the edge of the uppermost rule to the given line; press the edge of the lower rule tight to the paper with one hand, and with the other move the upper rule, till its edge coincides with the given point; and a line drawn along the edge through the point is the line required.

A very useful and convenient instrument is the *rolling parallel rule* (*see* Fig. 138, *d*), which is a flat rectangular rule of ivory or electrum. It is supported by two small wheels, which are connected together by a long axis, the wheels being exactly of the same size, and their rolling surfaces being parallel to the axis. When they are rolled backwards or forwards, the axis and rule will move in a direction parallel to themselves. The wheels are somewhat indented, to prevent their sliding on the paper. In rolling these rules, one hand only must be used, and the fingers should be placed nearly in the middle of the rule that one end may not have a tendency to move faster than the other. The wheels only should touch the paper when the rule is moving, and the surface of the paper should be smooth and flat.

16. MARQUOIS' SCALE. (Fig. 147.) The set of *Marquois' Scales*, usually made of wood, consists of a right-angled triangle (of which the hypotenuse or longest side is three times the length of the shortest) and two rectangular rules. The figure represents the triangle and one of the rules, as being used to draw a series of parallel lines. Either rule is a foot long, and has parallel to its edges, two scales, one placed close to the edge, and the other immediately within this, the outer being termed the *artificial*, and the inner the *natural*, scale. The divisions upon the outer scales are three times the length of those upon the

inner scale, so as to bear the same proportion to each other that the longer side of the triangle bears to the shorter. Each inner, or natural scale, is in fact a simply divided scale of equal parts, having the primary divisions numbered from the left hand throughout the whole extent of the rule. In the artificial scales, the zero point is placed in the middle of the edge of the rule, and the primary divisions are numbered both ways from the centre point outwards. Each division on this scale is three times the length of a corresponding division on the natural scale. The triangle has a short line drawn perpendicular to the hypotenuse near the middle of it, to serve as an index or pointer; and the longer of the two sides has a bevelled edge. The rectangular rules have numbers 25, 30, 35, 40, 45, 50, 55, 60, marked on each scale; these numbers simply show how many divisions the inch is divided into on the natural scale; the artificial divisions being three times the natural division, we are enabled by the method shown below, to draw parallel lines from $\frac{1}{25}$th to $\frac{1}{60}$th of an inch apart, or any multiples of these fractions.

To draw a line parallel to a given line at a given distance from it.

1. Having applied the given distance to one of the natural scales, which is found to measure it most conveniently, place the triangle with its sloped edge coincident with the given line, or rather at such a small distance from it, that the pen or pencil passes directly over it, when drawn along this edge.

2. Set the rule closely against the hypotenuse, making the zero point of the corresponding artificial scale coincide with the index upon the triangle.

3. Move the triangle along the rule, to the left or right, according as the required line is to be above or below the given line, until the index coincides with the division or subdivision corresponding to the number of divisions or subdivisions of the natural scale, which measures the given distance; and the line drawn along the sloped edge in its new position will be the line required.

17. SECTOR. (Figs. 144, 146, 149.) With the aid of the *sector* (*see* Fig. 144), all questions in proportion may be solved, lines may be divided either equally or unequally into any required number of parts; the angular functions, namely, chords, sines, tangents, etc., may be set off or measured to any radius whatever; plans and drawings may be reduced or enlarged in any proportion that may be desired; and, in short, every operation in geometrical drawing may be performed by the aid of this instrument and the compasses only. It is a ruler 12 inches long, and about ½ an inch broad, jointed in the centre, so as to allow of its being folded together in the direction of its depth, and made of either wood or ivory. The following are the various lines and scales which are marked on each side of the legs:—

(1) the *lines of logarithmic numbers, sines, and tangents,* marked N, S, and T, respectively;

(2) a *scale of 12 inches,* in which each inch is divided into ten equal parts;

(3) a foot divided into ten equal primary divisions, each of which is subdivided into ten equal parts, called the *decimal scale,* and placed on the edge of the instrument;

(4) a pair of scales of equal parts, called the *line of lines,* marked L;

(5) a pair of *lines of chords,* marked C;

(6) a pair of *lines of secants,* marked S;

(7) a pair of *lines of polygons,* marked Pol.;

(8) a pair of *lines of sines,* marked S;

(9) two pairs of *lines of tangents,* marked T, one being up to 45° and the other to a lesser radius, extending from 45° to 75°.

The lines marked from (4) to (9) are called *sectoral lines* and proceed in pairs from the centre, one line of each pair on either leg. (1), (8), and (9) appear on one face of the instrument; (2), (4), (5), (6), and (7) appear on the reverse face; and (3) appears on the edge.

A *lateral distance* is a distance measured from the centre along any sectoral line; a *transverse distance* is a distance measured from a point in one line of a pair of sectoral lines to the corresponding point in the other line. The most important scales, and the ones which are really of most service in geometrical construction, are the *line of lines*, the *line of chords*, and the *line of polygons*, which are below described.

LINE OF LINES. 1. *To determine $\frac{2}{7}$ths of a line 3 inches long.* (*See* Fig. 149.) Take the length of the line into the compasses, and having set one point in the primary division which is numbered 7, open the sector until the other point of the compasses meets the corresponding 7th primary division on the other limb; then the distance between the two points marked 2 on either limb, is that required; it will also be evident that from 1 to 1 would = $\frac{1}{7}$th, 3 to 3 = $\frac{3}{7}$ths, 4 to 4 = $\frac{4}{7}$ths, etc., of 3 inches.

2. *To find $\frac{9}{23}$ of a line 4·09 inches long.* (*See* Fig. 149.) Since there are only ten primary divisions, recourse must be had to the secondary divisions to solve this problem. In order to bring the construction some distance from the centre, which will ensure the accuracy of the result, multiply the numerator and denominator of the fraction by some number which will make the denominator, when so multiplied, near, but not greater than, 100: in this case 4 is a convenient multiplier; then $\frac{9}{23} = \frac{9 \times 4}{23 \times 4} = \frac{36}{92}$; having taken off 4·09 inches in the compasses make that length a transverse distance at the secondary division 92, then the transverse distance at 36 will give the part required.

LINE OF CHORDS. This scale is similar to the one marked Cho. on the protractor, and is used for the same purpose; but the double scales of chords on the sector are generally more useful than the single scale on the protractor; for on the sector the radius with which the arc is to be described may be of any length between the transverse distance of 60 and 60 when the legs are closed, and that of the transverse of 60 and 60 when the legs are opened as far as the instrument will admit of; but with the scale on the protractor, the arc described must always be of the same radius.

1. *To lay down an angle of 46° (being less than 60°).* (*See* Fig. 146.) Make the transverse distance of 60 and 60 equal to the length of the radius of the circle, and with that opening, from any point A as a centre, describe the arc BC. Take the transverse distance of the given degrees 46, and lay this distance on the arc from the point B to C. Join AC, AB; the angle CAB is the one required.

2. *To lay down an angle of 148° (being more than 60°).* (*See* Fig. 146.) Describe the arc BCD, making the radius equal to the transverse distance of 60 and 60, as before. Take the transverse distance of $\frac{1}{2}$ or $\frac{1}{3}$, etc., of the given number of degrees, and lay this distance on the arc twice, or thrice, as from B to *a*, *a* to *b*, and *b* to D. Join BA, AD; BAD is the angle required.

LINE OF POLYGONS. This scale is chiefly used for the ready division of the circumference of a circle into any number of equal parts from 4 to 12; it forms, therefore, a ready means of inscribing regular polygons in a circle. To do this, set off the radius of the given circle (which is always equal to the side of the inscribed hexagon) as the transverse distance of 6 and 6 upon the line of polygons. Then the transverse distance of 4 and 4 will be the side of the inscribed square; that of 5 and 5 the inscribed pentagon, that of 7 and 7 the inscribed heptagon, and so on.

18. CONTENTS OF A POCKET-CASE OF INSTRUMENTS. (Figs. 133, 138, 168, 185.) A small serviceable pocket-case of mathematical drawing instruments generally contains (1) a pair of compasses with fixed points (*see* Fig. 133), (2) a rectangular protractor, 6″ (*see* Figs. 168, 185), and (3) a parallel ruler 6″ (*see* Fig. 138, *a*). These are all that are necessary for ordinary field use.

19. BEAM COMPASSES. (Fig. 183.) The beam is of ebony or other well-seasoned wood, from 2 to 4 feet in length, to which are adapted two brass boxes, one of which is fixed to the end of the beam, while the other can be made to slide up and down, and be clamped to any part of it by a mill-headed screw. The fixed box carries a steel point, and the moveable box a tube for holding a steel pen or pencil point. Along the beam is usually inlaid a thin slip of brass or box-wood flush with the wood, on which is engraved a scale of inches: unless,

however, the points attached to the beam are truly parallel to each other, it is clear that the correct length will not be indicated on the scale, and it is preferable therefore to measure off the required distance on an independent scale. Beam compasses are very useful for setting off perpendiculars and measuring long distances required in laying down co-ordinate or graticule lines for the projection of large maps.

20. Plotting Instruments. For a description of plotting instruments and scales, refer to (Chap. VI).

21. Drawing Pins. (Fig. 440.) There are several methods of attaching paper to the drawing board; one of the most simple is by means of *drawing pins*, which are merely pressed through the paper into the board. The *Drawing Pin* is a small flat circular piece of metal, with a moderately stout needle point screwed into the centre of one side. The upper side of the head of the pin should be a portion of a sphere, the edges being thin, so as to lie close to the paper, otherwise it will injure the T square when passing over it. The steel point of the pin should not be too much tapered, or it will be continually flying out. For securing tracing paper, pins with large heads should be used.

CHAPTER VI.

Scales.

SIMPLE SCALES.

1. Scale defined. A *Scale* is a collection of equal parts used for measuring straight lines and laying down distances, each part answering for one foot, one yard, one chain, one mile, etc., according to the unit assumed. The greater the number of parts contained in an inch, the smaller will be the plan; for instance, if the scale is 1 mile = 1 inch, then a distance between two points which is 1 mile will be represented on the map by a distance of one inch, and so on; but if the scale is 2 miles = 1 inch, then a distance of 1 mile will be represented on the map by half an inch, and so on.

2. Division of Scales. All scales should be divided to units, tens, hundreds, etc., or else tenths, hundredths, etc. (decimals)—whether of miles, chains, yards or feet, as may be required. They ought always to represent a distance which is a multiple of 10 lineal units of some kind; as 80 miles, 50 yards, 100 feet, etc.

3. Subdivisions. These divisions are again subdivided according to the degree of minuteness sought and are called *subdivisions*. When one of the divisions is a foot, it may be divided to measure inches or tenths of decimals, as the distances required are in feet and inches or in feet and tenths. Subdivisions are made as small as can be clearly drawn and distinguished.

4. Uniformity a great requisite. Scales should be uniform and convenient; for example, 4 inches to 1 mile and not 3¾ inches to 1 mile. In a map, a square inch should represent an integral number of square miles or acres. When areas are required, the number of chains to the inch should be considered. A little care in the selection of a scale at the commencement increases the value of the map.

5. Primary Divisions. The *primary divisions* of a scale are the largest divisions marked thereon.

6. Simple and Diagonal Scales defined. Scales are of two kinds, either *simple* or *diagonal*. In the former the subdivisions are made on one line and to the limit of distinctness; in the latter they are made on several lines constructed on the principle explained in Art. 26, and may be carried to any degree of minuteness.

7. Construction of Simple Scales. Simple scales consist of any extent of equal divisions, which are numbered and reckoned forwards from the second division on the left hand, to the right: 0 (zero), 1, 2, 3, etc., and from the

same division backwards to the left: 0 (zero), 1. The primary division on the left is then subdivided into ten equal parts, if it is desired to construct a scale a multiple of 10. If then, these subdivisions be taken as units, each to represent one mile, one chain, or one foot, etc., the primary divisions will be so many tens of miles, or of chains, or of feet, etc.; if the subdivisions are taken as tens, these primary divisions will be hundreds; and, if the primary divisions be units, the subdivisions will be tenths.

8. MANNER OF DRAWING SCALES. Scales are first drawn in pencil, then inked in.

9. LENGTH OF SCALE. In constructing a scale consider first what will be a good length for it, so as to be able to take off it, at one measurement, long lines, though not necessarily the longest in the drawing. This is done by means of simple proportion, as follows:—

(*a*) If in a scale of 5 feet to an inch, a length of 30 feet is required to be shown, the necessary length will be—

as 5 feet : 30 feet : : 1 inch : $\frac{30 \times 1}{5} = 6$ inches, the required length.

(*b*) If in a scale of 2 miles to an inch, it is required to show 15 miles, then—

as 2 miles : 15 miles : : 1 inch : $\frac{15 \times 1}{2} = 7{\cdot}5$ inches, the required length.

(*c*) If in a scale of 6 inches to the mile, it is required to show distances up to 6,000 feet, then—

as 1 mile or 5,280 feet : 6,000 feet : : 6 inches : $\frac{6{,}000 \times 6}{5{,}280} = 6{\cdot}82$ inches.

10. IVORY SCALE. (Fig. 168.) A mathematical instrument box generally contains a brass or *ivory scale* with eight simply divided scales marked on it. To adapt them to feet and inches or feet and tenths, the first primary division on the left is subdivided into twelve equal parts on the lower line for the inches, and into ten equal parts on the upper line for the tenths. The scales are of the following lengths*: $\frac{1}{8}$, , $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and 1 inch; that is,

1 inch = 8 feet, or 1 inch = 8 feet, or 1 inch = 8 feet.
2 ,, = 8 ,, ,, 1 ,, = 4 ,, ,, 1 ,, = 4 ,,
3 ,, = 8 ,, ,, 3 ,, = 8 ,, ,, 1 ,, = $2\frac{2}{3}$,,
4 ,, = 8 ,, ,, 1 ,, = 2 ,, ,, 1 ,, = 2 ,,
5 ,, = 8 ,, ,, 5 ,, = 8 ,, ,, 1 ,, = $1\frac{3}{5}$,,
6 ,, = 8 ,, ,, 3 ,, = 4 ,, ,, 1 ,, = $1\frac{1}{3}$,,
7 ,, = 8 ,, ,, 7 ,, = 8 ,, ,, 1 ,, = $1\frac{1}{8}$,,
8 ,, = 8 ,, ,, 1 ,, = 1 foot ,, 1 ,, = 1 foot.

To lay down 360, or 36, or 3·6, etc., from any one of these scales, extend the compasses from the primary division numbered 3 to the 6th upper subdivision, reckoning backwards, or towards the left hand. To take off any number of feet and inches, 6 feet 7 inches for instance, extend the compasses from the primary division numbered 6, to the 7th lower subdivision, reckoning backwards.

Some ivory scales have decimal diagonal scales also marked thereon, the primary divisions of which are divided into inches, $\frac{1}{2}$-inches, or $\frac{1}{4}$-inches, while on the reverse side there appears a 6-inch rectangular protractor.

11. REPRESENTATIVE FRACTION. The ratio which the measurements of an object surveyed in the field bear to the corresponding measurements on the plan is denoted by a fraction called a *Representative Fraction*. Thus, suppose the actual distance between two objects A and B was 60 inches, and they were shown on the plan 1 inch apart, the representative fraction would be $\frac{1}{60}$, that is, the representation of the distance on paper is one-sixtieth of its magnitude in the

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OBSERVATION.

* Other scales are marked with the numbers 10, 20, 30, 40, 50, 60, 70, and 80, which simply represent the number of parts into which the inch is divided; that is, on the scale of 10, tenths of an inch can be taken off; on the scale of 20, twentieths of an inch; and so on.

field. So also, in a scale of 4 chains to the inch, or 4 chain lengths in the field = 1 inch on the paper, the representative fraction would be $\frac{1}{4 \times 66 \times 12} = \frac{1}{3,168}$, that is, the representation on paper is one-three thousand one hundred and sixty-eighths of the measurement in the field. In other words, to find the representative fraction of a scale, reduce both terms to a like quantity, as inches, place the lesser upon the greater, and simplify the fraction; and, conversely, if the fraction is given, the scale may be found.

EXAMPLES IN SIMPLE SCALES.

12. EXAMPLE I. *Construct a scale of 5 feet to an inch, to measure 30 feet* (*see* Art. 9, *a*), *and reading to a foot.* (Fig. 163.)

Draw two parallel lines 6 inches in length and $\frac{1}{15}$th of an inch apart, taking the measurements from a brass or ivory scale of inches. Divide the lower line into 3 parts, or primary divisions of 10 each, then subdivide the left-hand one of these into ten equal parts (by Prob. V of Chap. I, Art. 92). The primary divisions will then represent 10 feet and the subdivisions 1 foot, so any length up to 30 integral feet may be taken off. The line at the right hand end of the secondary divisions is marked 0 (zero); the primary divisions are numbered 10, 20, from zero to the right of it, and of the subdivisions to the left of it, 5 at the middle and 10 at the end; the word expressing the units of the measure (in this case, *feet*) is written on the right, immediately after the last number, as without this the scale would be useless. Now, to take 26 feet off the scale: Place one leg of the compasses on the primary division marked 20 and the other leg on the subdivision marked 6; the length included between the points of the compasses represents 26 feet.

13. EXAMPLE II. *Construct a scale of 2 miles to an inch, showing 15 miles* (*see* Art. 9, *b*), *reading to furlongs.* (Fig. 166.)

Set off a line 7·5 inches long, and divide it into 15 equal parts (by Prob. V of Chap. I, Art. 92); each division will equal one mile. Subdivide the left-hand primary division into 8 equal parts, as there are 8 furlongs in a mile, to show furlongs, and the scale is completed. Now, to take off the scale 4 miles 7 furlongs: Place one point of the compasses on the primary division marked 4 and the other point on the subdivision marked 7.

14. EXAMPLE III. *Construct a scale of 6 inches to the mile, showing 6,000 feet* (*see* Art. 9, *c*), *reading to 100 feet.* (Fig. 167.)

Draw a line 6·82 inches in length, and divide it into 6 equal parts (by Prob. V of Chap. I, Art. 92); also subdivide the left-hand division into tenths; each of the subdivisions shows 100 feet.

15. EXAMPLE IV. *Construct a scale of 6 feet to the inch.* (Fig. 169.)

Since 6 feet = 1 inch, the length of a primary division, or 10 feet, is equal to—

$$\text{as } 6 \text{ feet} : 10 \text{ feet} :: 1 \text{ inch} : \frac{10 \times 1}{6} = \frac{5}{3} = 1\cdot\dot{6} \text{ inches.}$$

If 30 feet, or 3 primary divisions, are required, they will equal $1\cdot\dot{6} \times 3 = 5$ inches.

Draw a line, and lay off on it 5 inches, taken from a scale of inches. Divide it into 3 equal parts, each will therefore equal 10 feet; subdivide the left-hand segment into 10 equal parts (by Prob. V of Chap. I, Art. 92), each of which will equal 1 foot. Number as shown in the figure, and we have a scale of tens of feet, with the left hand primary division divided into units.

16. EXAMPLE V. *Construct a scale of 4 inches to the mile, to read hundredths.* (Fig. 170.)

Take AB = 4 inches and divide it into 10 equal parts (by Prob. V of Chap. I, Art. 92), then each part is $\frac{1}{10}$ or 0·1 of a mile. Again divide the left-hand segment geometrically into 10 equal parts, each of which will be $\frac{1}{10} \times \frac{1}{10}$, or 0·01 of a mile. Lay off on AB produced, BC = one primary division. The scale is constructed as required.

17. EXAMPLE VI. *Construct a scale of 2 chains to 1 inch, in links, and also in chains.* (Fig. 172.)

Since 1 inch represents 2 chains, 10 chains will be equal to—

as 2 chains : 10 chains : : 1 inch : $\frac{10 \times 1}{2} = 5$ inches.

Take AB = 5 inches, and divide it into 10 equal parts (by Prob. V of Chap. I, Art. 92), each of which will therefore be 1 chain or 100 links. In AB produced lay off BC = 1 of these parts, and from C to the right lay off distances of 1,000 links or 10 chains, as many as may be necessary. Divide the left-hand primary division into 10 parts, then each subdivision will be $\frac{1}{10}$ of 100 links or 1 chain, that is, 10 links or $\frac{1}{10}$ chain, according to the upper and lower scales required. Number the divisions as in the figure.

18. EXAMPLE VII. *Construct a scale of 5 chains to 1 inch, in feet.* (Fig. 173.)

5 chains are equal to $5 \times 66 = 330$ feet, and these are to be represented by 1 inch. It is necessary to ascertain what length will represent a primary division of 100 feet.

∴ as 330 feet : 100 : : 1 inch : $\frac{100 \times 1}{330} = \frac{10}{33} = 0{\cdot}303$ inch;

and if 1,000 feet are required to be shown, its length on the paper $= 0{\cdot}303 \times 10 = 3{\cdot}03$ inches. Mark off, from a scale of inches, BC = 3·03 inches, and subdivide it into 10 equal parts, each part will therefore = 100 feet; produce CB to A, making AB = a primary division of 100 feet, and subdivide it into 10 equal parts, each of which will equal 10 feet.

19. EXAMPLE VIII. *Construct a scale of 100 feet to 1 inch, in chains and links.* (Fig. 174.)

Since 100 feet = 1 inch, ∴ a primary division of 1 chain or 66 feet =

as 100 feet : 066 feet : : 1 inch : $\frac{66 \times 1}{100} = \frac{33}{50} = 0{\cdot}66$ inch.

If a distance of 6 chains is required to be taken off the scale, its length on paper will $= 0{\cdot}66 \times 6 = 3{\cdot}96$ inches.

Lay off BC = 3·96 inches, and divide it geometrically into 6 equal parts (by Prob. V of Chap. I, Art. 92), each of which will therefore equal 1 chain or 100 links. Produce CB the length of 1 primary division to A, and subdivide it into 10 equal parts, each of which will be 0·1 of a chain, or 10 links. Number as in the figure, and the scale will be complete.

20. EXAMPLE IX. *Construct a scale of 5 inches to a mile, in feet.* (Fig. 175.)

Since 5 inches = 1 mile or 5,280 feet, ∴ a primary division of 1,000 feet is equal to—

as 5,280 feet : 1,000 feet : : 5 inches : $\frac{1{,}000 \times 5}{5{,}280} = \frac{125}{132} = 0{\cdot}948$ inch.

If a distance of 5,000 feet is required to be shown, its length $= 0{\cdot}948 \times 5 = 4{\cdot}74$ inches. From a scale of inches, lay off a length BC of 4·74 inches which divide into 5 equal parts, each of which therefore equals 1,000 feet. Produce CB the length of one primary division to A, and subdivide this left-hand segment into 10 equal parts, each of which will equal 100 feet.

21. EXAMPLE X. *Construct a scale of 6 inches to a mile, in yards.* (Fig. 176.)

If 6 inches = 1 mile or 1,760 yards, ∴ a distance of 1,000 yards, if that is the length required to be measured, is equal to—

as 1,760 yards : 1,000 yards : : 6 inches : $\frac{6 \times 1{,}000}{1{,}760} = \frac{75}{22} = 3{\cdot}41$ inches.

From a scale of inches lay off AB = 3·41 inches and divide it equally into 10 parts, each part ∴ equalling 100 yards; on BA produced to the left mark off AC equal to a primary division or 100 yards, subdivide this into 10 equal parts, each of which will equal 10 yards, and the scale is constructed.

22. EXAMPLE XI. *Construct a scale of 3 inches to a mile, in chains.* (Fig. 177.)

Since 3 inches = 1 mile or 80 chains, if 100 chains is to be measured off the scale, it will equal—

as 80 chains : 100 chains :: 3 inches : $\frac{100 \times 3}{80} = \frac{15}{4} = 3·75$ inches.

Lay off a length BC of 3·75 inches, and divide it into 10 equal parts, each of which therefore represents 10 chains; produce CB the length of one primary division to A, and subdivide this left-hand segment into 10 equal parts, each of which will be equal to 1 chain. Number the divisions as in the figure, and the required scale will be made.

23. MEASUREMENT BY INSPECTION. (Fig. 178.) In measuring any distance, say, of 3·64 feet, off a simple scale divided into divisions of feet and tenths only, the hundredths can be obtained *by inspection;* for instance, in the figure, place one point of your compasses at the primary division 3, and the other between the 6th and 7th subdivisions, conceiving that subdivision to be divided off into 20 equal parts, at the 4th conceived division from the 6th to the left, as if such a division had existed.

DIAGONAL SCALES.

24. USE OF DIAGONAL SCALES. When a line is to be divided into equal parts, so numerous and minute that they would be indistinct, it is usual to construct a *diagonal scale,* which is simply a plain one having the primary and secondary divisions constructed in the usual way, with the addition of lines drawn according to the diagonal method, hereafter explained, for further subdivision; for instance, in the simply divided scales one of the primary divisions is subdivided only into 10 equal parts, and the parts of any distance which are less than tenths of a primary division cannot be accurately taken off from them; but, by means of a diagonal scale, the parts of any distance which are the hundredths of the primary divisions are correctly indicated, as will be understood from the examples that follow.

25. CONSTRUCTION OF DIAGONAL SCALES. (Fig. 179.) To construct a diagonal scale—for instance, a scale which will read to tenths and hundredths. Draw 11 parrallel equidistant lines containing 10 spaces of arbitrary length (the whole usually measuring one inch); divide the uppermost of these lines into equal parts of the intended length of the primary divisions; and through each of these divisions draw perpendicular lines, which are consequently parallel to each other, cutting all the 11 parallels, and number these primary divisions 1, 2, 3, etc., to the right beginning from the second; also write the numbers 2, 4, 6, 8, *upwards,* at the left hand ends of the alternate horizontals. Subdivide the first primary division, which is on the left hand side, into 10 equal parts, both upon the highest and lowest of the 11 parallel lines, and let these subdivisions be reckoned in the opposite direction to the primary divisions, namely, to the left from zero, as in the simply divided scales. Draw the diagonal lines from zero point below to the first subdivision above, from the first below to the second above, from the second below to the third above; and so on, till a line from the ninth below to the tenth above is arrived at. Now since any one of the subdivisions on the upper parallel is equal to any one of the subdivisions on the lower parallel, and the diagonal lines decline, in their entire lengths from the perpendicular by intervals equal to one of the secondary parts, and because they are cut transversely into 10 other equal parts by equidistant parallel lines, therefore each subdivision is further subdivided into 10 equal parts measured along its corresponding diagonal line; so that every diagonal line, as it reaches its successive parallel, separates further from its perpendicular by one-tenth of the extent of a subdivision, or one-hundredth of the extent of a primary division.

26. MODE OF MEASUREMENT. (Fig. 179.) Taking the primary divisions to be *units,* to set off the number 5·74 by the diagonal scale. Set one foot of the compasses on the point where the fifth parallel cuts the eighth diagonal line, and extend the other foot to the point where the same parallel cuts the fifth

vertical line, namely, from o to o. Reckoning the primary divisions as *tens*, to take off the number 46·7. Extend the compasses from the point where the eighth parallel cuts the seventh diagonal to the point where it cuts the fourth vertical namely, from * to *. The primary divisions being *hundreds*, to take off the number 253. Extend the compasses from the point where the fourth parallel cuts the sixth diagonal to the point where it cuts the second vertical, namely, from × to ×

27. GENERAL RULE TO TAKE OFF ANY NUMBER TO THREE PLACES OF FIGURES UPON A DIAGONAL SCALE. On the *parallel* indicated by the *third* figure, measure from the *diagonal* indicated by the *second* figure, to the *vertical* indicated by the first.

EXAMPLES IN DIAGONAL SCALES.

28. EXAMPLE I. *Construct a scale of 9 miles to the inch, showing furlongs diagonally.* (Fig. 180.)

Proceed as in the plain scale. Since 9 miles = 1 inch, 60 miles is equal to—

as 9 miles : 60 miles : : 1 inch : $\frac{60\times 1}{9} = \frac{20}{3} = 6·67$ inches.

Lay off from a scale of inches a line equal to 6·67 inches, which divide into 6 equal parts; each part will therefore be equal to 10 miles; divide the left-hand primary division into tenths, each of which represents one mile. Above this line, since it is required to show furlongs or eighths of a mile, draw 8 equidistant parallel lines about one-eighth or one-tenth of an inch apart. From the bottom to the top line draw perpendiculars through the primary points of division. Subdivide the left-hand segment on the uppermost line into 10 equal parts. Draw diagonal lines from zero below to the first subdivision above, from the first subdivision below to the second above, from the second below to the third above, and so on, successively, till you arrive at the line drawn from the ninth below to the tenth above. Write the numbers 2, 4, 6, 8, *upwards*, at the left hand ends of the alternate horizontals, and the word "furlongs" beside the figure 8; and below the lowest horizontal line insert the usual figures, printing the word "miles" on the extreme right. To measure 26 miles 5 furlongs, and 44 miles 3 furlongs, place the points of your compasses at *a* and *b*, and *x* and *y* respectively.

29. EXAMPLE II. *Construct a scale of 9 feet to the inch, showing inches diagonally.* (Fig. 181.)

Proceed as in (Art. 28), drawing, in addition, four more horizontal lines equidistant as before. To measure 37 feet 9 inches, place the compasses at *p* and *q*.

30. EXAMPLE III. *Make a diagonal scale of 2 inches to a foot, to read to* $\frac{1}{100}$ *of a foot.* (Fig. 195.)

If 2 inches = 1 foot, 2 feet, if that is the greatest length required,—

as 1 foot : 2 feet : : 2 inches : $\frac{2\times 2}{1} = 4$ inches.

Lay off a length of 4 inches, and divide it at B into 2 equal parts, each of which is a primary division of 1 foot. Subdivide the left-hand segment A B into 10 equal parts, each of which = $\frac{1}{10}$ or 0·1 of a foot. Parallel to and equidistant from AB and each other draw 10 lines; also erect perpendiculars AC and BD from the points of primary division. Divide CD similarly to AB, and join the diagonals. Then the smallest division will be $\frac{1}{100}$ or 0·01 of a foot; and a scale should be numbered as shown in the figure.

To take off 1·35 feet, place the points of the compasses at *x* and *y*.

31. EXAMPLE IV. *Construct a diagonal scale of 4 inches to the mile, in yards, to read to 10 yards.* (Fig. 182.)

Since 4 inches = 1 mile, or 1,760 yards, 2,000 yards, if that is the distance required to be shown,—

as 1,760 yards : 2,000 yards : : 4 inches : $\frac{2{,}000\times 4}{1{,}760} = \frac{50}{11} = 4·54$ inches.

Lay off a length of $4 \cdot \dot{5}\dot{4}$ inches, and divide into 2 equal parts, each of which is a primary division of 1,000 yards. Draw parallel to and equidistant from this line and each other 10 lines. Erect perpendiculars AC, BD, from the primary points of division; divide CD similarly to AB, and draw the diagonals. Then the smallest subdivision will be 10 yards, and the scale should be numbered as shown in the figure. To take off 1,230 yards, place the points of the compasses at *m* and *n*.

32. EXAMPLE V. *Construct a diagonal scale o 5 chains to the inch, or 16 inches to the mile, in links.* (Fig. 126.)

If 5 chains or 500 links = 1 inch, 3,000 links, if that is the greatest length required to be shown, =

$$\text{as } 500 \text{ links} : 3{,}000 \text{ links} : : 1 \text{ inch} : \frac{3{,}000 \times 1}{500} = 6 \text{ inches}.$$

Lay off a length of 6 inches, and divide it into 3 equal parts, or primary divisions of 1,000 links. Draw 10 parallel lines, as before; erect the perpendiculars through the primary points of division; subdivide the upper and lower left-hand segments into 10 equal parts, and draw the diagonals. Write in the numbers, and the scale is ready.

To take off 27·60 chains or 2,760 links, place the points of the compasses at *a* and *b*.

33. MEASUREMENT BY ESTIMATION. (Fig. 182.) Any distance less than 10 yards can be *estimated* with sufficient accuracy on small scales. To obtain 1,375 yards, for instance, place the compasses at *r* and *s*.

VERNIER SCALES.

34. USE AND CONSTRUCTION OF VERNIER SCALES. Vernier Scales are sometimes used instead of diagonal scales. Their construction is similar to that of the verniers applied to the graduated arcs of surveying and astronomical instruments (*see* Chap. XIII, Art. 11), the only difference being that a vernier attached to a plain scale is usually fixed, and not moveable as in surveying instruments. The nature of these scales will be best understood from their construction.

EXAMPLES IN VERNIER SCALES.

35. EXAMPLE I. *To construct a scale of* $\frac{1}{100}$, *to show feet and tenths.* (Fig. 200.)

Let the scale be drawn in the ordinary way, but subdivided throughout its entire length; each subdivision shows one foot; set off to the left, on the upper line, a distance equal to 11 subdivisions, commencing from the zero of the scale; divide this into 10 equal parts as in the figure. Since 11 subdivisions of the plain scale have been divided into 10 equal parts on the vernier scale, each division on the vernier will represent $\frac{11}{10}$ = 1·1 of the subdivisions on the plain scale, and as these show feet, each division on the vernier will show 1·1 foot; consequently the several distances from the zero of the scale to the successive divisions on the vernier will show 1·1, 2·2, 3·3, 4·4, 5·5, 6·6, 7·7, 8·8, 9·9, and 11 feet. The scale is used thus:—

Let it be required to take off 26·7 feet. Subtract 7·7 from 26·7, the remainder is 19; place one point of the compasses on the 19th subdivision on the upper line of the plain scale, and the other on the 7th division of the vernier; this distance represents 26·7 feet. The reasoning is obvious. Besides having to take off 26 feet, 0·7 of a foot more is required.

As above shown, the distance from zero to the 7th division of the vernier represents 7·7 feet. Take off this distance and add to it whatever may be required (in this case 19) to make up the distance sought.

36. EXAMPLE II. *To construct a scale of* $\frac{1}{100}$, *to show feet and inches.* (Fig. 201.)

The plain scale required for this will be precisely the same as that drawn in the above example (*see* Art. 35), but the vernier is different as twelfths instead of tenths have to be shown.

Here take 13 subdivisions on the plain scale, divide them into 12 equal parts; the value of each vernier division will be $\frac{13}{12} = 1\frac{1}{12}$th of a foot, *i.e.*, 1 foot 1 inch, so the several distances on the vernier measured from zero will show 1 foot 1 inch, 2 feet 2 inches, 3 feet 3 inches, and so on. To take off 22 feet 10 inches. Subtract 10 feet 10 inches from 22 feet 10 inches, 12 feet are left; then the distance from the 12th subdivision on the plain scale to the 10th division on the vernier will represent 22 feet 10 inches.

PROTRACTORS, PLOTTING INSTRUMENTS, ETC.

37. PLOTTING SCALES. (Fig. 441.) *Plotting scales, also called bevelled-edge scales*, are straight rules, made of ivory or boxwood, and are usually about 10 or 12 inches in length by about an inch in breadth. Each rule has scales of equal parts, decimally divided, placed upon its edges, which are made sloping, so that the extremities of the strokes marking the divisions may be close to the paper. The primary divisions represent chains, and the subdivisions, consequently, ten links each, as there are 100 links on the surveying chain. Plotting scales may be procured in sets, each with a different number of chains to the inch. The advantages of this arrangement are, that the distances required can be transferred with great expedition from the scale to the paper by the aid of the pricking point alone, and the marks denoting the divisions are in no danger of becoming defaced, as upon the plain scale, by the frequent application of the compasses. They are also made on card-board. Smaller scales, consisting of the same material as their corresponding plotting scales, are frequently provided in the sets above referred to. They are about two inches long and an inch broad, with the divisions commencing at the extreme end of the scale, and are similarly divided to the plotting scales. Thus on a scale of 2 chains to an inch, the offset will consist of four primary divisions of 1 chain, or 40 subdivisions of 10 links each.

For setting off offset distances, the larger (or plotting) scale is laid on the chain line, while the smaller (or offset) scale is made to slide up and down, to the right or left, according as the plotting scale is fixed, and the required points thus plotted.

38. OFFSET SCALE. (Fig. 194.) The figure represents a contrivance for an *offset* scale; the cross scale B slides along the scale A, the whole length of the groove, and at right angles to it. The graduations on both the scales represent either feet or links or other unit of measurement. To apply it to use: place the scale AA on the paper, parallel to the line on which the offsets are to be plotted, and at such a distance that the zero division on the cross scale B (which is placed about its middle) may coincide with it as the scale slides along, and also that the zero of the scale A may be exactly opposite that end of the line at which the measurement commenced; then, in sliding the scale B from the beginning of the line, stop it at every divisional line on A corresponding to the distance on the station line at which an offset was taken, and lay off the exact length of the offset from the edge of the scale B, either to the right or left of the station line, to which it will be at right angles as taken in the field; the instrument thus gives both dimensions at the same time.

39. GUNTER'S SCALE. *Gunter's Scale* is a brass scale 2 feet in length, on which is placed a scale of inches, with a combined inch and half inch diagonal scale. It is convenient to use it in plotting long distances which cannot be measured on the ordinary 6-inch protractor.

40. PROTRACTOR. (Fig. 185.) The *Protractor* is an instrument used to lay down an angle containing any number of degrees, or to find how many degrees are contained in any given angle; it is so called as it protracts or lengthens the lines which form the angle. It is made in two shapes; the first, that of a *semicircle*, as ABC, divided in its arc into 180 equal parts or degrees, having a double row of figures numbered to the right and left; the second and more modern, that of a *rectangle*, as *abcd*, usually made of ivory or electrum, 6 inches long by $1\frac{3}{4}$ inches broad. The divisions forming the degrees are laid as accurately on the one as on the other, as will be seen on an inspection of the figure. In the middle of the chord forming the diameter is a fine notch ***a***, the centre of the circle. The

edges are bevelled off fine so as to admit of accuracy in use. Protractors are also made *circular*. Those on card-board, known as the "Ordnance pattern," and divided to quarter-degrees, are the best description.

41. To construct a Semicircular Protractor. (Fig. 184–185.) On any diameter AB (*see* Fig. 184) describe a semicircle, and divide it into two quadrants by drawing the radius OC perpendicular to AB. From the points A, B, and C, with the radius of the semicircle, intersect the circumference in D, E, G and H, thus dividing it into six equal parts of 30° each. Draw the chord AC and perpendicular to it the radius OK, bisecting the arc GD as also the quadrant in K. Set off GK or KD round the circumference, and thus divide it into arcs of 15°. By trisecting any one of these (by trial), arcs of 5° will be obtained, one of which must be subdivided (also by trial) into five equal parts to arrive at arcs of 1°. The protractor may then be completed, as shown in (Fig. 185.) It will have been observed that after getting arcs of 15°, the lower subdivisions are obtained by trial; this, if carefully done, would be sufficiently accurate; but less risk of error, with probably greater exactness, would result by laying off the arcs by means of their chords obtained from a scale of chords. From an inspection of (Fig. 185), it will be seen that a rectangular protractor is constructed by simply drawing the different radii so as to intersect the edge of a rectangle placed on the diameter of the semicircular protractor, symmetrically with reference to the centre *a*.

APPLICATION OF THE PROTRACTOR TO USE.

42. Problem I. *A number of degrees being given, to protract or lay down an angle whose measure shall be equal thereto.* (Fig. 123.)

To lay down an angle of 130 degrees from the point O of any line OA, apply the diameter of the protractor to the line OA, so that the centre thereof may coincide exactly with the point O; then, with a pencil or pin make a fine dot at D, against 130, upon the limb of the protractor; now remove the protractor, and draw a line from O through this dot, and the angle AOD contains the given number of degrees.

43. Problem II. *To find the number of degrees contained in a given angle AOD.* (Fig. 123.)

Place the centre of the protractor upon the angular point O, and the diameter exactly upon the line OA; then the degree upon the limb that is cut by the line OD will be the measure of the given angle, which, in the present instance, is found to be 130 degrees.

44. Problem III. *From a given point O, in the line OE, to erect a perpendicular on that line.* (Fig. 123.)

Apply the protractor to the line, OA, so that the centre may coincide with the point O, and the division marked 90 may be cut by the line OE; then a line AB drawn against the diameter of the protractor will be perpendicular to OE.

45. Circular Protractor. (Fig. 186.) The *circular protractor* is a complete graduated circle AA connected with its centre by 4 radii, *a*, *a*, *a*, *a*. It carries 2 moveable bars *c*, *c*, at one extremity of which is a pinion *d* working in a rack, and at the other a fixed vernier, which subdivides the primary divisions on the protractor to single minutes, and by estimation to 30 seconds. Each bar has a branch *e*, with a steel pricker at its end. These prickers make the necessary punctures in the paper. To use the instrument, set the vernier to zero on the limb and then place it on the paper with the centre over the station and the 2 steel points on the given line (from whence other angular lines are to be drawn). Now turn the pinion *d* till the opposite vernier reads the required angle, then press the branches *e*, *e*, lightly, which will cause the steel points to make punctures in the paper at opposite sides of the circle In this manner, at one setting of the instrument, a great number of angles, or a complete circular protractor, may be laid off from the same point.

46. Circular Talc Protractor. (Fig. 196.) A very serviceable protractor can be made in the following manner, especially for field-work :— Draw on

a piece of paper a protractor of the same size as in the figure; cut out the circle and the part marked A, and stick the remainder with a little clean paste on a thin piece of talc; when dry, cut the talc with a pair of scissors to the shape of the exterior circle. Draw on the piece of talc by scratching, with a needle or point of a pricker, the N and S, and E and W lines, intersecting one another in the centre, and also all the degree lines and circles. Rub these lines over with a mixture of black-lead and oil which will make the marks indelible; and the protractor becomes ready for use.

47. CIRCULAR CARD PROTRACTOR. (Figs. 355, 385.) This protractor (*see* Fig. 355) may be obtained from any instrument-maker, and is very convenient to use.* It is engraved on a square piece of card-board (about 2 or 3 inches in excess, all round, of the divided circle), and is divided into 360 parts or degrees, the larger ones having these subdivided to half and quarter degrees. It is usually made from 6 to 12 inches in diameter, the edge of the *innermost* circle being neatly cut out with a penknife. It has *two* sets of figures, one beginning at the *north* point from 0 back to 360°, and the other, immediately above the former, beginning at the *south* point, also from 0 back to 360°. This description of protractor is very serviceable in plotting, but only small estates and plans as come within the central cut out circle of the card†; and by its use inaccuracies and errors of plotting, otherwise pretty certain, are obviated, since all the points of the survey are fixed from a single position of the protractor on the paper. For this protractor it is best to employ a parallel ruler on wheels.

Suppose it was required to plot a survey (traverse‡) on a scale of 500 links = 1 inch, of which the following are the bearings‡ and distances of the several chain lines: –

Lines.	Bearings.	Distances.	Lines.	Bearings.	Distances.
AB	52° 00′	917 links.	EF	207° 30′	1,440 links.
BC	106 15	1,204 „	FG	235 15	1,850 „
CD	355 45	1,400 „	GA	172 15	2,114 „
DE	40 00	1,100 „			

Place the protractor on the paper, fastening its corners down with drawing pins to keep it from shifting, and within the central space choose a convenient point, as A, for the first station. Lay the edge of the ruler against the *two* opposite points of the graduated circle marked 52°, which is the bearing of the line AB, and roll it *down* to A, from which draw a fine pencil line in the N.E. direction, and on it set off the distance 917 links according to the scale on which the map is being protracted (namely, 500 links = 1 inch) for the point B. Next apply the ruler against the *two* opposite points marked 106° 15′, which is the bearing of the line BC, and roll its edge *down* to the point B, last fixed, from which rule a pencil line in the S.E. direction, and lay off on it from B 1,204 links, thus fixing the point C. Again apply the ruler against the *two* opposite points marked 355° 45′, and roll its edge *up* to the point C, last fixed, from which rule a pencil line in the N.W. direction, and set off on it 1,400 links, which fixes the point D. Repeat this process until all the stations have been plotted, when at the close it will

OBSERVATIONS.

* It can be procured from the Government Mathematical Instrument Department, Calcutta, and is known there as the "Ordnance pattern, Circular Card Protractor" of 12-inch diameter. Such a protractor may also be made by the Surveyor himself, by the method given in (Art. 41).

† Sometimes the plot is made on the card-board itself, when a larger drawing can be effected. In such a case the inner circle of the card must not be cut out. From the plot so made a trace of it can be taken on the ordinary drawing paper.

The protractor may also be used for laying down large estates and plans by cutting away the space outside of the *outermost* circle, and placing the protractor on the paper: then assuming the starting-point of the survey to be at the intersection of the NS and EW lines, mark off in succession the bearing of the several lines AB, BC, CD, etc., on the paper. Next lay aside the protractor, and obtain the position of A. Finally, proceed to lay down the positions of the other stations with the aid of a parallel ruler, and by setting off the distances as explained in the body of the article.

‡ These terms are explained further on, in (Chap. X).

be noticed that the bearing from the last point fixed, or G, will fall exactly into the first, or A, the length of the last line GA also coinciding with the measured distance, which closes the plot.

The smaller size card protractors (having the degree divisions marked inwards along the border of the *outer* circle, instead of being drawn outwards along that of the *inner* circle, as shown in Fig. 355) may be usefully employed in laying down minor surveys on separate sheets of paper. For let it now be required to plot an unclosed traverse survey (*see* Fig. 385) on the same scale as for the closed one, namely, 5 chains = 1 inch, the bearings and distances of the various chain lines being as follows:

Lines.	Bearings.	Distances.	Lines.	Bearings	Distances.
No. 1—AB	106° 05′	10·40 chains.	No. 5—EF	105° 25′	8·75 chains.
„ 2—BC	21° 30′	10·76 „	„ 6—FG	109° 35′	11·83 „
„ 3—CD	138° 25′	7·34 „	„ 7—GH	21° 55′	9·02 „
„ 4—DE	241° 40′	9·50 „	„ 8—HJ	171° 00′	10·27 „

Take a small, say, 4 or 6-inch card-board protractor, and cut out neatly with a penknife the edge of the outermost circle, making at the same time a tiny hole in the centre of it, so that the point over which it is placed may be visible. On the sheet of paper on which the plot is to be made select a convenient spot for your starting-point A, and draw a north and south line through it, also one east and west at right angles to this. Place the paper protractor in position over A, and lay some heavy weight on it that it may not be disturbed. With a pricker or fine-pointed pencil mark off the bearing of the 1st line AB on the paper against the *two* opposite points of the protractor at the reading of 106° 05′. In the same way lay off the *two* opposite points for each of the readings 21° 30′ and 138° 25′, the bearings respectively of the 2nd and 3rd lines, BC and CD. Remove the protractor, and draw straight lines through 1 and 1, 2 and 2, and through 3 and 3; if the work has been accurately done, all these lines will pass exactly through the centre. Next, on the first line 1 and 1 plot off your measured distance AB, of 10·40 chains, to scale; get your parallel ruler on 2 and 2, and run it down to B; draw a straight line through this point in the direction of C, which protracts the bearing of BC, and set off on it, according to scale, the length BC, or 10·76 chains, which fixes the point C. Again set your parallel ruler to 3 and 3, and run it down to C; draw a pencil line through this point in the direction of D, which gives also its bearing; on the line CD lay off the measured distance 7·34 chains, with scale, by which the position of D will be found. This carries us beyond a convenient range for our starting-point at which the protractor was first laid. Now through the last found point D, draw north and south, and east and west lines, as before, parallel respectively to the ones first drawn, and lay down the protractor in the same manner. Repeat here the process gone through at the station of origin, thus obtaining the positions of the points E and F. Lastly, lay down the protractor at F, go through the operation as before, until the positions of the points G, H, and J are obtained. In this manner a small compass survey may be both expeditiously and accurately plotted.

48. T SQUARE AND SEMICIRCULAR PROTRACTOR. (Figs. 187, 199.) By means of a T square and semicircular protractor (*see* Fig. 187), a survey can be most expeditiously as well as accurately plotted. The T square is made with 2 narrow pieces of wood, one of about 2 feet in length and the other 1 foot, being well fixed together in the form of the letter T. The shorter end of the rule slides along the edge of the drawing board, it having a projecting shoulder for the purpose. It is a most perfect kind of parallel ruler, and by the straight edge of the protractor (*see* Fig. 199) being applied against its long edge, is capable of carrying the protractor over the whole of a sheet of plotting paper of any size, and may be used upon a table of any form. It is convenient to suppose the work on the left-hand and the upper edge of the blade to represent the meridian of the station. It is clear therefore that angles taken with a theodolite or other instrument can be transferred to the plot as accurately as the protractor can be set.

49. PANTAGRAPH. (Fig. 198.) The *Pantagraph** is an instrument used for copying maps on the scale of the original, or for reducing and enlarging them. It consists of four rules, AB, AC, DF, and EF, usually made of brass. The two longer rulers, AB and AC, are connected together by, and have a motion round a centre at A. The two shorter rulers are connected in like manner with each other at F, and with the longer rulers at D and E, and, being equal in length to the portions AD and AE of the longer rulers, form with them an accurate parallelogram, ADFE, in every position of the instrument. The arms AB and DF are graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, etc., and have each a sliding index, which can be fixed at any of the divisions by a milled-headed clamping screw, seen in the diagram. The sliding indices have each of them a tube, adapted either to slide on a pin rising from a heavy circular weight called the *fulcrum*, or to receive a sliding holder with a pencil or pen, or a blunt tracing point, as may be required. The *fulcrum* is a leaden weight about which the whole instrument turns, the arms being supported by ivory castors which run freely in any direction. Above the pencil socket is a small cup into which shot can be thrown to increase the pressure of the pencil on the paper if required, but this ought to be avoided, if possible, as heavy pencil marks cannot be easily rubbed out and only disfigure the plan. When the instrument is correctly set, the *tracing point, pencil* and *fulcrum* will, from the property of similar triangles, be in *one* straight line, as shown by the dotted line in the figure, which is divided at G in the ratio required. While the point C, therefore, is carried along the boundaries of any figure, the intermediate point G will trace out a similar figure, reduced in the proportion of BC to BG or of BA to BD, the proportion required. The diagram represents the pantagraph in the act of *reducing* a plan to the scale of half the original. For this purpose the sliding indices are first clamped at the divisions upon the arms marked $\frac{1}{2}$; the tracing point is then fixed in a socket at C, over the original drawing; the pencil is next placed in the tube of the sliding index upon the ruler DF over the paper to receive the copy, and the fulcrum is fixed to that at B, upon the ruler AB. The instrument being now ready for use, if the tracing point at C be passed delicately and steadily over every line of the plan, a true copy, but of one-half the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread represented as passing from the pencil quite round the instrument to the tracing point at C, enables the draughtsman at the tracing point to raise the pencil from the paper, whilst he passes the tracer from one part of the original to another, and thus prevent false lines from being made on the copy. If the object were to *enlarge* the drawing to double its scale, then the tracer must be placed upon the arm DF, and the pencil at C, and if the copy were required of the same size as the original, then the sliding indices still remaining at the same divisions upon DF and AB, the fulcrum must take the middle station, and the pencil and tracing point those on the exterior arms, AB and AC, of the instrument.

GENERAL RULE.—*To reduce a drawing.* Fulcrum on AB; pencil on FD; tracer on EC.

To enlarge a drawing. Tracer on FD, pencil on EC; fulcrum on AB.

To make a copy on the same scale as the original. Fulcrum on DF; pencil on BD; tracer on AC.

Errors are so much increased by enlarging plans with the Pantagraph, that it ought not to be resorted to where accuracy is required.

The Pantagraph may be set to reduce drawings in two ways, termed technically, the *Erect manner* and the *Reverse manner*.

In the *erect* manner of setting the Pantagraph, the reduced copy will appear erect; that is, the same way upwards as the original. The fulcrum is placed on the long rule AB, and the pencil on the short rule DF. Thus set, the instrument will reduce in any of the given proportions engraved upon the rules, not exceeding half size. The following table gives the readings of the instrument which accord with the erect setting:

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* The original inventor of this useful instrument is not exactly known; [illegible] tract, published about the year 1631, by the Jesuit father Scheiner, entitled "Pantagraphice sive Ars nova Delineandi." The principles are self-evident to every geometrician; the mechanical construction was first improved by Mr. W. Jones, instrument-maker, Holborn, in the year 1750. Another instrument used for the same purpose as the Pantagraph is the Eidograph, for a description of which (*see* Art. 50).

Table of reductions by the Pantagraph in the Erect *manner.*

Readings given upon the scales.	Reduces in the proportion of	Readings given upon the scales.	Reduces in the proportion of
1 — 2	1 to 2	2 — 3	2 to 5
1 — 3	1 ,, 3	3 — 4	3 ,, 7
1 — 4	1 ,, 4	4 — 5	4 ,, 9
1 — 5	1 ,, 5	5 — 6	5 ,, 11
1 — 6	1 ,, 6	6 — 7	6 ,, 13
1 — 7	1 ,, 7	7 — 8	7 ,, 15
1 — 8	1 ,, 8	8 — 9	8 ,, 17
1 — 9	1 ,, 9	9 — 10	9 ,, 19
1 — 10	1 ,, 10	10 — 11	10 ,, 21
1 — 11	1 ,, 11	11 — 12	11 ,, 23

In the *reverse* manner of setting the Pantagraph, the reduced copy will appear reversed, or upside down, to the original. The fulcrum is placed on the short rule DF, and the pencil on the long rule AB. Thus set, the instrument will reduce in any of the given proportions engraved upon the rules, up to full size. The following table gives the readings of the instrument which accord with the reverse setting:—

Table of reductions by the Pantagraph in the Reverse *manner.*

Readings given upon the scales.	Reduces in the proportion of	Readings given upon the scales.	Reduces in the proportion of
1 — 2	1 to 1	2 — 3	2 to 3
1 — 3	1 ,, 2	3 — 4	3 ,, 4
1 — 4	1 ,, 3	4 — 5	4 ,, 5
1 — 5	1 ,, 4	5 — 6	5 ,, 6
1 — 6	1 ,, 5	6 — 7	6 ,, 7
1 — 7	1 ,, 6	7 — 8	7 ,, 8
1 — 8	1 ,, 7	8 — 9	8 ,, 9
1 — 9	1 ,, 8	9 — 10	9 ,, 10
1 — 10	1 ,, 9	10 — 11	10 ,, 11
1 — 11	1 ,, 10	11 — 12	11 ,, 12

If it were required to produce an enlarged copy with the Pantagraph, the pencil and tracer would have to change places, as before said; the proportions given in the above tables would of course read the same.

50. EIDOGRAPH. (Fig. 39.) The *Eidograph* is a far superior instrument to the Pantagraph, being more exact in its work, and is used for precisely the same purpose. The point of support when the instrument is at work is a heavy weight shown at H, from the under side of which three or four projecting needle points fix the instrument firmly to the drawing paper. Springing from this weight is a short standard or fulcrum, attached to a sliding box K, in which slides the centre beam C, and to any part of which it may be clamped by means of a clamping screw. At the ends of the central beam are two pulley wheels JJ, the centre pins of which revolve in sockets at the ends of the beam. Two steel bands, II, attached to the pulley wheels give them an exactly simultaneous motion, and these bands have a screw adjustment L, by means of which they may be tightened. The arms A and B are made to slide through boxes under the pulley wheels, and may be clamped at any proportion of their lengths in the same manner as the central beam C may be made to slide and clamp in the box K. The arm B carries a tracing point G, and the arm A carries a pencil point D. The pencil holder may be raised by means of a cranked lever F, attached to a cord E, which passes over the centre beam, and thence to the tracing point G. The two arms and the beam are divided into 200 equal parts, which are figured 100 each way from the centre, and may be read to 1,000th by means of the verniers on the sliding boxes. There is a loose weight which may be attached when the instrument

has been set, and the object of which is to steady it when there is a great difference in the proportions to which the instrument is being worked. The pulley wheels, which are of exactly equal diameter, give the easiest possible motion. To bring the instrument into adjustment, let the verniers be set to zero, which will bring them to the centres of the arms and of the central beam; place the arms at right angles to the beam, as near as can be guessed with the eye, and make a mark with the tracer and pencil point, and turn the instrument round so as to bring the pencil point into the mark made by the tracer; by doing this the tracer will be made to move exactly to the mark previously made by the pencil, if the instrument is in adjustment; if otherwise, the error in difference should be bisected, and the adjusting screws on the band should be moved until the tracing point comes exactly into the bisection.

The divisions upon the Eidograph do not positively indicate the reductions required to be performed by the instrument, but merely give a scale, which, with the assistance of the vernier, divides the beam and arms into 1,000 parts. To obtain the quantity to which the verniers are to be set, it is necessary to apply to the following table of proportions relative to divisions, or to the Rule immediately thereafter.

Table for reducing or enlarging Proportions.

Proportions.	Divisions on Bars.	Proportions.	Divisions on Bars.
As 1 is to 2	33·333	As 2 is to 3	20
„ 1 „ 3	50	„ 2 „ 5	42·857
„ 1 „ 4	60	„ 3 „ 4	14·285
„ 1 „ 5	66·667	„ 3 „ 5	25
„ 1 „ 6	71·428	„ 4 „ 5	11·111
„ 1 „ 7	75	„ 5 „ 6	9·091
„ 1 „ 8	77·778	„ 6 „ 7	7·692
„ 1 „ 9	80	„ 7 „ 8	6·667
„ 1 „ 10	81·818	„ 8 „ 9	5·882

Rule for finding the quantity equal to any given proportion for the setting of Eidograph. Subtract one sum of the proportion from the other, and multiply this difference by 100 for a dividend. Add the two sums of the proportion together for a divisor. The quotient from the working of this will give the number to which the beam and arms are to be set.

Example 1: Let it be required to reduce a drawing in the proportion of 3 to 5.

$$5 - 3 = 2$$
$$\times\ 100$$
$$5 + 3 = 8, \qquad 8)200(25$$

The centre beam C is to be set to 25 on the side nearest the pencil point D, the pencil arm A is also set to the 25 nearest the pencil point D, and the tracer arm B is set to the 25 furthest from the tracer G. If it were required to enlarge in the same proportion, each side would have to be set at the opposite 25.

Example 2: Let it be required to reduce a plan of 5 feet to the mile to a scale of 3 chains to the inch.

First, there must be like terms; hence to reduce both proportions to feet to the inch, which will in this instance be the simplest way, proceed thus:—

5 feet to the mile = 88 feet to the inch;
3 chains „ „ inch = 198 „ „ „ „

$$198 - 88 = 110$$
$$\times\ 100$$
$$198 + 88 = 286 \qquad ,286)11,000(38{\cdot}392$$

CHAPTER VII.

Map Drawing.

1. MAP. A *map* is a representation of the surface of the earth, or of any part of it, drawn on paper, on which all the lines or distances, as far as the difference of the surface permits, bear the same proportion to one another as those in nature do.

2. PLAN. A *plan* is a drawing which shows an object in its horizontal dimensions only. The term is usually applied to large scale maps, such as are constructed for engineering or other purposes. The common maps of countries and cities are plans. From a map of Burma you can find, by the scale on it, the distance between Rangoon and Moulmein, or the length of the Pegu-Yoma range of mountains, but no application of the compasses will enable you to obtain the height of those mountains or the depth of the sea.

3. CONSTRUCTING A MAP. To construct a map of a district or country, consists in making on paper a figure similar to the district, and representing, on a smaller scale, the prominent objects on its surface.

4. SIZE OF MAP DEPENDANT ON SCALE. On a sheet of paper 2 feet square a map of the whole world may be drawn, showing the countries, principal towns, rivers and mountains; and on another sheet of paper of the same size, a map of only a circle, a small portion of a district of country (for instance, the Yun circle of the Rangoon district), may be drawn, on which can be shown, in detail, the villages, cultivations, patches of jungle, creeks, roads, etc. The scale of the maps must of course necessarily be very different; the former being on a scale of 1,000 miles to an inch, and the latter of 1 mile to an inch.

5. LAND-SURVEYING. A plan cannot be drawn until actual measurements of country have been made. The making of these measurements by means of a chain or angular instrument is termed *Surveying*, and *Land-surveying* is the measuring, with a view to subsequent delineation on a map or plan, the boundaries and forms of natural and artificial objects on the surface of the ground, ascertaining and fixing their relative positions, and finding the superficial content or area of each part, and of the whole.

6. ACCURATE DETAILS RENDER PLAN VALUABLE. All surveys are essentially composed of details, and the greater the accuracy of the detail the more valuable the survey. Hence great care is necessary in transferring on paper the portion of country surveyed, and in the management of the drawing instruments.

7. VARIOUS DESCRIPTIONS OF SURVEY. There are several descriptions of survey. The *Trigonometrical Survey*, which constitutes an extended system of Triangulation, or a network of triangles considered either plane or spherical, according to the nature and importance of the work under survey. The term *Topographical Survey* implies the measurement and delineation of the natural features of a country and the works of man thereon, with the object of producing a complete and sufficiently accurate map. Being free from boundaries of properties, the principal lines of operations must conform to the features of the country and objects to be surveyed. The peculiar province of a *Revenue Survey* is to define boundaries of estates or properties, with a view of obtaining accurate areas of land for revenue purposes. The *Cadastral Survey* is a *field-by-field* survey, or a Revenue Survey on a larger scale, purely for revenue purposes. The *Town Survey*, for the survey of a town. A *River Survey*, for the survey of a river, and a *District Survey*, for the survey of a district.

8. REPRESENTATION OF HEIGHT. Objects having elevation can only be expressed on a flat surface, such as paper, by means of shade, and this can only be done of bodies whose forms present either slopes or curves. A hill which presents slopes can therefore be expressed on a ground-plan so as to convey an approximate idea of its elevation; but it is not possible to give the appearance of elevation to a building because its walls are perpendicular. In reality, this is a

matter of no consequence whatever, for the mind at once connects the idea of height with churches, houses, etc.; and the method of shading hills enables us, at the utmost, only to form a loose judgment of their height as compared to each other, for there cannot be determined by it the actual elevation of any single hill. When, for any particular object, it becomes necessary to determine the actual elevation of any point above the sea, a river, etc., it can be ascertained either by *levelling* or by vertical observations with a theodolite.

9. THEORY OF SHADING. The theory of shading supposes the light to fall vertically upon the hill in parallel rays; according to which steep slopes, receiving those rays at a more oblique angle than more gentle ones do, are illuminated in a less degree than the latter, and must be shown in a plan by a darker shade; while such portions of the ground as are horizontal, and consequently receive the rays of light perpendicularly to their planes, being thus illuminated in the greatest degree, are left without shade in a plan. *Practically*, the shading can never be so exact as to enable us to measure by it the positive height of a hill.

10. HORIZONTAL DISTANCES SHOWN ON PLAN. (Fig. 189.) All distances shown upon a plan are *horizontal* ones. For instance, the sloping line HK of the section, which is the hypotenuse of the right-angled triangle HKN, is represented on the *ground-plan* below by the horizontal line HK, which is equal to HN in the section, and in the same way, MB, a precipitous fall in the section, only occupies the space from M to B on the ground-plan. Thus it is seen that the height of mountains or the depression of valleys exercises no influence upon the situations of objects in a plan.

11. HEIGHTS REFERRED TO SEA LEVEL. The mean level of the sea (*see* Chap. XV, Art. 33) is, in large surveys, considered as the horizontal plane, to which all measurements are reduced.

12. EXPRESSION OF SLOPES BY SHADE. (Fig. 189.) Slopes are expressed by means of shade—made light or dark according as they are gentle or steep. The section in the figure represents ground of varied character; A is on a level with the sea; from that point the hill has a steep rise to C, from where it is more gentle as far as D; at E a descent begins, and continues to F, from where there is a steep slope to G, and so on. The shading of the ground-plan agrees with the section: for instance, that from C to A is darker than between D and C. From D to E the ground is level, and therefore no shade appears. The slope from E to F, being greater than that from D to C, is shaded darker; FG being steeper, is made darker still, and the deep shade from M to B is equal to the shading of both of the slopes LK and KH.

13. HILL SHADED BY PEN OR BRUSH. The shading of hills may be performed by using a black-lead pencil, with a pen, by washes of Indian ink.

14. VERTICAL AND HORIZONTAL METHODS OF EXPRESSING INCLINATION. (Fig. 192.) There are two modes of expressing inclinations of ground with the pen or lead pencil, namely, the *vertical* and *horizontal* manners. Both styles are employed in India and Burma; but the Atlas Sheets of these countries will eventually be shaded vertically. The horizontal style marks the contours of hills by waving lines, each line continuing on the same *level* while following every undulation of ground.

15. RAPID WAY OF REPRESENTING HILLS. The most rapid way of expressing hills upon paper, is by shading with Indian ink, by means of two camel-hair brushes, one to lay on the tint, and the other for softening it down.

16. DISPOSITION OF LIGHT AND SHADE. As before said (in Art. 9), the rays of light are supposed to fall vertically upon the ground, and the degree of shade used for expressing hills depends on the greater or less gradations of their declivities; that is, the more the slope of a hill recedes from the horizontal, the darker must be the shade.

17. KIND OF PAPER SUITED FOR DRAWINGS. The paper on which the drawing is to be made should be very hard, new, of good quality, but not too highly glazed, and should present as smooth a surface as possible. Aything that tends to destroy the surface, such as erasures, excessive rubbing witn India

rubber, washing, etc., should be avoided as much as possible. India-rubber should be used sparingly, and pressed very lightly on the paper.

18. DIMENSIONS OF DRAWING PAPER.* *Drawing paper*, properly so called, is made to certain standard sizes, as follows :—

Demy	20	inches	by	15¼	Inches.
Medium	22¾	,,	,,	17½	,,
Royal	24	,,	,,	19¼	,,
Super-royal	27¼	,,	,,	19¼	,,
Imperial	30	,,	,,	22	,,
Elephant	28	,,	,,	23	,,
Columbier	35	,,	,,	23½	,,
Atlas	34	,,	,,	26	,,
Double elephant	40	,,	,,	27	,,
Antiquarian	53	,,	,,	31	,,
Emperor	68	,,	,,	48	,,

Of these, Double elephant is generally the most useful size of sheet. Demy and Imperial are the other useful sizes.

19. RIGHT SIDE OF DRAWING PAPER. There is hardly any perceptible difference between the two sides of drawing paper. To tell therefore the *right* side of it, upon which plans, etc., are to be made, hold it up to the light, and the side from which the name or trade mark of the maker, impressed thereon in water lines, is seen *direct*, is the right side.

20. SELECTION OF LEAD PENCILS. In selecting black-lead pencils for use, care must be taken not to take ones that are too soft, nor so hard that their traces cannot be easily erased by india-rubber. Either an F or an H should be employed, with a moderately fine point, and when used should be gently pressed on the paper, and slightly inclined in the direction in which the line is being drawn, care being taken to keep it, throughout the operation, in the same position with reference to the ruler.

21. PRESERVATION OF DRAWING PAPER. Drawing paper should always be wrapped in flannel, and kept closed up in a tin case, to prevent its being affected by the dampness of the atmosphere ; otherwise it soon spoils.

22. RESTORATION OF DAMAGED DRAWING PAPER. The following will be found a good recipe for restoring damaged drawing paper. Take a wash composed of one drachm of isinglass steeped in two ounces of water for 12 hours. Then simmer it for 15 or 20 minutes over a fire. When nearly ready add of common alum, in powder, twenty grains ; strain through linen for use ; apply it when the paper is on the drawing board, and damp, and work it on, with a flat brush ; when dry, wash the paper over with water, which will indicate whether a second wash of the above is necessary. When the paper is thoroughly recovered, wash it well with plain water and a flat brush to take off any superfluous isinglass, absorbing the superfluous water with a clean linen rag. When thus prepared, the colours can be thrown in as safely as on good paper.

23. PREPARATION OF INDIAN INK FOR USE. Indian ink, free from grit, carefully rubbed up with water, then with a cork, and above all not too thick nor too thin, is to be used. One or two trials, by drawing two or three lines on a piece of

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OBSERVATION.

* Sizes of books are as follows :—

4 pages = 2 leaves = 1 sheet Folio, 2°.
8 ,, = 4 ,, = 1 ,, Quarto, 4to.
16 ,, = 8 ,, = 1 ,, Octavo, 8vo.
24 ,, = 12 ,, = 1 ,, Duodecimo, 12mo.
32 ,, = 16 ,, = 1 ,, Sixteen mo, 16mo.
36 ,, = 18 ,, = 1 ,, Eighteen mo, 18mo.
48 ,, = 24 ,, = 1 ,, Twenty-four mo, 24mo.
64 ,, = 32 ,, = 1 ,, Thirty-two mo, 32mo.
96 ,, = 48 ,, = 1 ,, Forty-eight mo, 48mo

The size of the page will, of course, depend upon the size of the sheet, which may be either—Double elephant, Atlas, Columbier, Elephant, Imperial, Super-royal, Royal, Medium, or Demy.

waste paper, while the ink is being prepared, will ensure a proper consistency and the requisite degree of blackness. Indian or Chinese ink, which alone is used for the printing and inking up of maps, should always be rubbed up fresh for use.

24. MOUNTING DRAWING PAPER ON THE BOARD. Before commencing a drawing the first thing to be done is to prepare the paper, that is, stretch it evenly upon the smooth and flat surface of a drawing board. The paper must prior to this be thoroughly and equally damped with a sponge and clean water *on the opposite side from that on which the drawing is to be made.* When the paper absorbs the water, lay it on the drawing board with the wetted side downwards, placing its edges as nearly as possible parallel with those of the board; otherwise in using a T square, an inconvenience may be felt. Next, lay a straight flat ruler on the paper, with its edge about half an inch from, and parallel to, one of its edges. Hold the ruler firm and turn up the projecting half inch of paper along its edge; then pass a brush with good strong paste once or twice along the turned-up edge, after which by sliding the ruler over the pasted border the paper will again be laid flat, and the ruler being pressed lightly on it for a little time, that edge of the paper will adhere to the board. Similarly paste the opposite edge of the paper on its side of the board; and so also with the remaining third and fourth edges.

25. CAUTION AS TO THE DRYING OF PAPER. The wetting of the paper is for the purpose of expanding it; and the edges being fixed to the board in its enlarged state, act as stretchers on the paper, while it contracts in drying, which it should be allowed to do gradually in the shade. By this means all creases or undulations disappear from the surface, and forms a smooth plane to receive the drawing. If possible, the paper after being stretched on the board should be allowed to stand for 3 or 4 days before use.

26. JOINING SHEETS OF PAPER. In joining two sheets of paper together, by overlapping, it is necessary, in order to make a neat joint, to feather-edge each sheet; this is done by carefully cutting with a knife half-way through the paper near the edges, and on the sides which are to overlap each other; then strip off a feather-edged slip from each, which being done dexterously, the edges will form a very neat and efficient joint when put together.

27. METHOD OF HOLDING COMPASSES. (Fig. 136.) In using the compasses, they should be held at the top between the forefinger and thumb, with one or more fingers under the hinge to increase or diminish the distance between the points gradually or without a jerk. Care must be taken that no pressure is employed with the compasses, so as to make big holes in the paper, especially so in the drawing of circles with regard to the centre.

28. MANNER OF USING THE DRAWING PEN. In order to use the Drawing Pen, first of all, dip the blades in water and then wipe the outside surfaces dry; then with another clean pen or piece of paper take up some ink and insert it between the nibs. Hold the pen firmly against the ruler, slightly inclined in the direction of the line to be drawn; be careful to make both nibs touch the paper, to preserve an even pressure and the same position of the pen with regard to the paper and ruler throughout, and a slow but equal motion along the ruler. The pen should always be cleaned and dried before being laid aside.

29. DAMPING OF PAPER NOT A NECESSITY. Paper is not invariably stretched on a drawing board for the purpose of executing a map or other drawing on it. In rough plans it will be sufficient to pin the drawing paper on an even table, without putting it through the process of damping, pasting, etc.

30. SCALE OF SURVEY TO BE FIRST FIXED. The first thing to be done towards the execution of a drawing on paper, is to determine the *scale* to which a survey is to be plotted; that is, the proportion which the actual dimensions of the drawing shall bear to those of the object it represents. Drawings are made on various scales, according to circumstances, depending upon convenience for portability and reference, and also upon the minuteness or largeness of individual parts of the object.

31. SCALES IN GENERAL USE. The following are among the scales usually adopted for the special objects referred to :—

1 *inch* = 8 *miles** ... 1 ,, = 4 ,, ...	For geographical maps on small scales.
1 *inch* = 1 *mile*† or 80 *chains* 2 *inches* = 1 ,, ,, 40 ,,	For extensive surveys of country showing general topographical features.
4 *inches* = 1 *mile* or 20 *chains* 8 ,, = 1 ,, ,, 10 ,,	For surveys of towns, cantonments, large estates, and for village surveys as now conducted by the Revenue Survey of India.
16 *inches* = 1 *mile* or 5 *chains* 32 ,, = 1 ,, ,, $2\frac{1}{2}$,,	For a field-by-field survey of cultivation, for cities, estates, villages, etc., where pathways, tanks, roads, houses, gardens, wells, etc., are distinctly delineated. These scales are adopted by the Cadastral Survey of India.
40 *inches* = 1 *mile* or 2 *chains* 64 ,, = 1 ,, ,, $1\frac{1}{4}$,, 80 ,, = 1 ,, ,, 1 *chain*	For plans of buildings and grounds, where great clearness is required and when the work is of limited extent.
$26\frac{2}{5}$ *inches* = 1 *mile* or 200 *feet.* $52\frac{4}{5}$,, = 1 ,, ,, 100 ,, $105\frac{3}{5}$,, = 1 ,, ,, 50 ,, 132 ,, = 1 ,, ,, 40 ,,	For engineering purposes, owing to the facility they afford for decimal calculations.

32. SCALES REFER TO LINEAL MEASUREMENTS. The *scale* of a drawing has reference to the proportion of its *lineal* magnitudes to those of the object; it has nothing to do with *superficial* magnitude or area. This distinction should be borne in mind, for superficial magnitude increases or diminishes much more rapidly than lineal magnitude. In drawing done to a scale of one-half full size, for example, the lineal dimensions are drawn one-half their actual magnitude, while the superficial area of each part is reduced to one-fourth.

33. REASON FOR DRAWING SCALE ON PLAN. (Fig. 115.) The scale should *always* be drawn on the plan, because the paper is liable to alter its dimensions through hygrometrical changes of the atmosphere, especially so after it has been removed from the board; but such changes would not signify if the scale were drawn as mentioned, since the scale and drawing would be *equally* effected. With moisture paper expands, whereas with heat it contracts. Let us take, for instance, two points A and B on the paper 4 miles apart, the distance between which is measured from off a brass scale of 1 mile = 1 inch. Now let us suppose the paper exposed to night dew, it becomes moist, the paper thereupon expands, and A and B are situated as appears in the lower dotted line of the figure. If the distance be now measured with a pair of compasses between the two points and referred to the brass scale, it will be found to be 4·2 inches. Again, suppose the paper exposed from its damp state to great heat, it at once contracts and A and B appear as shown in the upper dotted line of the figure, and if the compasses be applied as before, it will be found to measure 3·8 inches. If, on the contrary, the scale is drawn on the map, the scale and drawing are then equally effected by any hygrometric changes in the atmosphere. Hence, the necessity for drawing the scale on the plan it refers to, is apparent.

34. WHERE SCALE IS TO BE LAID DOWN. Having determined on the scale, it should be laid down for use in a convenient spare part of the paper.

35. SPACE THE DRAWING WILL OCCUPY TO BE ESTIMATED. A rough estimate should be formed of the space which the drawing will occupy; and for this purpose, it is sufficient to take the extreme dimensions of the object in the plans in which it is to be drawn.

36. PLACING OF NORTH POINT. (Fig. 114.) In plotting plans, the

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OBSERVATIONS.

* The scale determined upon by the International Geographical Congress for the preparation of a map of the world was of 1 : 1,000,000, equivalent to 15·78 miles = 1 inch.

† This is the natural scale of $\frac{1}{63360}$.

north point should be placed upwards, and a drawing representing the direction of the Magnetic Needle should be made on either the West or East edge of the paper, as may be convenient.

37. VARIATION OF THE COMPASS TO BE SHOWN. Maps and surveys may also have the direction of the meridian laid down, with the variation of the compass.

38. SURVEY OF DETAIL REGULATED BY SCALE DETERMINED ON. The scale to be adopted in plotting a survey depends upon the amount of detail it is desired to show. For ordinary chain and compass surveys, a scale of 400 feet to an inch will be found quite large enough to show all objects of importance. Using this scale, an area of about 2 square miles will fit into a sheet of paper measuring 40 inches by 27 inches. In a Village Survey, or the like, a scale of 5 chains to the inch = 16 inches to the mile, will give an area of $2\frac{1}{2}$ acres in a square inch on the plan; that is, a sheet of paper 30 inches by 22 inches, allowing an inch round for the border, will contain an area of exactly 1,400 acres = $2\frac{3}{16}$ square miles; so that an area of 2 square miles, provided of course it is not a long narrow strip of land, can be well shown upon it. The scale should invariably be fixed before the survey is commenced, as upon the scale will depend the degree of minuteness to be observed in measuring offsets, etc., and the class of objects to which offsets will be taken.

39. PLOTTING. The next thing to be done is to make a drawing of the survey on paper, which process is called *plotting*.

40.* PLOTTING A CHAIN OR THEODOLITE SURVEY. (Fig. 113.) In a chain or theodolite survey, as all the surface has been divided into triangles, each triangle is separately plotted. Next, fix upon a part of the paper whereon to lay down your starting points, so that your other points may all fall within the paper. Take a convenient direction to represent the first line and mark on it the distance as measured in the field with the aid of a pair of compasses and your drawn scale; the two extremities will now be the first and second stations. In the case of a theodolite survey, the second station is laid down with reference to its bearing from the first station as described further on (*see* Chap. XIV, Art. 17). (Properly, the stations of a theodolite survey should be laid down according to their latitudes and longitudes checked by their intermediate distances, but as the object of this work is not intended to embrace this more advanced method of laying down points, allusion will only be made to their bearings, which is here treated of.) From the second station as centre, with the second chained or deduced line as radius, describe an arc of a circle, and from the first station as centre, with the third chained or reduced line as radius, describe another arc, and the intersection of these two arcs will be the third station. On each of the sides of this triangle construct other triangles in exactly a similar way, until all the stations are so plotted. If the lengths of the above lines exceed the span of common compasses, they should be laid down by means of a beam-compass (*see* Chap. V, Art. 19). All points of intersection of the sides of smaller triangles, and of all proof-lines, should be pricked off and marked in pencil with the corresponding letters taken from the rough diagram in the field-book. Any tie-lines, as measured in the field, should be drawn on the plan, and should of course measure the same on paper as they did in the field. As soon as the chain lines are completed, the offsets have to be plotted, each being drawn to the same side of the chain line on the paper as it originally was measured on the ground, and the extremities of these offsets being joined, a correct outline sketch of the area surveyed is obtained. The figure will render the above description clear.

41.† PLOTTING A COMPASS SURVEY. (Figs. 114, 474.) In a compass survey (*see* Fig. 114), select a convenient point on the paper to represent the first station on the ground, and so that the other stations of the survey may fall within the paper; and draw a line through this point, pointing to the top of the paper and

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OBSERVATIONS

* The subject of this Article, together with the terms used therein, may not be quite understood till Chapters IX and XIII have been read.

† The subject of this Article, together with the terms used therein, may not be quite understood till Chapter X has been read.

parallel to its edge, to represent the north and south line. From this mark off, by means of a rectangular or circular protractor, the bearing of the first line. Through the point so marked and the first assumed point draw a line, and measure on it a length equal to the length measured in the field, according to scale. This gives the second station. Through this, draw a line parallel to the north and south line at the first station, and lay off the bearing of the second line, and afterwards measure its distance as before. This gives the third station, and here and at every succeeding station up to the last the same process is to be repeated. Having arrived at the last station, join it with the first. Measure the distance and bearing of the line so drawn, and if they agree with those entered in the field-book, the survey has been correctly plotted; if not, the error must be looked for and corrected, and it will generally be found to have occurred through not drawing the north and south lines at the various stations accurately parallel to each other,* unless some gross error has been made in plotting some of the angles with the protractor. As soon as the errors in the plotting of the chain lines have been corrected, the offsets are plotted in exactly the same way as in chain surveying (*see* Art. 40), proceeding on the paper in the same order as that which was followed in the field. For a very accurate method of plotting a compass traverse by rectangular co-ordinates (*see* Chap. XIV).

If bearings have been taken from different points on the chain line to any conspicuous objects, they should be plotted, when, if correctly observed and plotted, the lines will be found to intersect in the same point; if they do not, and the chain lines have closed on the first station correctly as regards bearing and distance, an error must have been made in taking these bearings in the field, which can only be corrected by going to the field again and re-observing the bearings. The figures will render the above description clear.

42. DRAWING OF THE PLOT. The principal triangles and chain lines are next ruled in with red lines, exhibited as constituting the basis of the whole work. Generally, however, these are shown on a separate plan to avoid confusion of details. The various boundaries and objects are drawn in Indian ink with the steel drawing-pen: straight lines, rectangular boundaries, such as those of buildings, etc., are ruled; irregular boundaries, such as streams and winding roads, fences, etc., are drawn freely by the hand, the steel-pen being held upright, with its broad edge parallel to the direction of the line to be drawn.

43. PRINTING. (Fig. 336.) A good style of printing is an essential ingredient in a good Surveyor, hence the course of Printing, comprising letters and figures, given in (Fig. 336), should be thoroughly mastered. As a rule, Block Printing is decidedly the best for all kinds of Headings, being neat and legible. For Main Headings, Fancy Letters may occasionally be used. For detail matter within the drawing, Capital and Small Italic is employed. Parallel lines at a distance apart, equal to the height of the letters to be printed, should always be drawn for capital and small letters, though for the latter it is not so important if the draughtsman is skilful; for beginners, the parallel lines should in every case be drawn; and also a line to determine the height of these letters, which like *d, t, l, etc.*, reach above the original lines. For large Block Printing, vertical lines spaced at equal intervals may be used with advantage; and for inclined Capitals parallel lines at the required angle will prove of great assistance in securing uniformity. For large printing, a good plan is to draw in the outline, and then fill in with the pen, care being taken to fill up the corners so as to make them sharp and defined. The spacing of letters is best determined by the eye; but it is always advisable to sketch in the position of the letters of a heading or title with a pencil before proceeding to put them into ink.

44. DISPOSITION OF THE LETTERS AND FIGURES. The mass of printing on the plan should invariably run from west to east, parallel to the north and south edges of the paper, and it is usual to place the first letter of the name as close to the site as possible. Curved lines of printing are generally employed for

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OBSERVATION.

* Frequently *section paper* (*see* Fig. 474) is employed for the purpose of plotting. This paper has faint blue lines, an inch or so apart, running parallel to each other throughout the length and breadth of the paper. By its use the errors of plotting referred to are avoided.

the names of districts, circles, etc., and should run as nearly parallel to the general outline as possible, and when several districts or circles are exhibited, the name requires to be stretched across the space with the letters at *equal intervals*, to show the extent or connection of the same. The inked up details should, in all cases, be kept clear of the printing, not so far apart as to give the items a fragmentary appearance, nor so close as to intrude on and obscure the names. The continuity of streams and roads crossing names must of necessity be broken, but their course should be interfered with as little as possible.

45. CURVED WRITING. On winding rivers, canals, roads, lakes, coasts, chains of mountains, etc., the writing should be curved and adapted to their natural sinuosities; that is, parallel to their courses.

46. WRITING TO FACE TOP OF THE PAPER. (Fig. 119.) The general appearance of all the names and writing should invariably be to face the *north end* or *top* of the paper, so that it may be all legible at one view, by placing the map before you and not upside down, or requiring the plan to be shifted before it can be used. The eye is supposed to rest at the centre of the lower edge of the paper, whence the whole of the writing on the drawing is capable of being taken in at a glance. For this purpose, all printing which does not run due west and east, and has to be therefore curved or slanting, between the north and east lines, must be done outwards *from* the eye, and between the north and west lines inwards *to* the eye. A reference to Fig. 119 will render the meaning clear.

47. SIZE OF LETTERS. (Figs. 324—333.) The disposition of the writing requires taste; and it must be remembered that the utility of a plan depends very much on that facility of reference which is obtained by the relative keeping of the names. The size of the letters must correspond with the importance of the objects represented, and on the situations in which they are placed -the size of the scales not being lost sight of. It is more or less regulated by the different sizes set out in the figures referred to.

48. DIRECTION OF LIGHT ON A DRAWING. (Figs. 217, 274.) The light is generally supposed to come in at the left-hand upper or N. W. corner of the drawing at an angle of 45°; that is, the *east* and *south* sides of any object in *elevation* and consequently above the surface of the ground, such as a house (*see* Fig. 217); and the *west* and *north* sides of any object in *depression* and consequently below the surface of the ground, such as a tank or river (*see* Fig. 274), will be in shade, which is shown by the lines on those sides being drawn thick.

49. NECESSITY OF GOOD PRINTING. On the style and character of the *printing* of a map depends its general appearance and degree of usefulness. Generally this is a most difficult part of a map to *do well*, and there are many persons who can colour and draw in a superior manner, but who fail in printing; hence the necessity of learning to print. In printing, patience is an essential ingredient in the draughtsman; he must with his pen move slowly, but surely. In printing as well as drawing there must be uniformity of character; for instance, hamlets are written in small italic, ordinary villages in large italic, large towns and villages in small erect letters, chief towns of districts in capital erect letters, and so on. The printing and drawing in a map should as much as possible correspond with the style laid down in the Conventional Signs given in (Art. 50).

50. CONVENTIONAL SIGNS. (Figs. 217—333.) The following are the Conventional Signs most generally in use in plan-drawing, and should be strictly adhered to in the filling in of details, etc., in a map. They are intended for maps on a scale of 16 inches to the mile, but for smaller scale drawings the same signs are to be employed, only smaller. In maps intended for photography the symbols should be drawn in Indian ink only, and no colour used.

CONVENTIONAL SIGNS.

No.	Description.	Sign.	Colour.
		Fig.	
1	City or Town ...	217	Lake.
2	Village ...	218	Burnt Sienna.
3	Market ...	219	Lake.
4	Fort ...	220	„
5	Church ...	221	„
6	Hindu Temple ...	222	„
7	Musjid or Mahd. Mosque.	223	„
8	Burmese Pagoda ...	224	„
9	„ Monastery	225	„
10	Post Office ...	226	„
11	Salt Golah ...	227	„
12	Staging Bungalow...	228	„
13	Rice Mill ...	229	„
14	Saw „ ...	230	„
15	Indigo Factory ...	231	„
16	Silk „ ..	232	„
17	Sugar „ ...	233	„
18	Distillery ...	234	„
19	Court House ...	235	„
20	Police Station ...	236	Indian ink.
21	Signal Staff ...	237	Lake.
22	Azimuth Station of Survey.	238	Blue.
23	Light House ...	239	Indian ink.
24	Telegraph Tower ...	240	Lake.
25	Boundary Pillar ...	241	„
26	Station of the G. T. Survey.	242	Indian ink.
27	Bench Mark ...	243	Lake.
28	Masonry Well ...	244	„ (Blue Centre.)
29	Mud „ ...	245	Lake. (Blue Centre.)
30	Burial Ground ..	246	Indian ink.
31	Water Mill ...	247	„ (Water Blue.)
32	Telegraph Line ...	248	Indian ink.
33	Metalled Road ...	249	Lake. (Lake wash.)
		Fig.	
34	Unmetalled Road ...	250	Burnt Sienna. (Bt. Seinna wash.)
35	Sunk „ ...	251	Indian ink.
36	Raised „ ...	252	„
37	Village „ ...	253	Burnt Sienna.
38	Footpath ...	254	„
39	Single Line Railroad	255	Indian ink.
40	Double „ „	256	„
41	Bund or Embankment.	257	„
42	Rail Fences or Palings.	258	„
43	Hedge with trees ...	259	„ (Green patches.)
44	„ without trees	260	Indian ink. (Green patches.)
45	Masonry Bridge ...	261	Indian ink. (Water Blue.)
46	Wooden „ ...	262	Indian ink. (Water Blue.)
47	Iron „ ...	263	Indian ink. (Water Blue.)
48	Suspension „ ...	264	Indian ink. (Water Blue.)
49	Draw „ ...	265	Indian ink. (Water Blue.)
50	Pontoon „ ...	266	Indian ink. (Water Blue.)
51	Boats (of) „ ...	267	Indian ink. (Water Blue.)
52	Ford ...	268	Indian ink. (Water Blue.)
53	Sluice Gate ...	269	Indian ink. (Water Blue.)
54	Ferry ...	270	Indian ink. (Water Blue.)
55	Old bank of River...	271	Indian ink. (Dark Blue wash.)
56	Direction of Current	272	Indian ink.
57	Ebb and Flow of Tide.	273	„
58	River and Stream ...	274	„ (Water Blue.)
59	Reef of Rocks ...	275	Indian ink.
60	Lake and Tank ...	276	„ (Water Blue.)

CONVENTIONAL SIGNS—*concluded.*

No	Description.	Sign.	Colour.
		Fig.	
61	Marsh or Jheel ...	277	Indian ink. (Blue patches.)
62	Sand Bed ...	278	Indian ink. (Light Red wash.)
63	Salt Pans ...	279	Indian ink. (Blue lines.)
64	Brick Fields ...	280	Indian ink.
65	Iron Works ...	281	„
66	Glass „ ...	282	„
67	Lime Kilns ...	283	„
68	Gravel Pits ...	284	„
69	Quarries ...	285	„
70	Nullah ...	286	„ (Water Blue.)
71	Canal ...	287	Indian ink. (Water Blue.)
72	Garden ...	288	Indian ink. (Green wash.)
73	Betel-leaf Garden ...	289	Indian ink. (Green wash.)
74	Gardens and Ornamental Grounds.	290	Indian ink. (Green wash.)
75	Mango Tope ...	291	Indian ink. (Green patches.)
76	Sal Tree ...	292	Indian ink. (Green patches.)
77	Tamarind or other large trees.	293	Indian ink. (Green patches.)
78	Palmyra Trees ...	294	Indian ink. (Green patches.)
79	Cocoanut „ ...	295	Indian ink. (Green patches.)
80	Tamarisk „ ...	296	Indian ink. (Green patches.)
81	Date „ ...	297	Indian ink. (Green patches.)
82	Betel-nut „ ...	298	Indian ink. (Green patches).
83	Ravines „ ...	299	Indian ink.
84	High Ground ...	300	„
85	Hill (Ridge with Peaks).	301	„
86	Ridge ...	302	„
87	Sand Hill ...	303	„ (Light Red wash.)
88	Bamboo Jungle ..	304	Indian ink. (Green patches.)

No.	Description.		Sign.	Colour.
			Fig.	
89	Forest or Tree Jungle.		305	Indian ink. (Green patches.)
90	Low or Bush Jungle		306	Indian ink. (Green patches.)
91	Jhow Jungle ...		307	Indian ink. (Green patches.)
92	High Grass Jungle...		308	Indian ink. (Green patches.)
93	Low Grass „ ...		309	Indian ink. (Green wash.)
94	Salt Waste ..		310	Indian ink. (Blue wash.)
95	Salt Waste with jungle intermixed.		311	Indian ink. (Blue wash and Green patches.)
96	Cultivation ...		312	Indian ink. (Bt. Sienna wash.)
97	Lately thrown out of Cultivation, or Fallow land.		313	Indian ink. (Bt. Sienna wash.)
98	Rocky or Stony Ground.		314	Indian ink.
	Boundaries.			
99	Province ...		315	Indian ink.
100	District ...		316	„
101	Circle ...		317	„
102	Village ...		318	„
103	Village in centre of Stream.		319	„
104	Village in centre of Road.		320	„
105	Village on side of Stream.		321	„
106	Village on side of Road.		322	„
107	Cultivated Fields ...		323	„
	Style of Printing.			
108	Province ...	Outer Names.	324	Indian ink.
109	District ...		325	„
110	Circle ..		326	„
111	Large River, Forest Reserve, etc.	Inner Names.	327	„
112	Chief Town of District.		328	„
113	Chief Town of Circle.		329	„
114	Small River, Hill Range, etc.		330	„
115	Town ...		331	„
116	Village ...		332	„
117	Hamlet ...		333	„

51. PLAN TO BEAR SURVEYOR'S NAME. The name of the Surveyor and date of execution of the survey should invariably be entered in a corner of the plan.

52. EFFECT OF A GOOD PLAN. A good plan conveys to the mind a more perfect image than can be obtained by looking at the ground itself. With it the great features of a country can be examined and compared, the directions of lines and coasts, rivers, mountains, forests, etc., can be traced, together with their respective distances from one another, and the comparative heights of mountains, etc.

53. QUALITIES ESSENTIAL IN A PLAN. In plans there are three things to be desired; *1st, correctness*, without which a plan is worse than useless, for it misleads; *2nd, clearness*, in order that every part may be understood; *3rd, beauty of execution*, which adds a great deal to the value of a map.

54. COLOURING. In using cake colours, they must be rubbed down with soft water, mixing them well together till the desired tint of colour is obtained, then with a brush apply the wash steadily and quickly to the space to be coloured; but do not let the proper limits be passed, nor more quantity of colour used than will evenly cover the space, for when too much is used, deeper tints appear in one place than another. The map should be damped with a clean, moist sponge, before colouring. If a printed map is to be coloured, the back of the paper should be wetted with a solution of four ounces of pounded alum in a pint of spring water, to keep the colours from sinking, allowing the paper to dry before the colours are laid on. The printing of names of places should be done when the colours on the map are perfectly dry. Pencilled plans are first coloured, then inked. On tracing cloth, colours should be applied heavily on the side reverse to that of the drawing, that is, on the dull side.

55.* LITHO WORK. Where lithographic work is undertaken for the reproduction of maps, care must be taken, in the copying of the litho sheets,† to use a clean pen and clean hands. The litho ink should be prepared freshly sufficient for the day's use, and rain or distilled water must be employed in grinding down the ink. A piece of cork ought to be used in rubbing down the litho ink and not the fingers. It is well to keep the prepared ink while in use covered, so as to prevent dust and other foreign matter from getting into it. On no account must the smallest admixture of Chinese ink with the litho ink be allowed; the former does not take on the litho stone, and comes up as a dirty splash removeable by water, whereas the litho ink having wax in its compositon is not so removed and takes well on the porous litho stone. Cleanliness is essential in all respects.

56. TYPING. Typing is sometimes, and not infrequently, resorted to in the completion of original sheets. Where this is done, all type used for typing work during the day should be cleaned and sorted into trays; the hand ink-rollers

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OBSERVATIONS.

* A new process, known as the "direct zinc printing process," by means of which drawing or tracings in black ink can be transferred to zinc without first making a negative, has been recently introduced in the Survey of India Office, which, for certain classes of work, has many advantages over the ordinary process of photo-zincography. By this process work is executed at less than one-quarter of the rate charged for photo-zincographic work, and with better results, provided that the original drawings sent in conform to the following conditions: (1) The process is only adapted to the reproduction of drawings to the same scale and drawn on a single sheet. Two or more sections cannot be joined together as in ordinary photo-zincography. (2) Tracing paper or cloth is the most suitable form of original. Failing this, any smooth surfaced paper may be used, no matter how thick. Rough surfaced drawing papers are unsuitable and should not be used. (3) It is necessary that the work should be drawn in good, black, freshly prepared Indian ink. With this proviso, any drawing will reproduce satisfactorily no matter how fine the lines may be. Colour should not be used. (4) Nothing whatever should be pasted on either the front or back of the original. Slips of type or other matter pasted on the front will not reproduce, and consequently have to be drawn on the zinc afterwards. Nothing should be written on the back of the original. (5) Headings, footnotes, and other type matter should be neatly printed on the original in their proper places, as the rates charged do not admit of such matter being put on zinc by means of type transfers. (6) Originals should not be folded or creased if it can be avoided, but either left flat or rolled up when despatched.

† The following is an excellent recipe for making transparent transfer paper for litho map work; it assists the tracing to be done considerably. Boil ½ oz. of gelatine or isinglass in about a pint of water and coat twice ordinary tracing paper. Boil ½ lb. gamboge and ¼ lb arrowroot separately in about ½ a pint of water each. Mix together and strain through a thin cloth; if found too thick, add a little water and give one or two coats to the paper already coated with gelatine. The paper must be allowed to dry between the coatings. After the paper has been passed through a litho press on a clean stone face downwards or otherwise rolled, it is ready for use.

are also to be rolled dry. It is essential that the impressions on a map should be fine, yet clear and black. Black and clear they will not be unless the old and dry ink is removed from the face of the type; and fine they cannot be if the face and hair-lines of the type are destroyed during the process of cleaning by rough usage. A hard brush or old tooth-brush should on no account be used. If it is necessary to clean the fresh ink off the type when in the holder, a leather should be used and not a rag; the hairs from a rag get into the ink and spoil the impression, rendering it woolly. Ink should be new and rubbed up daily. After an impression is taken, and before the type is removed from the holder, it can be cleaned with a soft, small camel's hair brush dipped in turpentine lightly passing over it; then dab it gently with a ball of cotton wool. Type is thus always kept clean in the tray. The dabbing may be dispensed with, as the type will probably not be wanted till the turpentine has dried. The typer should have by his side, *away from the map*, a little pot of turpentine; it is no trouble using it. The rollers must not on any account be touched with water. Turpentine or kerosene oil should be used to clean them; the former is the best. The turpentine should be applied with the finger or a soft rag, and the roller dried by running it over a piece of clean paper.

57. REDUCING, COPYING, AND ENLARGING PLANS. (Figs. 198, 392.) Plans may be *reduced*, *copied*, or *enlarged* by means of the *pantagraph* (*see* Fig. 198) described in (Chap. VI, Art. 49), or the eidograph (*see* Fig. 392) described in (Chap. VI, Art. 51), and their use is to be adopted in preference to the mode by squares hereinafter described (*see* Art. 60). It gives the outline of any drawing, however crooked, with the utmost exactness, and may be used for copying figures, plans, marine-charts, maps, profiles, landscapes, etc. The process of *enlarging* does not admit of the same accuracy as that of *reducing* or even copying of the same size, and ought not to be resorted to where great precision is required.

58. DUPLICATES OF DRAWINGS. Duplicates of drawings may be made, when the lines and boundaries are regular, by laying the plan on the sheet of paper on which the copy is to be drawn, and pricking through all the necessary angles and points with a fine needle: if the lines in the original are straight, join the punctures by pencil lines; if curved, transfer by means of tracing paper or set off with the aid of compasses. Next draw the pencil lines in Indian ink and the plan is finished.

59. TRACING DRAWINGS. (Fig. 504.) Drawings may also be *copied* on the same scale by *tracing* over *tracing cloth*, *tracing paper*, or on a *copying glass*. In the case of *tracing cloth*, lay it over the map and trace the whole of the figure at once in Indian ink; in the case of *tracing paper*, which is very thin and generally intended only to transfer a drawing on to stout paper, lay it on the drawing and trace the plan on it in pencil, remove the tracing paper, rub the under side over with black lead and lay it on the stout paper intended to receive the drawing. Fix the two well together with a number of weights, and pass a fine smooth tracing point over each boundary or mark on the tracing with a slight pressure of the hand; thus a clear pencilled mark will be left on the fair sheet, which is afterwards drawn in Indian ink in the usual way. In the case of a *copying-glass* (*see* Fig. 504), which is an instrument consisting of plate glass set in a frame of wood, which can be inclined to any angle, just as a reading desk. On this glass the original plan is laid and over it the fair sheet of paper. The frame is then fixed in such a position, as before the window of a house, that a strong light may fall upon the glass from behind and shine through it, whereby every line in the original plan is seen distinctly through the fair sheet. The copy then is at once made in Indian ink.

60. REDUCTION BY SQUARES. (Fig. 116.) Plans may also be ***reduced***, ***copied***, or ***enlarged*** from one scale to another, by the method of squares (*see* Fig. 116). For this purpose divide the original plan, by pencil lines, into as many equal squares as may be convenient, then divide the fair sheet into the same number of equal squares, according to the proportion of the scales, whether greater, equal or less. Draw by the hand, or with the help of proportional compasses, in every square what is contained in the corresponding square of the given figure,

and by this means a very accurate copy will be obtained. The smaller the squares the more correct will the copy be.

61. ENTERING DIMENSIONS ON PLAN. When it is necessary to enter dimensions on a plan, they should be written intermediately between the extreme points, the distance between which they represent, and in the same direction as the measurement has been taken. When it is not necessary to enter the dimensions in writing, the same may be obtained by measurement off the scale.

CHAPTER VIII.

Calculation of Areas.

BY ACRE-SQUARES.

1. THE PURPOSE OF MEASUREMENT BY ACRE-SQUARES. There are several methods of calculating the areas of fields and other enclosures, but that by *Acre-Squares* is particularly adapted for accuracy and despatch of work for *small* parcels of land, and particularly offsets. The areas might be worked out by the rules of Mensuration, but where the scale of the map or plan happens to be small, or the boundaries of the figure distorted, this mode of calculation would be needlessly laborious.

2. CONSTRUCTION AND USE OF THE TALC-SQUARE. (Fig. 124.) The process is very simple. Take a piece of strong transparent horn paper or talc (if these cannot be had, a good substitute, for ordinary purposes, is a bit of tracing cloth), about 6 inches square, upon which describe a rectangle about 5 inches square, and divide it by measurements off a brass or ivory scale into squares of acres and decimals, on whatever scale the work is protracted on the map. For instance, if the map was drawn on a scale of 5 chains to the inch = 16 inches to the mile, it would be necessary to construct a square of 5 inches on the transparent substance, and then divide each side of this square into 5 equal parts, and from these points of division rule lines parallel to the sides of the square. The 5-inch square will thus become divided into 25 smaller squares. Now, since the scale of the map is 5 chains to the inch, therefore each square inch on both the map and the talc contains 5 × 5 or 25 square chains = $2\frac{1}{2}$ acres, and the big 5-inch square will contain $2\frac{1}{2} \times 25 = 62\frac{1}{2}$ acres. If, then, the sides of each square inch on the talc is subdivided into five equal parts and from the points of subdivision lines are drawn parallel to the sides of the square, then each inch square will become made up of 5 × 5, or 25 little squares; but, as before said, the square inch contains 25 square chains; hence each subdivisional square will represent 1 square chain of area, or the one-tenth of an acre.

3. METHOD OF CONSTRUCTING AN ACRE-SQUARE. Let us suppose that it was desired to make an acre-square on any given scale corresponding to that of the plan, the Surveyor could easily construct one for himself by adopting a simple rule of proportion, as will be clear from the explanation in the examples given below. It will also be seen that all that is required to be obtained is the exact length, in inches or portions of an inch, of a side of the square the area of which = 1 acre, but the reason for finding the length and breadth of a large rectangle or square, containing a number of these little squares, is based on the principle of working *from a whole to a part*, instead of from a part to a whole, thereby avoiding accumulative errors which would otherwise necessarily exist.

EXAMPLE 1. *Construct an acre-square, on a scale of 4,000 yards = 3 inches, to measure 1,000 acres.*

Since 4,840 square yards = 1 acre,

∴ the side of an acre-square on the ground = $\sqrt{4{,}840} = 69{\cdot}57$ yards,

and ∴ " " " " plan =

4,000 yards : 69·57 yards : : 3 inches : $\frac{3 \times 69{\cdot}57}{4{,}000} = 0{\cdot}052178$ inches ;

Hence the length of 100 such squares = $100 \times 0{\cdot}052178 = 5{\cdot}2178$ inches;

and " " 10 " " = $10 \times 0{\cdot}052178 = 0{\cdot}5218$ inches.

Construct a rectangle with a length = 5·218 inches and a breadth = 0·522 inches, off a scale of inches, on talc or other transparent substance. Divide the length and breadth into 100 and 10 equal parts respectively; and join the corresponding points of division on the opposite sides, thus forming 1,000 little squares, each of which = 1 acre in area.

EXAMPLE 2. *Construct an acre-square, on a scale of 600 feet = 1 inch, to measure 100 square chains.*

Since 4,356 square feet = 1 square chain,

∴ the side of a chain-square on the ground = $\sqrt{4{,}356} = 66$ feet,

and ∴ the side of a chain-square on the plan =

$600 \text{ feet} : 66 \text{ feet} :: 1 \text{ inch} : \frac{1 \times 66}{600} = \frac{11}{100} = 0{\cdot}11$ inches;

Hence the length of 10 such squares = 10 × 0·11 = 1·1 inch.

Construct a square with 1·1 inch sides, off a scale of inches, on talc or other transparent substance. Divide each of its sides into 10 equal parts, and join the corresponding points of division on the opposite sides, thus forming 100 little squares, each of which = 1 square chain in area.

EXAMPLE 3. *Construct an acre-square, on a scale of 4 inches = 1 mile, to measure 600 acres.*

Since $\frac{1}{640}$ square miles = 1 acre,

∴ the side of an acre-square on the ground = $\sqrt{\frac{1}{640}} = \sqrt{0{\cdot}0015625} =$ 0·03953 miles,

and ∴ the side of an acre-square on the plan =

$1 \text{ mile} : 0{\cdot}03953 \text{ miles} :: 4 \text{ inches} : \frac{4 \times 0{\cdot}03953}{1} = 0{\cdot}15812$ inches;

Hence the length of 30 such squares = 30 × 0·15812 = 4·74360 inches;

and „ „ 20 „ „ = 20 × 0·15812 = 3·16240 „

Construct a rectangle with a length = 4·744 inches, and a breadth = 3·162 inches, off a scale of inches, on talc or other transparent substance. Divide the length and breadth into 30 and 20 equal parts respectively, and join the corresponding points of division on the opposite sides, thus forming 600 little squares, each of which = 1 acre in area.

4. SIZE OF ONE ACRE. (Fig. 356.) The figure shows the actual space occupied by *one acre* (the customary unit of land measure), laid out in the form of a square, on maps drawn to the following scales:—

2 chains = 1 inch (side of square = 1·581 inches)
$2\frac{1}{2}$ „ = 1 „ („ „ „ = 1·265 „)
3 „ = 1 „ („ „ „ = 1·054 „)
4 „ = 1 „ („ „ „ = 0·791 „)
6 „ = 1 „ („ „ „ = 0·527 „)
10 „ = 1 „ („ „ „ = 0·316 „)

A reference to the figure will facilitate the choice of the most suitable scale for the plot of a survey.

5. THE MODE OF USING THE TALC-SQUARE. (Figs. 124, 126.) To use this *talc-square*, as it is called, lay it over the field (*see* Fig. 124) the area of which is sought, and keep it fixed by means of a weight. The ruled talc should be so applied that two opposite salient points of the field, such as *m*, *n*, may touch two of the parallel lines on the left and right of the computer.* Next, proceed to measure *vertically* every column containing any portion of the field, beginning at either the right or left-hand side and moving to the side opposite. If the boundary line of the field does not exactly coincide with any horizontal line of the talc, nor is parallel thereto, equalize any irregularities at the extremities *by the eye*, so that the space *within* the field and that *outside* of it may be equal to, and com-

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OBSERVATION.

* This is not an essential requisite, but is convenient. If the side *m* did not, for instance, touch one of the parallel lines, but was situated as shown in the right-hand lower corner of the figure, then additional horizontal measurements with the compasses would have to be made from *o* to *r*, *o* to *s*, and *o* to *t*; the last distance *o* to *t* giving the area of the portion lying outside of the vertical line *o o o*.

pensate, each other. Hold a pair of compasses in the right hand, place one of its points at *1* (*see* Fig. 124), and open it out till the other point reaches *2*; then place the *upper* leg of the compasses at *3* and the *lower* at *a*, and open them *out* to *4*; again place the upper leg at *5*, and the lower at *á*, and open them out to *6*; again place the upper leg at *7*, but as the lower leg goes beyond the piece of talc, measure the distance between the points of the compass off a diagonal scale of 5 chains to the inch (*see* Fig. 126), corresponding with the scale of the map, and place the result down on a bit of paper. Now close your compasses and begin afresh at the point *7*, proceeding as before, till the area of the whole figure is obtained as follows: –

1 to 2 + 3 to 4 + 5 = *á* to 6 = 15·18 square chains.
7 ,, 8 + 9 ,, 10 ... = *b* ,, 10 = 15·15 ,, ,,
11 ,, 12 + 13 ,, 14 ... = *c* ,, 14 = 16·66 ,, ,,
15 ,, 16 ... ... = 10·94 ,, ,,
17 ,, 18 ... ... = 13·95 ,, ,,
19 ,, 20 ... ... = 13·37 ,, ,,
21 ,, 22 + 23 to 24 ... = *d* to 24 = 17·88 ,, ,,
25 ,, 26 + 27 ,, 28 ... = *e* ,, 28 = 15·00 ,, ,,
29 ,, 30 + 31 ,, 32 ... = *f* ,, 32 = 10·28 ,, ,,
33 ,, 34 ... ... = 3·48 ,, ,,

∴ Total area of figure = 131·89 ,, ,
= 13·189 acres.

6. DESCRIPTION OF RECTANGULAR PIECE OF TALC. The area of a figure can also be measured by means of a rectangular piece of talc. Describe upon it a rectangle, on whatever scale the work is protracted, of 80 chains by 72, divide each side into 24 equal parts, and draw lines across joining the divisions; then will each small space, rectangular in shape, be equal to one acre, for 80 chains multiplied by 72 chains = 576 acres, and the square of 24 is also 576; on one side of the rectangle these rectangular spaces may be divided off into roods or square chains, if necessary.

7. METHOD OF MEASURING THE AREA OF AN OFFSET. (Fig. 125.) In order to measure the area of an offset, place any of the lines drawn across the talc-square, on the station of the survey, in such a manner that the offsets to the right and left of the line may be all brought within the small squares of the rectangle; then, holding the talc firm with the left hand and with a pair of compasses in the right hand, commence at one end of the line and measure the length of such portion of the offset from the chain line (balancing the irregular edges as near as possible) as may fall within each row of small squares, commencing with the first row on the right, adding this on, by opening *out* the compasses, to the portion contained in the second row, this again to the portion contained in the third row, and so on; if the offset passes to the left of the chain line, draw *in* the compasses from the chain line to the offset, so will the area of offset to the left of the line be deducted; then measure the distance in the compasses along one side of the rectangle, and the number of squares contained within the two legs will be the area, in acres and parts of an acre, of the offset. The station line YZ has placed over it a talc-square, divided off into acres, the crooked line, as seen through the talc, representing a boundary line. To calculate the area of the offsets on the above line YZ, commence at Y and measure the distance *ab*; keeping this distance in the compasses, place the *upper* leg of the compasses at *c*, and open them *out* to *d*; place the upper leg again at *e*, and open them out to *f*; again from *g* to *h*, and from *i* to *j*; here the offset going to the *left* of the line, place the upper leg of the compasses on the chain line at *k*, and draw them *in* towards *l*; then at *m*, drawing them in to *n*; at *o*, drawing them in to *p*; and at *q* to *r*; the offset again proceeding to the *right* of the line, the upper leg of the compasses must be placed on *s*, opening them out to *t*; from *u* to *v*; and from *w* to *x*; then will the number of squares contained within the compasses be the number of acres and parts of an acre contained in the offset on the line YZ. In the diagram the dots on the chain line, and on the offset, show the measurements to be taken with

the upper leg of the compasses, and the dots below show where the under leg of the compasses would fall in making the above measurements, observing that the distances from 1 to $c = ab$, 2 to $e = 1$ to d, 3 to $g = 2$ to f, 4 to $i = 3$ to h, 5 to $k = 4$ to j, and so on to the end of the line. The area of the offset, given in the figures, on a scale of 5 chains to the inch, is thus found to be 0·652 acres.

8. EMPLOYMENT OF TALC-SQUARE. The *talc-square* is an instrument which can be used for finding the content of irregular figures of any shape.

9. HOW TO MAKE TALC-SQUARES. The best method of making these talc-squares is to draw the rectangle and small squares correctly on a piece of drawing paper or card-board with a fine steel pen, keeping this as a pattern, and whenever a talc-square is required, to lay the talc over this, fastening it at the corners with a little gum to prevent its slipping, and with the point of a pricker to scratch the lines across the talc; when they are all drawn, a mixture of lamp black and oil should be rubbed well into the marks; round the edge of the talc, paste a narrow edging of paper to prevent its breaking.

10. CHECKING THE AREA OF A FIGURE. To check the area of a figure obtained by the application of the talc-quare, turn the plan round and apply the compasses in the *horizontal* columns, taking them now as verticals, and proceed as before till the area is determined.

11. THEORY OF THE WORK. (Fig. 127.) Let ABDKEC be the plan of the field. Draw on the plan equidistant parallel straight lines, thus dividing the figure into strips of equal width. Construct one of these strips, as BDEC. Draw the straight line *bd* at right angles to the parallels, so that the area of the strip may be the same whether BD or *bd* be regarded at its end; if BD can be regarded as a straight line, *bd* will pass through its middle point; if BD be not a straight line, the position of *bd* must be determined as well as possible by the eye. Similarly, draw *ce* at the other end of the strip, so as to leave the area unchanged. Then the area BDEC is equivalent to the rectangle *bdec*. Proceeding in this way a series of rectangles which are together equivalent to the original figure is obtained. The area corresponding to all these rectangles can be easily ascertained, and therefore the area of the original figure. Suppose, for example, that the parallel straight lines are drawn an inch apart, and that the sum of the lengths of all the rectangles is 29 inches: the area of the original figure is 29 square inches. Now, suppose that the plan has been drawn on the scale of 3 chains to an inch; then a square inch of the plan corresponds with 9 square chains of the field; therefore the area of the field is 9 × 29 square chains, that is, 261 square chains. In practice the process of forming the sum of the lengths of the rectangles is performed by an instrument called the *computing scale.*

12. DESCRIPTION OF THE COMPUTING SCALE. (Fig. 128.) A scale represented in the diagram is made equal in length to 50 chains as plotted on the plan: it is divided by transverse lines into 20 parts; each part being $2\frac{1}{2}$ chains in extent, would therefore, between parallels 1 chain in width, represent a space equal to 1 rood; two of the parts, being 5 chains in extent, between the same parallels, would represent a space equal to 2 roods; and so on for the whole length of the scale, which itself would, between parallels 1 chain asunder, represent a space equal to 20 roods or 5 acres. A slider is attached to the scale admitting of motion in the direction of its length; it bears a vernier scale (hereinafter described) with spaces equal in length to 1 part on the primary scale (or to $2\frac{1}{2}$ chains) divided into 40 parts, each of which spaces would, therefore, with a height of 1 chain, represent $\frac{1}{40}$ of a rood, or 1 perch The slider carries in a projecting frame a fine hair or wire drawn across its centre at right angles to the direction of the length of the scale.

13. MODE OF USING THE COMPUTING SCALE. (Figs. 129, 130.) To apply this scale to the measurement of areas, a piece of strong transparent paper (called horn paper), with parallel lines ruled 1 chain apart, is laid over the field the area of which is required, and kept fixed thereto. The ruled paper must be so placed that two opposite points of the field, such as x, y (*see* Fig. 129), may touch any two of the parallel lines. The scale is then placed upon the ruled paper in a direction parallel to the lines; and the vernier in the sliding frame, being set opposite to the zero point on the scale, is brought also to coincide with

or intersect that portion of the left-hand boundary fence which is enclosed between the upper pair of parallel lines. If the wire and boundary fence do not exactly coincide, the scale must be moved bodily to the right or left until the spaces *a*, *b* (*see* Fig. 130), intersected between the parallels by the wire and fence, are judged by the eye to be equal to, and compensate, one another. The scale is then kept fixed by the pressure of the hand, while the sliding frame is moved to the right-hand boundary of the field, and the cross-wire adjusted in a manner similar to before: so far, the number of divisions on the scale passed over by the cross-line, in its movement from left to right, indicates the number of roods and parts of roods included in the field between the upper parallel lines. The *slider* remaining fixed in position, the *scale* is moved downwards 1 chain, that is, to the next lower division of the parallel lines, and the vernier is brought, as before, by the movement of the *scale*, to coincide with or compensate the left-hand boundary. The coincidence or compensation being established, the sliding *frame* is moved to bring the cross line to coincide with or compensate the right-hand boundary; the division opposite the cross line then indicates the number of acres, roods, and perches comprised within the two upper parallel divisions. When the slider has by a repetition of the process, passed over the whole length of the scale, 5 acres have been measured. The operation is then continued by moving the slider from *right* to *left* (the equalization commencing on the right-hand side) until the slider having reached the starting-point or zero of the scale, 10 acres have been measured, the quantity measured being indicated by the scale itself. The mechanical operation is continued and repeated in a similar manner until the whole field has been measured. If the "computing scale" is taken in hand, and each step of the operation, as above described, followed, by measurement on a plan, there will be no difficulty felt in understanding the process, and in appreciating the rapidity and accuracy of the results.

14. THE ACRE-COMB. (Figs. 127, 128, 503.) The Calculating or *Acre-Comb* (*see* Fig. 503) is constructed on precisely the same principle as the talc-square. It is made of electrum, or card-board, measuring 6 or 7 inches by 2 or 2½ inches. The side of the acre-squares are accurately marked on each of the longer sides, and perforations made at the divisions; on one of these sides there is also a corresponding scale shown, off which the measurements for area are made. The card-board within the graduated rectangle is cut away, and the opposite divisions at the sides are joined up with horse-hair or fine silk, or other thread passed through the perforations and forming parallel lines. The comb is more handy and less liable to injury than the talc-square, and is generally preferred for small areas.

Example: Calculate the areas of the following figures on a scale of 5 chains to the inch, by means of the talc-square, or the acre-comb, and give the results in acres and decimals:—

1. Of (Fig 127) ... ... *Ans.* 5·74 acres.
2. Of (Fig. 128) ... ... *Ans.* 3·53 acres.

BY CASTING

15. CASTING DEFINED. *Casting* is a method adopted by surveyors to compute the areas of pieces of ground whose boundaries run in very irregular lines, and by which means the trouble of calculating numerous offsets is completely avoided, and with proper care equal accuracy is obtained.

16. METHOD OF CASTING EXPLAINED. (Fig. 340.) It consists in *equalizing* the boundaries *by the eye*, after first having laid down the figure correctly on paper, as surveyed by means of measured lines and offsets, by applying the *straight* edge of a transparent piece of horn, or talc or of gum paper, or a piece of glass, in such a manner that the small parts *cut off* by it from the crooked figure should be equal to those *taken in*. Pencil lines, which are represented by the dotted lines in the sketch, are then ruled along the edge of the horn or other substance, and the figure afterwards reduced to triangles, the contents of which are easily computed by the rules of Mensuration.

Example: Find the area of the irregular figure ABCD, it having been plotted on a scale of 200 links = 1 inch.

First equalize the irregular boundaries by the straight lines AB, BC, CD, DA; next join AC, and draw DE, DF perpendiculars on AC; then, by Mensuration (*see* Chap. II, Art. 33) area $= (DE + BF) \times \frac{AC}{2} = (173 + 176) \times \frac{372}{2} = 349 \times 186 = 64{,}914$ square links.

17. CASTING WITH THE AID OF A PARALLEL RULER. (Fig. 360.) Casting may be more efficiently performed with the aid of a parallel ruler, in accordance with the following general rules:—

(1) Draw a pencil line, as B3, at right angles, or at any other angle, to the chain line, as AB.

(2) Lay the ruler from the first to the third angle, and move it parallel to the second angle; then make the first mark on the pencil line.

(3) Lay the ruler from the first mark to the fourth angle, and move it parallel to the third angle; then make the second mark on the pencil line.

(4) Lay the ruler from the second mark to the fifth angle, and move it parallel to the fourth angle; then make the third mark on the pencil line.

(5) Lay the ruler from the third mark to the sixth angle, and move it parallel to the fifth angle; then make the fourth mark on the pencil line; and so on, the process of casting may be effected to any number of angles.

(6) Great care must be taken to prevent the least slipping of the parallel ruler during the operation, otherwise the result of the work, if the error is not discovered and corrected immediately, will be wrong according to the magnitude of the slip.

PROBLEMS IN CASTING.

18. PROBLEM I.—*To reduce an irregular offset-piece AabcB to a right-angled triangle of equal area.* (Fig. 360.)

Draw the indefinite pencil line B3 perpendicular to BA. Lay the parallel ruler from B to *b*, and move its further side to *c*, which will cut B3 at 1, where a mark must be made. Lay the ruler from 1 to *a* and bring its near side to *b*, marking B3 at 2. Lay the ruler from 2 to A, move it parallel to *a*, marking B3 at 3. Join A3; then BA3 is the right-angled triangle required.

19. PROBLEM II.—*To reduce a curved offset-piece AabcdeB to a right-angled triangle of equal area.* (Fig. 361.)

Divide the curve by points, *a*, *b*, etc., so that the parts A*a*, *ab*, etc., may be nearly straight, and draw A5 perpendicular to AB. Lay the ruler from A to *b*; move it parallel to *a*, and mark A5, which it cuts at 1. Lay the ruler from 1 to *c*; move it parallel to *b*, and mark A5 at 2. Lay the ruler from 2 to *d*; move it parallel to *c*, and mark A5 at 3. Lay the ruler from 3 to *e*; move it parallel to *d*, and mark A5 at 4. Lay the ruler from 4 to B; move it parallel to *e*, and mark A5 at 5. Join B5; then AB5 is the right-angled triangle required.

20. PROBLEM III.—*To reduce the irregular figure ABCDEFGHK to a trapezium of equal area.* (Fig. 213.)

Produce the line AK at pleasure. Lay the ruler from K to G; move it parallel to H, and mark AK produced at 1. Lay the ruler from 1 to F; move it parallel to G, and mark AK produced at 2. Lay the ruler from 2 to E; move it parallel to F, and mark AK produced at 3. Draw a line 3 to E and produce it. Lay the ruler from E to C; move it parallel to D, and mark 3E at 4. Lay the ruler from 4 to B; move it parallel to C, and mark 3E produced at 5. Draw a line from 5 to B; then AB53 is the trapezium required.*

OBSERVATION.

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* In this manner the crooked sides of a field may be successively reduced to straight ones. Thus, if the side AB had been crooked, the operation of straightening might be continued by producing the dotted line 5B, and finding successive points therein, corresponding to assumed angles, till the last angle was brought thereon, and so on with respect to the side AK, had it also been crooked. When the sides of a field are curved, the method of reducing them to straight lines is the same as shown in Prob. II, Art. 19

21. PROBLEM IV.—*To reduce the irregular figure AbCDEFGHA to one triangle equal in area thereto.* (Fig. 373.)

Produce any line of the figure, as AH, both ways. Lay the ruler from A to C; move it parallel to B, and mark AH at *b*. Lay the ruler from *b* to D; move it parallel to C, and mark HA produced at *c*. Join *c*D. Lay the ruler from H to F; move it parallel to G, and mark AH produced at *g*. Lay the ruler from *g* to E; move it parallel to F, and mark AH at *f*. Lay the ruler from *f* to D; move it parallel to E, and mark AH produced at *e*. Join D*e*; then *c*D*e* is the triangle required.

22. PROBLEM V.—*To draw an equalizing line through the crooked fence abcde, so that the two fields* AB*edcba, DCedcba, may be four-sided and unchanged in area.* (Fig. 339.)

Lay the ruler from *a* to *c*; move it parallel to *b*, and mark AD at 1. Lay the ruler from 1 to *d*; move it parallel to *c*, and mark AD at 2. Lay the ruler from 2 to *e*; move it parallel to *d* and mark AD at 3. Draw the line *e*3, and it will divide the two fields as required.*

BY THE PLANIMETER.

23. PLANIMETER DESCRIBED. (Fig. 500.) The *Planimeter* † is an instrument, about 9 inches in length, used for calculating the contents of irregular figures on paper. It is most simple in its application, and gives the result with the greatest accuracy. The *rod* A, bearing the *tracer* T, is graduated to the following units of measure (but there are other kinds differently graduated, and so give the contents in different denominations, whether English, French, Russian, etc.): —

0·1 □ *f* = 0·1 square foot.
10 □ *in* = 10 square inches.
0·5 □ *dcm* = 0·5 square decimeter (French measure).

On taking the instrument out of the case, the rod B must be shifted from under rod A to the side on which is fixed the *clamping screw* C, or to the side opposite that on which the graduations are marked. To find the area of any figure, push the rod A into the *tube* O till the graduation marked 0·1 square foot, 10 square inches, etc., as required (but as the generality of maps are drawn to some proportion of the inch, the index would have to be set to 10 square inches), coincides with the small line, or *index*, of the bevelled edge of the tube O. Clamp C, and by means of the *slow motion screw* S, make the coincidence perfect. When this is done, place the instrument on the paper and see that the *roller* D, the *tracing point* T, and the *needle point* P, upon all three of which it rests, touch the paper. Press the point P gently into the paper at a convenient place within or without the figure to be measured, so that the *tracer* T may traverse the entire periphery of the figure without the point P being shifted. A little experience and thought will show where the point P can be most conveniently placed, so that the greatest area within easy span of the instrument may be obtained at one operation. That the point P when fixed may not move during the operation, a small circular weight is given to be inserted in the hole over it. Having fixed P, place the tracer T at any point on the outline of the figure about to be measured, marking the point clearly, so that the tracer may return to it; note the readings of the two wheels G and D; the readings given by the horizontal disc G are whole numbers or integers, and those given by the roller D are decimal parts. For example, suppose the horizontal disc G shows 3, the roller D 87, and the *vernier*,‡ which is attached to the roller D, 4; then the reading of the instrument

OBSERVATIONS.

* Had the fence *abcde* been curved, the equalizing line might have been found as in Prob. II, Art. 19.

† Invented by Professor J. Amsler in 1854.

‡ *See* the enlargements (*a*) and (*b*) of the two scales shown in Fig. 500. In these, the length of the vernier scale is made equal to nine divisions on the limb or primary scale, and it is subdivided into 10 equal parts; there is consequently a difference in length of one subdivision between the scale on the limb and the vernier scale—since

10 Vernier subdivisions = 9 divisions on limb;
1 Vernier subdivision = $\frac{9}{10}$ " " ;

therefore a reading off the limb of the roller D of the planimeter (*see* Fig.) is by means of the Vernier capable of being read off to $\frac{1}{10}$th of a division marked on the primary scale. Of the two examples given in the Fig., the reading of (*a*) = 0·300, and of (*b*) = 0·367.

before starting is to be set down as 3·874. Follow the periphery or boundary line of the figure, *in the direction of the hands of a watch*, as accurately as possible, observing also that the point T *never* approaches the point P *within* an angle of 20°, as the area becomes unreliable by so doing, until you arrive at the starting-point (straight lines may be followed along a ruler); then read off the numbers on the disc G and the roller D, which suppose to be 9·761. Notice must be taken whether the disc G has made an entire circuit (*i. e.*, where the zero of the disc passes the index) *forward* or *backward*; if so, 10 must be added for every revolution to the whole number of the *second* or *first* reading according as the motion is forward or backward; thus, if the disc had gone once round *forward*, the *second* reading would have been 19·761; if twice round, 29·761; and so on; but if the disc had gone once round *backward*, the *first* reading would have been 13·874; if twice round 23·874; and so on. When the two readings are obtained, there are two cases to be observed in computing the area:—

1st Case.—When the point P is fixed *outside* the figure.

Rule: *Subtract the* first *reading from the* second *reading* (increased by 10 for every *forward* revolution of the disc), *and multiply the remainder by the unit of measure* (as 0·1 sq. ft., 10 sq. in., etc., to whichever graduation the index is set).

Suppose the unit of measure to be 0·1 sq. ft.

9·761	2nd reading.
3·874	1st „
5·887	Difference.
0·1	Unit of measure.
0·5887	Square feet.

Suppose the unit of measure to be 10 sq. in.

9·761	2nd reading.
3·874	1st „
5·887	Difference.
10	Unit of measure.
58·87	Square inches.

2nd Case.—When the point P is *inside* the periphery of the figure.

Rule: *Add to the* second *reading the constant number engraved on the rod A above the corresponding unit of measure, subtract from this sum the* first *reading* (increased by 10 for every *backward* revolution of the disc), *and multiply the remainder by the unit of measure* * (as 0·1 sq. ft., 10 sq. in., etc., to whichever graduation the index is set).

Suppose the unit of measure to be 0·1 sq. ft.

9·761	2nd reading.
20·708	Constant number on rod over 0·1 sq. ft.
30·469	Sum.
3·874	1st reading.
26·595	Difference.
0·1	Unit of measure.
2·6595	Square feet.

OBSERVATION

* The constant number, being the area of a circle of which P is the centre, and the distance from P to the axis O, the radius, is in this case 20·708, the unit of measure being 0·1 sq feet, and 22·009, the unit of measure being 10 sq. in. It is well to recollect that these constant numbers vary in different planimeters.

Suppose the unit of measure to be 10 sq. in.

9·761	2nd reading.
22·069	Constant number on rod over 10 sq. in.
31·830	Sum.
3·874	1st reading.
27·956	Difference.
10	Unit of measure.
279·56	Square inches.

Having found the area, let us suppose, in square inches, it remains to multiply this by the number of square miles, or square feet, which a square inch of the map or plan operated upon represents.

To take another example, suppose it was required to obtain the area of a figure off a plan constructed on a scale of 16 inches = 1 mile. Adjust the planimeter as before, and set the index to the graduation on the rod marked 0·1 sq. ft. Now, suppose the first and second readings were found to be respectively 4·728 and 7·699; the area would be calculated as follows:—

7·699	2nd reading.
4·728	1st ,,
2·971	Difference.
36	(Constant multiplier for scale of 16″ = 1 mile, to obtain area, *in acres*.)
106·956	Acres.

It is of no consequence whether the roller D is inside or outside the figure provided it is on the same level. In measuring small figures, the point P is fixed *outside* the periphery; for large figures, the point P is fixed *inside* the periphery. For the contents of very large figures, they can be subdivided by drawing straight lines through them, and the contents of the several parts computed separately and added together. Great care is necessary in using and handling this delicate and useful little instrument, and no force should be employed in shifting the rods, etc., for the slighest injury to the axles of motion will ruin it hopelessly. The roller D should be kept perfectly free of rust, or true results will not be obtained. The axles may be occasionally oiled. If, after being some time in use, the instrument does not give accurate results, when the rod A is set to one of the marked divisions, it should be corrected by the tangent screw, elongating or shortening the arm to the required length. To test its accuracy, draw a small square carefully to scale, and see that the results, as calculated by the known value of the sides, and by the instrument, coincide.

CHAPTER IX.

Chain Surveying.

1. CHAIN DESCRIBED. (Fig. 349.) The *Chain* is an instrument used for measuring the distances required in a survey. There are two descriptions of it, one is 100 feet long, divided into 100 equal divisions, called *links*, each link being, therefore, equal to 1 foot; the other is the *Gunter's chain*,* which is 66 feet long, divided, like the other, into 100 equal divisions, also called *links*, each link being, therefore, equal to $\frac{66}{100}$ of a foot, or 7·92 inches.

2. CONSTRUCTION OF CHAIN. (Fig. 349.) Both descriptions of chain are constructed in the same way, but differ only in their length. They are usually made of strong iron or steel wire about $\frac{1}{8}$ of an inch in diameter; each end of the

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OBSERVATION.

* So named after the inventor, Dr. Edmund Gunter, an eminent mathematician, in 1620. It is also sometimes called a 4-pole chain.

link is bent round and joined to the next link by 2 or 3 small rings,* which can be removed or more entered when necessary; the last link at each end is shorter than the others, and has a small brass swivel-handle attached to it, and this handle being counted as a part of the link, makes its length the same as that of the others.

3. NUMBERING OF LINKS. (Fig. 349.) The links are numbered from each end of the chain towards the middle. At every tenth link from each end is attached a piece of brass with points;† that at the 10th link has one point, that at the 20th link has two points, that at the 30th link has three points, that at the 40th link has four points, and that at the 50th link, or the middle of the chain, has a round piece of brass; so that it is evident that the 10th link from one end of the chain is the 90th link from the other, and the 20th link from one end is the 80th from the other, and so on; hence, any distance on the chain line may be readily counted.

4. ARROWS DESCRIBED. Each chain is provided with 10 pins called *arrows*,‡ made of iron wire and each about 12 or 15 inches long. They are pointed at one end, so as to be easily pressed into the ground, and turned at the other, so as to form a ring to serve for a handle. They are used in succession to mark the chain length in measuring a line.

5. DIFFERENT USES OF THE TWO CHAINS. For general use, where *linear* measurement is alone the object, the 100 feet chain is most convenient, but for work in the hills or for measuring offsets of a chain line, one that is 50 feet long, which is in every way similarly made to the former, except that it has brass notches at every 5 links, may be used with advantage. When the sole object is to obtain the *areas* of fields *in acres*, the Gunter's chain of 66 feet is specially adopted.

Since 66 feet = 1 chain,
∴ 66 × 66 = 4,356 square feet = 1 square chain,
and ∴ 4,356 × 10 = 43,560 „ „ = 10 „ „ ;
but 1 acre = 43,560 „ „
hence 1 acre = 10 square chains.

Thus, the area of any field being found in square chains, the same can be reduced to acres by dividing it by 10.

Since there are 100 links in one chain,
1 square chain = 100 × 100 = 10,000 square links,
∴ 10 „ „ = 10,000 × 10 = 100,000 „ „

and since to divide by 100,000 is the same thing as to point off 5 places of decimals from the right, the area of a field being given in square links, it can be reduced to acres by simply inserting a decimal point between the 5th and 6th figures from the right, thus leaving the figures towards the right hand for the decimals of an acre and the remaining figures towards the left hand for the acres. For the reduction of the decimal part to square annas and pies (Burmese measure, *see* Table LXI, "Calculating Tables," 5th edition): multiply the decimal by 16 and cut off five figures of the product from the right, thus leaving the square annas on the left hand; again multiply the last decimal by 12 and cut off five figures from the product as before, for the square pies.

Example: Find the area of a field of which the breadth is 6·78 chains and length 8·32 chains, the measurements having been made with a Gunter's chain.§

6·78 chains = 678 links.
8·32 „ = 832 „
1,356
2,034
5,424
∴ area = 564,096 square links.

OBSERVATIONS.

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* By the use of these rings the chain is less liable to twist and get entangled or *kinked*.

† Also called "Chain *numbers*."

‡ The reason why 10 is the number adopted is that 10 Gunter's chain = 1 furlong and 8 furlongs or 80 chains = 1 mile; also because, apart from its being easily remembered, the work of tabulating up every 10 chains' distance travelled of a measured line is facilitated. Again, an acre of land = 10 square chains.

§ When the word chain alone is made use of, Gunter's chain is always meant

Cutting off five figures from the right hand
gives ... ... 5·64096 acres;
Multiply the decimal by 16 16

gives ... ... 10·25536 sq. annas;
Multiply the decimal by 12 12

gives ... ... 3·06432 sq. pies.

Hence area = 5 acres, 10 sq. annas, 3·06 sq. pies.

And since 1 acre = 43,560 square feet, if the above field had been measured by a 100 feet chain, its length and breadth being multiplied together, the product would have to be divided by 43,560 to reduce it to acres, which is evidently not nearly so simple a method of arriving at the area, *in acres*, as pointing off five places of decimals.

6. LIABILITY TO ERRORS IN CHAINS. The chain is liable to many errors: *1st*, in itself; *2nd*, in the method of using it; and *3rd*, in the uncertainty of pitching the arrows; every possible precaution must therefore be used. If the chain is pulled too tight, the rings between the links will open and make the chain too long, and the arrows will incline, thus making the measured line shorter than it is. For, suppose the chain to be 1 inch too long, in every chain measured there will be 1 inch of ground included which ought not to be included, and therefore the measured line will be *shorter* than it ought to be. If, on the other hand, the chain were not drawn sufficiently tight, or were not long enough, the measure obtained would be *longer* than it really is.

7. PROVISION FOR ERROR OF CHAIN. If a line has been measured with an incorrect chain (and such is often the case), the true length of the line will be obtained from the following proportion:—

length of correct chain : length of chain used :: measured distance : true distance.

For example, suppose the chain (Gunter's) to have stretched 3 inches, and that the measured distance is 2,080 links, then:

$$66 \text{ ft.} : 66\tfrac{1}{4} \text{ ft.} :: 2{,}080 \text{ links} : \frac{265 \times 2{,}080}{66 \times 4} = 2{,}088 \text{ links, true distance.}$$

To take another example: suppose the distance between two places was measured and found to be 2 miles, but that the chain with which the distance was measured, and which was supposed to be 100 feet long, was really only 99 feet 11 inches long; what is the true distance between the places?

2 miles = 5,280 × 2 = 10,560 feet.

$$\therefore\ 100 \text{ feet} : 99\tfrac{11}{12} \text{ feet} :: 10{,}560 \text{ feet} : \frac{1{,}199 \times 10{,}560}{100 \times 12} = 10{,}551 \text{ feet, true}$$

distance.

The day's operations should be invariably started with a *correct* chain; but if from any cause an *incorrect* chain is used at the commencement of a piece of work it must be again measured at the finish (as laid down in Art. 10), and the mean of the two errors taken for the length of the incorrect chain for the purpose of the calculation above given; for the obtaining of the true length of a measured line, or for that of the finding of the true area of a survey (*see* Art. 49). Thus, for example, at the start of a measurement a chain was 3 inches too long and at the finish 1 inch too short; then the *mean* length of the chain used would be $\frac{3-1}{2}$ inches = 1 inch in *excess* of a *true* chain's length. Or, suppose at the start the chain used was 2 inches too short and at the close it was short by 4 inches; then the *mean* length would = $\frac{4+2}{2}$ inches = 3 inches in *defect* of the length of a *true* chain.*

OBSERVATION.

* Sometimes, in order to ensure reliable results, double measurements of the lines are made by the employment, independently of each other, of two chains, a 66 feet and a 100 feet (which, being of different lengths, act besides as a check upon each other), the mean value of both being manifestly more to be depended on than if a single chain were alone used. In such a case it becomes necessary to convert the measurements made in one denomination into their equivalents of the other; that is, of chains into feet, and *vice*

8. TESTING CHAIN'S LENGTH. (Fig. 443.) To provide against errors of the above kind, the chain should be frequently tested. If the chain is a new one, it should invariably be measured *daily* until it has stretched to its utmost; if an old one, once in every seven days is sufficient; for suppose a chain to have stretched 3 inches, then, although apparently trifling, in a day's work of about 300 chains the error would be 75 feet in the whole distance measured. The comparison should be made with two *Standard steel bars*, shown in the figure, which are each one inch square in thickness and 6 feet long, having feet and inches correctly marked on them; if these are not available, the ordinary offset rod will, for general purposes, be sufficiently accurate; two should be used, thus: Stretch the chain moderately tight on a level piece of ground, fixing two stout wooden pins at each end in the handle of the chain; then lay down two rods in succession from one end, keep the second stationary, and taking up the first, place it beyond the second; then keeping that stationary, take up the second and place it beyond the third, and so on, until the end of the chain or 11 rods is reached, when the deficiency or excess of the chain may be measured with any scale of inches and the correction made accordingly. If the deficiency or excess is great, for the former straighten some of the links, which are sure to be found bent; for the latter remove from each extremity of the chain one or more of the rings as necessary. The same correction, whatever it be, should be made equally at both extremities, in order that the middle point of the chain may be in its true position, in which case the error will be spread throughout the chain. It is a good plan for the Surveyor, when he has once got a correct chain which has been duly tested with some known standard, to himself mark out the true chain's length in the vicinity of his residence, to which his chain could be referred when required. For this purpose, let him select a tolerably level piece of ground and stretch the chain thereon to its fullest extent; then mark the points *outside* each of the two handles of the chain, where drive two stout wooden pegs about a cubit long well into the ground and nearly flush with the surface. A strong nail might advantageously be driven into the centre of each of these pegs at top, into which the handles of the chain could be inserted at the time of testing its length. The distance from the centre of one peg to the centre of the other should correspond *exactly* with the true chain's length measured from the outer side of the handle at one end to the outer side of the handle at the other. Any difference thereafter detected in the length of the chain from this fixed length will be the error *plus* or *minus*, as the case may be.*

9. HORIZONTAL AND NOT ACTUAL DISTANCES REQUIRED. (Fig. 350.) When the ground over which the measurement is taken rises or falls, or both alternately, it must be recollected that the *horizontal* distances are what are required for the survey, and *not* the actual distances measured along the surface. For ordinary purposes, the horizontal measurement may be obtained by holding that end of the chain upon which the ground slopes, so as to keep it, as nearly as can be judged, horizontal, the arrow being placed vertically under it, and so on for any distance; the sum, as AB, of all these horizontal distances, C*a*, *bc*, *de*, *etc.*, will be the total horizontal length required. Where greater accuracy is wanted, the angles of depression or elevation are measured with an angular instrument and the horizontal distance calculated from a table of slopes (found in Table XII, "Calculating Tables," 5th edition), but this does not yet concern us. In measuring along the *steep* sides of hills, chaining up and down the slopes may be accomplished with sufficient accuracy for all practical purposes by what is known as ***stepping***, which consists of short lengths of the chain being held in a horizontal

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versâ. This may be best done by the aid of Tables I and II of the "Calculating Tables" (5th edition), but in the absence of tables, the required conversions can be effected by the following simple rules, namely:—(i) *To reduce links to feet*, subtract from the number of links as many units as it contains hundreds; multiply the remainder by 2 and divide by 3. Thus, 6,823 links = (6,823 − 68) × 2 ÷ 3 = 4,503 feet. (ii) *To reduce feet to links*, add to the given number half of itself, and add one for each hundred (more exactly, for each ninety-nine) in the sum. Thus, 417 feet = $417 + \frac{417}{2} + 6 = 632$ links.

* To *fold* or "do up" the chain, take the middle of it in the left hand, and with the right hand take hold of the doubled chain just beyond the second link; double up the two links between your hands, and continue to fold up two double links at a time, laying each pair obliquely across the others, so that when it is all folded up the handles will be on the outside easy to tie up and to carry.

position, and the extremity transmitted to the ground by dropping thereon the arrow from the end of the chain, or that part of it held by the leader. The greater the angle of the slope so much less with the horizontal distance, and *vice versâ*. In this simple manner, chaining along very sloping ground may be accomplished both accurately and expeditiously.

10. THE LEADER AND FOLLOWER, AND THEIR DUTIES. Before a line can be measured, its direction must first be clearly laid down by means of flags. A flag is placed upright at each end of the line, and if the line is very long, intermediate flags must be placed so that the chained line may not deviate from the line required. This is called *ranging* the line.

To measure a line with the chain, two men are required, one called the ***leader*** who drags the chain forward, and the other the *follower*. Having first set flags up at the places whose distances are to be obtained, the leader takes the 10 arrows* and proceeds with one end of the chain in the direction of the line to be measured, while the follower holds the other end at the ***first*** station, and directing the leader in a straight line with the forward flag, by signalling to him with the hand to move laterally right or left as required, sees that the chain is properly stretched, that there are no knots in it, etc.; the leader puts one arrow in the ground, and pulls on the chain again; but if the ground is too hard to admit of the arrow being driven in, a cross should be scratched on the ground, and the arrow laid down pointing to the intersection. The follower, having come up to this arrow, puts the handle of the chain close against it on the ***outside***, and again directs the leader, who, as before, puts an arrow into the ground against the ***inside*** of the handle at his end. The follower takes up the first arrow, and the same process is repeated until the 10th arrow has been put into the ground. Here the follower comes up to the leader, and marking accurately the position of the 10th arrow on the ground, returns the 10 arrows to the leader, and the Surveyor notes in the margin of his book (1,000), which means that 10 chains, or 1,000 links, have been measured. The number of arrows in the hand of the follower added to 10 for each time the arrows have been returned by the follower to the leader, will always show the number of chains that have been measured. So that when the leader arrives at the *second* station, the number of arrows in the hand of the follower (denoting so many 100 links), added to the odd links on the ground, and the number of changes (denoting so many 1,000 links) will make up the entire length of the line between the two stations.

Example: Suppose the follower has returned the arrows three times, and has 6 arrows in his hand, and that the extremity of the line is at the 34th link of the next chain; what is the whole length of the line?

Here the 10 arrows 3 times returned = 30 chains = 3,000 links.
6 ,, in his hand = 6 ,, = 600 ,,
34 links of the next chain = 34 ,,

∴ Total length = 3,634 ,,

11. CHAINMEN TO BE WATCHED. The Surveyor should keep a continual watch over the chainmen in measuring lines, and not allow them too much latitude, and to do that his position must be just in rear of the chain, otherwise the chances are that the measurements will be carelessly made and an incorrect account of the number of chains gone over kept, thus probably necessitating a remeasurement of the whole work.†

12. CHAINING OF LONG LINES. For long chain lines, especially over undulating ground, it is requisite to range out as many intermediate points as may be deemed necessary. To this end, flags having already been placed at the

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* (i) It would be as well to tie a piece of red cloth or tape in the handle of the arrows, so that they may be the more readily distinguished when planted in the midst of grass or jungle, etc.

(ii) Eleven pins are sometimes used, one being of brass, or otherwise different from the rest, and being used to mark the end of the eleventh chain; another being substituted for it before the leader goes on.

† As the *follower* is the more responsible person of the two men, the more intelligent of them should be put on to this duty. If inexperienced chainmen have to be employed, the Surveyor would do well to get them to measure the distance between any two points, and then remeasure it in the opposite direction. The difference of the two results ought to impress upon them the necessity for great carefulness.

extremities of the line that has to be chained, a man should be sent ahead to plant other flags in the chain line at intervals, as required, the Surveyor himself directing from 3 or 4 yards to the rear of the flag at the commencement of the line. In this way chaining operations of long lines are accurately performed, the intermediate flags acting as useful guides for both the leader and follower. Often it may not be possible to command a view of the forward station, which is situated perhaps in a valley, when it is manifest that unless intermediate points, such as those referred to, were previously established, it would be impossible to chain the line.

13. MAXIMUM ERROR ALLOWED. The maximum error allowed, in lineal measurement on the Revenue Survey, according to the test it is submitted to by traverse proof, a rigid mode of calculation of a theodolite traverse survey (*see* Chap. XIV, Arts. 9—10, 15) is 1 link in 10 chains, equal to 5·28 feet per mile.

14. SURFACE TO BE DIVIDED INTO TRIANGLES. (Figs. 337, 348, 369.) In making a survey with the chain only, we are confined to one, and the simplest geometrical figure, namely, the *triangle*. The surface to be measured is therefore to be divided into a series of triangles (*see* Fig. 348), and they are to be as large as is consistent with the nature of the ground and the surface to be surveyed. The sides of these triangles are first measured, and, as a necessary check on this part of the work, a straight line is in addition measured from one of the vertices to a point in or near the middle of the opposite side. This fourth line, as B*b* or *xy* (*see* Fig. 337), is called a *tie-line* or *proof-line*, and detects errors which may have been committed in the measurement of the sides of the triangle. Indeed, the triangle could not be considered correct unless such a tie-line was measured, for the three sides of a triangle, however carefully measured, are no guarantee of its accuracy; there must therefore be a proof or tie-line. Within the larger triangles as many tie-lines and smaller triangles are to be measured as may be necessary to determine the positions of all the objects embraced in the survey. The directions of the lines forming the sides of these secondary triangles must be so disposed that the offsets to be measured from them may be as short and as few in number as practicable.*

15. TO FIND THE THIRD STATION OF A TRIANGLE. (Fig. 351.) In using a measuring chain all three sides of a triangle are measured; hence having two stations † of a chain line given on paper, the third may be found thus: From A and B as centres, with the measured distances AC, BC as radii, describe two arcs of circles intersecting at C, which is the required point.

16. ROUGH SKETCH IN THE FIELD-BOOK. (Fig. 354.) The disposition and general combination of these triangles demand care and judgment. Previous to commencing any measurement make a careful reconnoitre, that is, walk over the ground for the purpose of obtaining a general knowledge of the surface, and of the relative positions of the most conspicuous objects. This is an important and absolutely essential part of the proceeding, for without it the work would in every probability be unsystematically laid out, and there would perhaps be some hitch or delay arising on account of some unforeseen circumstance, such as the encountering of obstacles not previously observed, and the like. Next draw a hand-sketch, or rough diagram in the Field-book (*see* the Figure, sheet No. 18) resembling a plan of the ground to be surveyed, showing some of the principal roads, streams, temples, etc., so as to assist the memory in distributing the surface into triangles. The after operations of plotting the survey from the field-book are made very much simpler if the positions of the measured lines and the stations be also shown in the rough sketch, which, it may be remarked, should at the same time be neat.

OBSERVATIONS.

* In laying out both principal and secondary lines, deep lakes and tanks should be avoided if possible; if this cannot be done, a rope may sometimes be carried round and stretched across, or the distance may be obtained by laying off four right-angles with the optical square (*see* Art. 27) as in Fig. 369, making *cd* = *fe* and taking the distance *df* equal to the inaccessible portion (*ce*) of the line. Chainmen should never be sent with a chain into water beyond their depth.

† Stations are certain marks left on the ground along various survey lines to which reference has to be made at a future opportunity. On extensive and important surveys it is necessary that these marks should be more or less permanent. In arable lands and for ordinary work, a stout peg well driven into the ground is all that is usually required. On plans, stations are denoted by means of a small equilateral triangle or a circle, drawn on the paper thus: △, ⊙.

17. CHAIN LINES TO BE RUN ALONG BOUNDARIES. (Fig. 338.) The sides of the larger triangles must pass as close as possible to the external boundaries to be surveyed, and the nearer their approach is to an eqilateral form the better; otherwise, if a slight error has occurred in measuring any one of the sides, the area of the figure is thrown out a great deal.

18. STATIONS DISTINGUISHED. (Fig. 338.) The triangles having been thus disposed, pegs are placed in the ground at each vertex of the triangles, and the stations are lettered or numbered, as the case may be, in the hand-sketch. Flags might at the same time be temporarily set up at all the points selected. These are usually made of bamboo, 10 feet or so in length, surmounted with a piece of red and white cloth, about 18 inches by 12 in size, so as to be readily discernible at long distances.

19. OFFSETS DESCRIBED. (Figs. 352, 501, 522.) *Offsets* are short perpendiculars, as *ab*, *cd* (*see* Fig. 352), measured from points on the chain line to the angles of a crooked or zigzag line, or to objects situated near the chain line. These distances are generally measured with a 50-link chain or with a *measuring tape* * (*see* Fig. 501), (such as Chestermann's, which is 66 feet long, having feet divisions on one side and links on the other), the latter being very useful for the measurements of the rears of houses and yards, and small enclosures, through which the chain cannot be taken; but if the distances are very short, they may be measured by a rod with an iron spike at one end, called an *offset rod* (*see* Fig. 122), and which is 10 feet or 10 links = 6 feet 7·2 inches in length, divided into feet or links, according to the description of chain used, whether the 100 feet chain or Gunter's chain. These parts or divisions should be painted white and black alternately, to distinguish the links easily.

20. OFFSETS DETERMINE THE POSITIONS OF OBJECTS. (Fig. 352.) It is clear that if from certain points in the chain line offset measurements are taken to the two corners of a rectangular house, its position becomes determined; thus: at a certain distance A*a* from A, the corner of a house is at a perpendicular distance *ab*, and at a distance A*c* from A, the other corner is distant *cd* from the chain line, the position of the house is exactly known. In the case of buildings, diagonal tie-lines, such as *a* to *d* and *c* to *b*, might also with advantage be taken, so that the angles may be fixed with exactitude. The right-angles for the offsets should be carefully set out, while the lengths of the frontage, sides, back and projection should be accurately measured and recorded. So also when the chain line runs through a town and cross-streets are met with; in such cases it would be well to take, in addition to the ordinary offsets to the four angles, diagonal tie-lines as a greater check on the work in the determination of the positions of the four points formed by the cross-roads.

21. MANNER OF SURVEYING A FENCE. (Fig. 353.) Suppose it were necessary to survey a fence, commence by measuring a main chain line AB along the side, and take offsets shown by the dotted lines perpendicular to it, to the several points or angles in the boundary, noting them in the Field-book, as hereinafter explained.

22. ONLY 100 FEET OFFSETS TO BE TAKEN. Offsets longer than 100 feet, or about 1½ Gunter's chain, should never be admitted in a chain survey. The main chain lines must be so adapted that no such long offsets will be necessary; if this cannot be managed, smaller triangles must be built on them, and so prevent the necessity of long offsets.

23. OBLIQUE OFFSETS DEFINED. (Fig. 341.) If a solitary object is at a considerable distance from the chain line, its position may be fixed by measuring two distances *ac* and *bc* from two convenient points in the main chain line and some distance apart. These distances are called ***oblique offsets***, as they are not perpendicular to the chain line.

24. INTERMEDIATE STATIONS ON CHAIN LINE TO BE MARKED. (Fig. 338.) In measuring a chain line all *intermediate* stations, as *b*, *x*, *y*, left on the line from which it is intended to run cross or tie-lines, or else from which to fix

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OBSERVATION.

* This should be used only under exceptional circumstances, as there are several objections to its constant employment for offset measurements. After much use or wet it elongates and is not very strong or durable either. The measuring tape is best suited for engineering purposes.

on other triangles, must be marked at the time of measuring the sides of the principal triangles, with either a peg driven into the ground at the point, or by some other means; in fact, it is a good practice to scratch a line at every point on a chain line whence an offset is taken.

25. DESCRIPTION OF CROSS-STAFF. (Fig. 342.) The *Cross-staff* is an instrument used for the purpose of taking offsets, and can be made up by any carpenter. It consists of a piece of wood, about 6 inches square and 1½ inches in thickness, fixed on the end of a staff about 5 feet long, with an iron spike at the end for the convenience of planting it in the ground. The square piece on the top has two grooves, *ab* and *cd*, in it, about half an inch deep and half an inch broad, made with a common saw, at right angles with each other in the centre of the board. This instrument being placed in a vertical position anywhere on the chain line, if one groove be directed to the forward or back station, the other will of course give the perpendicular to the chain line.*

26. OPTICAL SQUARE USED FOR OFFSETS. (Fig. 397.) Besides the cross-staff an instrument called the *optical square* is employed to obtain the perpendiculars on a chain line, and it gives them much more correctly than the cross-staff. For long offsets, especially, the cross-staff has been almost superseded by the optical square, which is much superior to it both for convenience and expedition; and it has also the advantage of greater portability.

27. DESCRIPTION OF THE OPTICAL SQUARE. (Figs. 389, 397.) The *optical square* (*see* Fig. 397) is an instrument for setting off right angles and raising perpendiculars on a chain line. It is made of brass and resembles a hollow wedge-shaped box of 2-inch sides and 1¼ inch depth, having a handle fixed below about 3 inches long. It contains two rectangular strips of looking-glass fixed at an inclination of 45°; hence, while viewing an object *by direct vision* through the opening on the hollow side and one of the two rectangular apertures over the mirrors, any other, forming a right angle with it at the place of the observer, will be referred *by reflection*, so as to coincide with the object viewed. Thus a line may be laid out perpendicular to a station-line, and from any point on it, by simply standing with the instrument over the given point, and looking through it along the line, having a person to go with a flag in the direction the perpendicular is required, and signing to him by hand to move to the right or left, until his flag is seen by reflection to coincide with some object on the line along which the observer is looking, when the place of the flag will be a perpendicular to the station-line at the place of the observer. If it be required to find on a line the place of a perpendicular from a fixed object, as a house, etc., the observer himself must move along the line until the image of the object appears, as before, in the direction of the line; and the place where he then stands will be the spot where such perpendicular would fall. Suppose AB (*see* Fig. 389) to be a chain line, proceeding in the direction of B, at which there is a flag set up, and D a tree to the *left* to which it is required to take an offset. Let the observer stand on the chain line looking in the direction of the flag and hold the optical square in his right hand, having its hollow side *abcd* facing the tree, and let him apply his right eye to the opening on the hollow side in line with the aperture over the mirror *nearer* him at the point *e*, and direct his vision through the aperture over the other mirror along the point *f*, to the flag. Now the image of the tree appears reflected by the mirrors to the observer, either to the right or to the left of the flag; if to the *right*, he must *advance* slowly along the line, keeping the instrument carefully in position all the time, until the tree coincides with the flag; if to the *left*, he must *recede* in the same way until the same thing occurs. The position of the observer when the coincidence takes place, is the point C at which the tree is perpendicular to the chain line, from which the required offset will of course be measured. Again, suppose on the observer's proceeding further on the chain line,

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OBSERVATION.

* To test the accuracy of the instrument, set it up on the chain line, sight through one slit to the forward flag, and plant an offset flag a chain or so's length, to the right or left, in the line of sight of the other slit. Next turn its head a quarter of the way around, so that the second slit, looked through, points to the forward flag. Then see if the other slit covers the offset flag again, as it will if correct. If it does not do so, but sights to some other point, to one or other side of the offset flag, put up another flag at such other point, and half the difference of the angle thus formed between the two offset flags will be the error of the instrument, which can be corrected accordingly.

he sees a pagoda to the *right* to which an offset must be taken. Let him hold the optical square, as before, making the hollow side *abcd* face the pagoda, then let him apply his right eye to the opening on the hollow side in line with the aperture over the mirror *nearer* him at the point *g*, and direct his vision through the aperture over the other mirror along the point *h*, to the flag. Now the image of the pagoda appears reflected by the mirrors to the observer, either to the right or to the left of the flag; if to the *right*, he must *recede* slowly along the line, keeping the instrument carefully in position all the time, until the pagoda coincides with the flag; if to the *left*, he must *advance* in the same way until the same thing occurs. The position of the observer when the coincidence takes place is the point E at which the pagoda is perpendicular to the chain line. In using the instrument hold it quite erect.

28. FIELD-BOOK DESCRIBED. (Figs. 343, 344.) The ***Field-book*** (*see* Fig. 343) is an oblong note-book in which the results obtained by measurement in the field are recorded. It should not only be accurate, but should also give as correct a sketch of all the objects along the various chain lines as possible, so that with the help of the measured distances the Surveyor might be enabled to plot all the details of his survey exactly as they are on the ground. Field-books should be paged from end to end. A convenient size for these books is about 7 inches long by 4 inches broad, opening at the end, and not at the side like ordinary books. Each page of the book should be divided into three columns, by drawing two parallel red lines ⅜ inch apart down through the centre. The space between these two lines represents the measurements on the chain line, and the margin represents the ground on either side of it, in which also all necessary observations are to be written, sketches made, etc. The entries are commenced at the *bottom* of the *last* page in the book, because if the book be held in the hand over the chain as it lies on the ground, and the bottom of the page taken to represent the starting-point A (*see* Fig. 344), and the top of this page the point B, then the offsets *ab* and *cd*, which lie to the right of the line on the ground, would be entered to the *right* of the centre column in the field-book; but if the entries had commenced from the top of the page, these offsets would have to be entered to the left, which would be inconvenient.

29. EXAMPLE OF FIELD-BOOK. (Figs. 344, 425.) The figure (*see* Fig. 344) shows how the line AB with the offsets *ab* and *cd*, in the plot by the side of it, would be entered in the field-book. The points A and B are called ***stations***, and are marked ⊙ in the field-book, and the line is entered. ⊙ A is then entered in the middle of the column, since the measurment of the line commences *exactly* at ⊙ A, and just above ⊙ A a circle with two ciphers is written thus (00) to show that the measurements were made from 0; next the distances on the chain line are all counted *from this point*, and entered in the centre column, one above the other, at equal distances apart, say, of ⅓ of an inch. Thus, suppose A*a* is 150 and A*c* 250; then, of course, *ac* is 250 - 150 = 100; but it is the *total* distance A*c* from A that is entered in the field-book, and *not* the intermediate distance *ac*. Proceed in this manner, writing the measurements along the chain line in the centre column, and the offsets beside them, on the right or left, as the case may be, opposite the distance at which they are taken, until ⊙ B is arrived at, which measures 400 links from ⊙ A; draw a circle round 400 to show that it is the measurement to ⊙ B which is written just above it; next draw a double line across the centre column to denote that the chain line from ⊙ A to ⊙ B is concluded, and start afresh for your next line from above the double line, as before. Another method of recording field measurements, adopted in some cadastral and other large-scale surveys, is the drawing of a large plot roughly showing the situation of the field and other boundaries under survey, and on which all the chainage on the main and subsidary lines, as well as the offsets, are duly recorded as the survey is proceeded with. An example of this system is to be found in (Fig. 425), which represents an extract from a cadastral* survey field-book roughly plotted on the 32″ scale, and for eventual plotting on the 16″ scale, or one-half that of the rough plot.

OBSERVATION.

* For an explanation of this term, *see* Chap. XIII, Art, 84.

30. PLOTTING IN THE FIELD. For a chain survey the distances, instead of being recorded in a field-book, may be at once plotted in the field as the survey is proceeded with; and for this purpose the Surveyor must provide himself with a common drawing board mounted with paper whereon to plot the work (*see* Chap. X, Art. 22).

31. MANNER OF ENTERING LINES CROSSING THE MAIN CHAIN LINE. (Fig. 344.) Suppose a fence shown by the dotted line *ef*, to cross the chain line at a distance from A of 300 links, it would be entered as shown in the field-book. Remembering that the space between the two lines in the field-book represents the chain itself, and is, therefore, supposed to be only *one* line, it will be evident that, at whatever point on the left the line *ef* intersects it, it must leave at a point ***exactly opposite*** this on the right. In fact, a single line down the middle of the page, instead of a space, would do as well to represent the chain line, but it would be very untidy to have to enter all the distances on one line instead of between two lines. The arrow point from ⊙ B shows approximately the direction of the next line to ⊙ C.

32. PLAN OF A FIELD, AND THE FIELD-BOOK FOR THE SAME. (Figs. 337, 338.) Fig. 338 shows a plan of a triangular field, and the lines that would be measured in order to make a survey of it, and Fig. 337 shows the field-book for the same. The lines B*b* and *xy* are tie-lines, measured as a check upon the work. Suppose the Surveyor to start from ⊙ A in the direction AB. The line AB would be chained, and the offsets taken to the boundary, as explained above (in Art. 21), and entered in the field-book. Having arrived at ⊙ B, two arrows are entered, one showing the direction of the line B*b*, and the other the direction of the next line BC. This line would be measured in the same way, and an arrow entered at ⊙*x* showing the direction *xy*, and another at C for the direction of CA.

33. FOUR-SIDED FIELDS SURVEYED AS TWO TRIANGLES. (Fig. 347.) If the field were a four-sided one, it would evidently be surveyed by constructing two triangles on opposite sides of one of its diagonals, and measuring the sides of each triangle, as in the figure.

34 FIELD-BOOK TO BE PAGED. The pages of a field-book should be numbered for facility of reference, and each day's work dated. It may be remarked that the entries in a field-book should be made both neatly and accurately—if possible, in ink— so that the work may be understood not only by the Surveyor himself, but also by any competent draughtsman who may afterwards be required to independently plot the survey, and who may never have even seen the ground of which a map has to be plotted.

35. SCALE OF PLAN. Before commencing the actual measurements in the field, the scale of the plan, or, in other words, the degree of accuracy should be considered. The $\frac{1}{100}$ of an inch can be shown on paper, so if the scale is to be 100 feet to an inch or larger, the chainage and offsets must be read accurately to the nearest foot; whereas if the scale was to be 500 feet to an inch, no distance under 5 feet in length would be perceptible on the plan. These considerations determine the number of offsets required, for the larger the scale, the greater the detail that can be shown.

36. EXAMPLES IN PLOTTING. (Figs. 337, 338, 346—348, 354, 425).

1. Lay down a field, on a scale of 400 links = 1 inch, from the following Notes (*see* Fig. 337), and find its area by mensuration for the large triangle, and by acre-squares for the offsets.

(i) For plot (*see* Figs. 338);*

(ii) area of large triangle

$= \sqrt{1{,}050\,(1{,}050-600)(1{,}050-700)(1{,}050-800)} = \sqrt{41{,}343{,}750{,}000}$

$= 203{,}332$ sq. links $= 2{\cdot}03332$ acres (by Chap. II, Art. 28),

area of offsets $= - - 13{,}750 + 19{,}650 + 24{,}000 = 29{,}900$ sq. links

$= 0{\cdot}299$ acres (by Chap. II, Art. 5),

$\therefore$ total area $= 2{\cdot}03332 + 0{\cdot}299 = 2{\cdot}33232$ acres

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* Having given the plot in this and the following examples, the entries in the field-book answering to the same might, for practice, be made, taking the measurement off with a pair of compasses. Of course the measurements thus taken off can only be very *approximate*, but it will be good exercise for the student if he has time for the purpose.

2. Plot 3 fields which have been surveyed, from the following Notes (*see* Fig. 346), on a scale of 250 links = 1 inch. For Plot (*see* Fig. 347).

3. Draw the plan of a country from the following Field-notes (*see* Fig. 354) on a scale of 200 feet = 1 inch. For Plan (*see* Fig. 348).

4. Make an accurate plot of the fields, on a scale of 16 inches = 1 mile, from the measurements recorded in (Fig. 425) and which are roughly plotted there on the 32″ scale.

USEFUL PROBLEMS.

37. PROBLEM I. *To draw upon the ground a straight line through two given points.*

Plant a picket, or staff, at each of the given points; then fix another between them, in such a manner that when the eye is placed at the edge of one staff, the edges of the other two may coincide with it. The line may then be prolonged by fixing up other staves. The accuracy of this operation depends greatly on fixing the staves upright, and not letting the eye be too near the staff from whence the observation is made.

38. PROBLEM II. *To walk in a straight line from a point to a given object.*

Fix upon some point, as a bush, or a stone, or any mark that you may find to be in a line with your given object, and walk forward, keeping the two objects strictly in line, and always selecting a fresh mark when you come within 20 or 30 paces of the one upon which you have been moving. Observe that to walk in a direct line, it is always necessary to have *two* objects constantly in view.

39. PROBLEM III. *To trace a line in the direction of two distant points.*

Let two persons separate to about 50 or 60 paces; then, by alternately motioning each other to move right or left, they soon get exactly into line with the distant objects; or for greater accuracy, they may hold up staves.

40. PROBLEM IV. *To lay off a perpendicular with the chain.* (Figs. 171, 214.)

(1) (*See* Fig. 214.) Let a be the point at which it is required to raise a perpendicular to the chain line AB. If we make $ab = 30$, $ac = 40$, and $bc = 50$,* the angle BAC will be a right angle, and ab consequently perpendicular to AB. To do this fix an arrow into the ground at A, through the ring of the chain, marking 20 links from one handle; measure 40 links on the chain line from a to c, and fix another arrow, through the other handle of the chain, into the ground at c; now 80 links are lying loosely between a and c; if then the circular piece of brass at the 50th link of the chain is held and pulled out so as to tighten the chain from it to a and c, the point b will be found; and thus ab will be the perpendicular required.

(2) (*See* Fig. 171.) Suppose it was required to raise a perpendicular at a on the chain line AB. Take any number of even links, say 40, fasten the two ends at b and c on the chain line, about 10 links (generally a fourth of the assumed number) on either side of a. Hold the centre of the 40 links and stretch the chain to d; ad will be the perpendicular sought.

41. PROBLEM V. *To avoid an obstacle in a chain line, which obstructs the sight, such as a house.* (Fig. 369.)

Let AB be the direction of the chain line, which is obstructed between a and d. From a, a', any two points on the chain line somewhere near the obstacle, erect *equal* perpendiculars ab, $a'b'$, both to the right or to the left as convenient, and of such a length that a line joining their extremities, b, b', when produced, may pass clear of the obstacle; join bb' and produce it in the direction of c until the obstacle is passed; and from any two convenient points in it, as c, c', erect equal perpendiculars cd, $c'd'$; return towards the chain line AB by laying off the perpendiculars cd, $c'd'$, each *equal* to ab; then dd' produced will evidently be the direction required. Check measurements from o, the bisection of the line df, to the points a and g should, however, invariably be applied in which $ao = og$, otherwise the direction of the line cg could not be relied on. The check

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* These three sides may be any multiple of 3, 4, and 5 respectively, such as 6, 8, 10; 9, 12, 15; etc.; r (by Theorem XII of Chap. I, Art. 79) $bc^2 = ab^2 + ac^2$, or $5^2 = 3^2 + 4^2$; but the longer the measure-ents, the more accurate will the perpendicular be.

lines *bc* and *eh* might also with advantage be measured and should be equal to each other.

42. PROBLEM VI. *To find the length of the line AB, obstructed by something that can be seen over, such as the bend of a river or a tank.* (Fig. 370.)

Fix upon a convenient point E, and measure AE and BE, prolonging them till ED = AE, and EC = BE; then measure CD, and it will be equal to AB, the distance required.

43. PROBLEM VII. *To measure the width of a river that is too broad for the chain to reach over.* (Figs. 371, 120.)

(*a*) Let AB be the chain line (*see* Fig. 371), part of which cannot be measured on account of the river. Send a flag across the river and have it set up in the line AB, as at *c*, somewhere near the opposite bank. At *a* erect a perpendicular of any convenient length. At *b* make *bd* perpendicular to the imaginary line *bc*, and measure the distance *ad*. Then *ac*, the distance required, will be equal to the square of the distance *ab* divided by the length *ad*; in other words, $ac = \frac{ab^2}{ad}$.

(*b*) Or, at a point C on the line AB (*see* Fig. 120) set out a convenient perpendicular CD, which bisects in E; at D set out the right angle CDF; put up flags at B and E, and range out the line BE until it intersects DF in F; measure DF, which will equal BC.

44. PROBLEM VIII. *To find the distance of an inaccessible object O by means of a rhombus.* (Fig. 372.)

With a chain or measuring tape, whose length is equal to the side of the intended rhombus, lay down one side BA in the direction BO; and let BC, another side, be in any convenient direction; fasten two ends of two of these lengths at C and A; then the other two ends (at D) being kept together, and the lines stretched on the ground, those lines AD, CD, will form the other two sides of the rhombus. Set up a mark at R, where the direction CO and AD intersect; and measure RD; then RD : DC : : CB : BO, in other words, $BO = \frac{DC^2}{RD}$, *i.e.*, $\frac{BA^2}{RD}$.

Example: Suppose the side of the rhombus is 100 feet, and RD = 11 feet 7 inches; then, $BO = \frac{100^2}{11\frac{7}{12}} = \frac{100,000}{139} = 863$ feet, nearly.

If the ground be nearly level, a rhombus, whose side is 100 feet, will determine distances to the extent of 300 yards within a very few feet of the truth.

45. PROBLEM IX. *To find, by symmetrical triangles, the length of a line of which both ends are inaccessible* (Fig. 121).

Let A and B be two inaccessible objects, the distance between which is sought. Take any convenient line CD, whence the objects A and B are visible. With C and D as centres draw the arcs *ade* and *b'cf'* on the ground, with a fairly large radius of, say, a chain's length. Set stakes on these arcs in the direction of the objects A and B from each of the points C and D; these stakes would be driven in at *a*, *b* and *a'*, *b'* respectively. Lay off from *d* and *c* equal chords to *da*, *db*, and *cb'*, *ca'* upon the adjoining sides of the respective arcs; these other chords would be *de*, *df*, and *cf'*, *ce'*. Range out the lines C*f* and D*f'* till they meet at F, likewise the lines C*e* and D*e'* till they meet at E; and set up stakes at the points of intersection. Then measure the line EF, which will be equal in length to the required distance AB.

46. PROBLEM X. *To divide a triangular field ABC in certain given proportions, so that a well, situated at one of its corners, may be common to all the divisions.* (Fig. 375.)

Let the given proportions be as the numbers 2, 3 and 5, and the well be at C; then—

as 2+3+5 = 10 : AB : : 2 : A*a*,
,, : ,, : : 3 : *ab*,
,, : ,, : : 5 : *b*B;

which are the portions of the base AB belonging to the respective triangles; therefore, if lines be drawn from *a* and *b* to C, the field will be divided in the required proportion.

Example: Let AB = 860, and the proportion be as before; then—

as 10 : 860 : : 2 : Aa = 172,
,, : ,, : : 3 : ab = 258,
,, : ,, : : 5 : bB = 430.

47. PROBLEM XI. *To divide a triangular field ABC in any given proportion, by fences running from a given point in one of its sides.* (Fig. 376.)

Let H be the given point in the side AB, and let the triangle be required to be divided into three equal parts. Divide AB into three equal parts in the points D, E; join HC, parallel to which draw DF, EG meeting AC, BC respectively, in F and G; and join HF and HG, along which lines fences being made, the same will divide the triangle as required.

48. PROBLEM XII. *A map with its area being given, and its scale omitted to be either drawn or mentioned; to find the scale.*

Cast up the map by any scale whatsoever, and it will be—

as the area thus found
: the given area of the map,
: : the square of the assumed scale
: the square of the scale by which the map was originally laid down,

the square root of which will give the scale sought.

Example: A map whose area is 20,296 square chains being given, and its scale omitted to be either drawn or mentioned; to find the scale.

Suppose the map was drawn on an assumed scale of 20 chains to an inch, and the content thereby produced be 5,074 square chains; then -

as 5,074 : 20,296 : : 20^2 : the square of the required scale = 1,600, the square root of which = 40.

The map was therefore originally laid down by a scale of 40 chains to an inch.

49. PROBLEM XIII. *To find the true area of a survey, though it be taken by a chain that is too long or too short.*

Let the map be constructed, and its area found as if the chain was of a true length, and it will be—

as the square of the true chain,
: the square of the chain surveyed by,
: : the area of the map thus found,
: the true area of the map.

Example: If a survey be made with a chain which is 3 inches too long, that is, with one whose length is 42 feet 3 inches, and the map thereof be found to contain 147,260 square chains; required the true area.

As $66^2 : (66\frac{1}{4})^2$: : 147,260 : the true area = 148,378 square chains.

50. PROBLEM XIV. *To determine the height* (*approximately*) *of an inasscessible object by means of shadows, when the sun shines.*

Set up vertically a staff of known length and measure the length of its shadow upon a horizontal plane; measure also the length of the shadow of the object whose height is required. Then it will be—

as the length of the shadow of the staff
: the length of the staff itself,
: : the length of the shadow of the object
: the object's height.

Example: The length of the shadow of the rod 10 feet long, set up vertically, is found to be 2·4 feet. Required the height of a pagoda the length of whose shadow from the centre of the base measures 58·5 feet.

As 2·4 : 10 : : 58·5 : the height of the pagoda = 244 feet, nearly.

51. PROBLEM XV. *Suppose that instead of measuring the distances in a sketch survey of a river or route, the intervals of time and the rate of travelling along each bearing were recorded. Required to construct a scale by which such a survey may be plotted without calculating the actual distances.* (Fig. 197.)

Let the rate of travelling per hour be m miles, and the scale on which the survey is required, n inches to a mile. Then 1 hour = 60 minutes = m miles, = $n \times m$ inches, or nm = 60, that is, if the product of the rate and the number of inches taken to represent one mile be divided into 60 parts, a scale of minutes is obtained, which can, of course, be subdivided to half or quarter minutes or less, as necessary.

Example: Let the rate $m = 3\frac{3}{4}$ miles per hour. The scale on which the survey is required $n = 2$ inches to 1 mile. Then $m \times n = 2 \times 3\frac{3}{4} = 7\frac{1}{2}$ inches.

Take AB equal to $7\frac{1}{2}$ inches, divide it in 6 equal parts (by Chap. I, Art. 93), and one of these divisions (equal 10 minutes) again into 10 equal parts for minutes. Then the sketch survey may be plotted from this scale direct, and so long as the rate remains $3\frac{3}{4}$ miles, the plot will be on the scale of 2 inches to the mile.

When the rate alters, a different scale must be constructed.

52. PROBLEM XVI. *To determine the area of a piece of ground, having the map given by weight.* (Fig. 374.)

Let parallels be drawn at half an inch asunder, and others at right angles to them at a like distance; each square will then be a quarter of a square inch in area. With a penknife, cut away all the squares marked a, which the boundary of the map does not reach; then all the whole squares contained within the body of the map, and the squares which the boundary passes through and which are marked z, z, z, etc., will remain. Add the number of whole squares in the body of the map, and those marked z, together, which number note down; then find with grains and tenths of a grain the weight of the paper. Call the number of squares, first area, and their weight in grains and parts, their first weight. Then cut the map close by the boundary and weigh it in grains and decimals of a grain, as before. Then say—

as the 1st weight : to the 1st area : : the 2nd weight : the 2nd area,

which gives the area of the map in squares and decimals of a square. Then by knowing by what scale it was laid down, the area of the each square is known and consequently the area of the whole map. The state of the atmosphere has a very sensible effect on this operation. The weighment should be made with as delicate scales as possible, and the mean of two or more trials adopted; the method, however, is a ***very rough*** one.

THE PERAMBULATOR.

53. PERAMBULATOR DEFINED. The *Perambulator* is an instrument for measuring distances, and consists of a wheel with an apparatus of clock-work and a dial-plate, upon which the distance travelled over is shown by an index.

54. ITS USE. The instrument is chiefly used for measuring on a level surface, distances such as roads, routes for surveys, etc., where expedition rather than strict accuracy is required, and is not wanted for a chain survey. In using it great care must be taken to keep it going in a straight line, otherwise the slightest deviation in its course, as going from one side of the road to the other, or carrying it over uneven ground, will cause it to register a ***greater*** distance than that actually travelled over.

55. THE VARIOUS KINDS OF PERAMBULATORS. Perambulators are of various patterns, the chief of which are (1) the *English* pattern, (2) the *Madras* pattern, (3) ***Everest's*** pattern, and (4) *Waugh's* pattern; but their principles are very much the same.

56. THE ENGLISH PATTERN. (Figs. 151, 153.) The "English Pattern Perambulator" (*see* Fig. 151) consists of a wheel of wood 8 feet 3 inches in circumference, or half a pole, lined with iron to prevent wear. This wheel works in a short axis which communicates motion to the wheel-work C. The number of revolutions taken by the wheel are recorded by an index on the box C. On the dial-plate (*see* Fig. 153) are three graduated circles, the outermost of which is divided into 220 parts, or the yards in a furlong; the next into 40 parts, the number of poles in a furlong; the third into 80 parts, the number of furlongs in 10 miles, every mile being distinguished by its proper Roman figure. For portability the wheel can be detached from the body of the instrument.

57. THE MADRAS PATTERN. (Fig. 150.) The "Madras Pattern 8-mile Perambulator" has a wheel 20 feet in circumference, and owing to its great height

is principally used for roads and military purposes. On one side a plate is graduated, in feet and inches, from 1 to 20 feet, in three circles, one within the other, each circle containing $6\frac{2}{3}$ feet. Suspended from the axle is a front plate graduated to furlongs and miles. The axle is furnished at both ends with a revolving handle. Two men are required to work this description of perambulator.

58. EVEREST'S PATTERN. (Fig. 148.) "Everest's Pattern 6-mile Perambulator" consists of a wheel 107·39 inches in circumference, and has handles like those of a wheel-barrow by which it is pushed forward. The machinery in this instrument is very simple. The dial-plates are fixed on the left hand side, the upper being divided into 100 parts, and the lower to 60 parts or 10 divisions to a mile. For portability the wheel is separable.

59. WAUGH'S PATTERN. "Waugh's 10 and 20-mile Perambulator" is somewhat similarly constructed to Everest's. Its circumference is 10 feet 9·306 inches, or $\frac{1}{490}$ part of a mile. The upper plate of the dial is graduated into 100 parts, each of which denotes of $\frac{1}{1,000}$ a mile, ten thousands being read by estimation. The tens on this plate represent $\frac{1}{100}$ of a mile. The lower plate is subdivided into 50 parts, each of which represents $\frac{1}{10}$ of a mile, or one revolution of the upper plate. Hence the lower plate measures as far as 5 miles, as well as subdivides to tenths. A pinion on the axis of the wheel is graduated into 20 parts, each representing one mile, which measures up to 20 miles. The index to the mile scale is in the centre of a small aperture at the word "miles"; the index to the lower plate or scale of miles and tenths is at the centre of the large aperture opposite the figure "100" on the upper plate, with which it revolves; the index of the hundredths and thousandths is on a projecting piece attached to the bottom of the dial. The dial, when not in use, is packed in the locker between the shafts, together with the attaching screw and key, to save it from unnecessary wear and tear in travelling The field-book is also carried in this locker. This instrument is much more strongly built than the English pattern one, and is better suited for the requirements of India and Burma, where it will too frequently meet with rough usage and be taken over rough ground.

CHAPTER X.

Compass Surveying.

1. USE OF PRISMATIC AND SURVEYING COMPASSES. In Chain Surveying, the sides of fields and the positions of objects are fixed by the measurement of lines only, whereas in Compass Surveying angles have, in addition, to be measured, and this is effected by either of two instruments—the *Prismatic Compass* or the *Surveying Compass*, both of which are here described; but as the former instrument is now generally used for the above purpose, since it is certainly the superior of the two, being much more accurate as well as convenient, only a brief description of the latter has therefore been thought necessary.

2. MAGNET DEFINED A *Magnet* is a loadstone, that is, an ore of iron which has the peculiar properties of attracting iron and of pointing to the poles when freely suspended. It has also the power of communicating these properties to iron and steel, thus forming *artificial magnets*.

3. MAGNETIC NEEDLE DEFINED. The *Magnetic Needle* is a small bar of iron or steel which is magnetized by one of several methods—usually by having its both ends rubbed several times with a pair of magnets; hence the properties of the magnet having thus been communicated to it, its magnetized ends point to the poles—the positive end to the north pole and the negative end to the south pole. When the needle begins to get feeble and loses its power of action, its magnetism may be renewed as follows: Obtain two bar magnets. Provide a board with a hole to admit of the axis, so that its collar may fit fairly, and that the needle may rest flat on it, without bearing at the centre. Place the board before you, with the north end of the needle to your right. Take a magnet in each hand, the left holding the North end of the bar, or that which has the mark across, *downwards*; and the right holding the same mark *upwards*. Bring the bars over the axis, about a foot above it, without approaching each other within two inches;

bring them down vertically on the needle (the marks as directed), about an inch on each side of its axis; slide them *outwards* to its ends with slight pressure; raise them up; bring them to their former position, and repeat this a number of times. That is to say, the southern pole of one bar is drawn along the northern pole of the needle from the centre outwards and the northern pole of the other bar along the southern pole of the needle.

4. DISTINCTION BETWEEN MAGNETIC NORTH AND TRUE NORTH. (Figs. 336, 348.) The direction in which the magnetic needle points is termed the ***magnetic north***, and differs from the ***true north*** by sometimes several degrees ***east*** or ***west*** of it, depending entirely upon the geographical position (and not local attractions) of the country where the needle is set up. This angular difference is termed the ***variation of the needle.**** Representations of the variation are shown in (Figs. 336, 348).

5.† MERIDIAN DEFINED. A ***meridian*** is a great circle supposed to pass through the poles of the earth and the zenith and nadir of any given place on it.

6. MAGNETIC MERIDIAN DEFINED. A ***magentic meridian*** is a great circle, parallel with the direction of the magnetic needle, and passing through its poles.

7. DEFINITION OF BEARING. (Fig. 364.) The ***bearing*** of a line is the angle which the line makes with the meridian at a point on it, and is usually reckoned from the north at 0°, by the east, south, west, back to 360° or north; thus, in the Figure, the bearing of OA = 35°, of OB = 168°, of OC = 237° 40′, and of OD = 312° 10′. The ***true*** bearing is the angle made with the true north; and the ***magnetic*** bearing is the angle made with the magnetic north.

8. EMPLOYMENT OF PRISMATIC COMPASS. The ***Prismatic Compass is*** an instrument with which horizontal angles to within 10 minutes (and by estimation in the larger instruments to even 5′) can be observed with great rapidity, and is often held in the hand, in reconnoitering or making a rough sketch of a country to take the bearing of an object, but much greater accuracy can be obtained by placing it on a tripod stand, which is almost invariably done. It is very useful for filling in the details of a theodolite survey, for sketching along a road, etc. It should be observed that the Prismatic Compass cannot be used in places or under any circumstances where there is the slightest metallic attraction, local or other, as the needle is so sensitive that the least thing will cause a variation.‡ Again the compass must not be relied upon for extensive triangulation, as from local and other causes slight errors are certain to occur.

9. DESCRIPTION OF THE PRISMATIC COMPASS. (Figs. 377—378.) The ***Prismatic Compass*** (*see* Fig. 378) consists of a round box about 4 inches in diameter, and is mounted on a stand having three legs each about 4½ feet long. In (Fig. 406) B represents the compass box, in the centre of which is a small pivot O, upon which swings a magnetic needle, one end (usually having a distinctive mark upon it) of which always points to the north, attached to a graduated circular silver rim instead of the obsolete card. This rim should not be less than 4 inches in diameter,§ and the magnetic needle should be powerful.‖ The silver

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* In other words, the *variation of the needle* is the angle which the magnetic north makes with the true north, at a point either *east* or *west* of it. In India and Burma the variation of the needle is about 2½° ***east***, whereas in London it is now about 17° ***west***. The laws of this variation are at present little known. It has been found from a series of observations taken at different places and at different periods that a slight annual change (increase or decrease), extending from 0 to 8 minutes of arc, takes place in the magnetic variation all over the globe. *Magnetic declination* measures the deviation of the "needle" from the true geographical north and south direction; ***magnetic inclination*** or "dip" is the angle which the "needle" makes with the horizon. There are two foci of attraction in the northern hemisphere, one in North America (λ. 70° N., L. 95° W.) and the other in Siberia (λ. 60° N., L. 130° E.), and two corresponding points in the southern hemisphere. A magnetic needle, freely suspended, would thus point to one or other of these foci by which most attracted. The "magnetic equator" occurs in the extreme south of India (λ. 10° N., L. 80° E.); below this line the needle will be found to dip towards the south pole.

† The astronomical terms used in this Article are defined in Chapter XVI.

‡ The Surveyor should even avoid wearing steel or iron on his person, as these too are liable to affect the needle.

§ There is, however, no advantage in having the instrument too large, say, beyond 5 inches diameter, as no additional power is gained by increasing the length of the needle. Long needles are apt to have their strength speedily diminished by use; short ones, made very hard, are therefore to be preferred.

‖ The needle should not come to rest very quickly. If it does, it indicates either that it is weakly magnetized or that the friction on the pivot is great. Its sensitiveness in indicated by the number of vibrations which it makes in a small space before coming to rest.

rim is graduated to ½ degrees (*see* Fig. 377) round the edge, commencing from North, and numbered at every tenth degree line 10, 20, 30, etc., in the direction in which the hands of a clock move, all the way round the circle to 360°; 90° representing East, 180° South, 270° West, and 360° North; so that the bearings can be read to 30′ and estimated with the eye to 10′ and even to 5′.* The figures and divisions are not all shown in the sketch of the graduated ring (*see* Fig. 377); but a portion of the edge is represented above it on an enlarged scale. *n* is a spring, which, being pressed by the finger and then released, acts upon the ring, and checks its vibrations, so as to sooner bring it to rest, when making an observation. A stop is likewise fixed at the other side of the box close to the hinge-joint of the sight-vane S, by touching which a little lever is raised and the needle thrown off its centre; which should always be done when the instrument is not in use, as the constant playing of the needle would wear the agate point upon which it is balanced, and upon the fineness of the point much of the accuracy of the instrument depends. S is the sight-vane, having a fine thread or wire stretched along its opening, by which the point to be observed with the instrument is to be bisected. The sight-vane is mounted upon a hinge-joint, so that it can be turned down flat in the box when not in use. P is a prism of glass in a little metal case attached to a plate sliding in a socket, and thus admitting of being raised or lowered at pleasure, and also supplied with a hinge-joint, so that it can be turned down into the box when not in use. In the plate to which the prism is attached, is a narrow slit to which the eye is applied in making an observation. On looking through this slit, and raising or lowering the prism in its socket, distinct vision of the divisions on the graduated ring is obtained, and these divisions and the perpendicular wire of the sight-vane appear *together* on looking through the prism, and the division with which the wire coincides, when the needle is at rest, is the magnetic azimuth or *bearing*, of whatever object the wire may bisect. R is a mirror which slides on and off the sight-vane S, with sufficient friction to remain stationary on any part of the vane; it can be inclined by means of a joint and is there to reflect the image of an object to the eye of the observer when the object is much above or below the horizontal plane. When the instrument is used to obtain the magnetic azimuth of the sun, a dark glass must be interposed, and the colored glasses represented at G are intended for that purpose; the joint upon which they act allowing them to be turned down over the sloping side of the prism box. Owing to the prism being placed at the reverse side of the graduated ring and exactly opposite the sight-vane, and also that everything seen through it is inverted, the zero of the ring is placed at the south, and the numbers of the graduations reversed and written upside down; by this means the number seen through the prism is the reading required. A cover fits on to the box, and the whole is packed in a leather case, which is slung over the shoulder by a strap for convenience in travelling.

10. METHOD OF USING THE PRISMATIC COMPASS. (Figs. 378, 432.) Place the instrument (*see* Fig. 378) on its stand (*see* Fig. 432) over the station of observation. Turn up upon their hinge-joints the sight-vane S, and the prism P. Bring the instrument in as horizantal a position as can be judged with the eye, by moving the legs of the stand, or if the box has a ball fitting tightly into a socket underneath, as the new pattern ones have, by adjusting the instrument as required with the hand. Next raise the prism in its socket till the divisions on the ring are distinctly seen; then look through the slit, turning the box round till

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OBSERVATION.

* (i) Beginners should guard against what is a very frequent source of error with them, namely, the reading from the wrong number of the two between which the needle points, such as taking 185° for 175° in a case where the needle points exactly between 180° and 170° (*see* Fig. 199).

(ii) Want of precision in the recording of the indications of the compass will give indifferent results; thus, inaccuracies of 5′ and 15′ respectively in an angle, *i.e.*, in the difference of two equidistant lines, causes them to separate from each other as follows: -

	For 5′ difference.	For 15′ difference.
At the end of 100 feet	= 0·15 feet	0·44 feet.
At the end of 1,000 feet	= 1·45 feet	4·36 feet.
At the end of 1 mile	= 7·68 feet	23·04 feet.

the wire in the sight-vane bisects one of the objects whose bearing, or angular distance from any other object, is required; then by slightly touching the spring *n*, bring the needle to rest, and the division on the ring which coincides with the wire of the sight-vane, will be the *magnetic bearing*, or bearing of the object from the North point of the magnetic meridian. Again, turn to any other object and repeat the operation; the difference between the bearing of this object and that of the former will be the angular distance of the objects in question. Suppose the former bearing to be 63° 18′ and the latter 15° 37′, both reading from the North point, eastward, hence the angle will be 63° 18′ - 15° 37′ = 47° 41′.

11. INSTRUMENTS DIFFERENTLY DIVIDED. The divisions in some instruments are numbered from 0° to 180° South, reckoned Eastward, and thence to 180° North, reckoned Westward; but these are more liable to error in recording the results in a field-book; hence those with the complete numbering, from 0° to 360°, are preferable.

12. VARIATION OF NEEDLE TO BE ATTENDED TO. The variation of the needle must always be attended to, for if the fixed points of a theodolite survey, above alluded to (*see* Art. 8), have been surveyed on the *true* meridian of the earth, the variation of the needle must be *added* to, or *deducted* from, the observed magnetic bearing, according as the variation is *east* or *west* of the true north, to obtain the true meridional bearing of the line. In ordinary maps the plotting is done with reference to the magnetic meridian, and a memorandum of the variation of the needle made in some corner of the paper.

13. DESCRIPTION OF THE SURVEYING COMPASS. (Figs. 365, 383.) The *Surveying Compass* (*see* Fig. 383) is in so many respects similar to the prismatic compass (*see* Art. 9), that the only parts of it which it is necessary to describe are those in which the two instruments differ. In the centre of the box is a small pivot, upon which swings, by itself, a magnetic needle, one end (the blue) of which always points to the north. In the bottom of the box is fixed a circular card which is divided into 360°, and numbered regularly 10°, 20°, 30°, etc. The north, south, east, and west points are also marked; but what would be ordinarily called the east is marked W, and similarly the west is marked E. The reason for this is clear: for suppose AB (*see* Fig. 365) to represent the needle, free to turn on its centre C, and pointing due north and south; the east and west points being marked as they would be indicated on a map, that is, the east to the right, and the west to the left of north. Again, suppose the box to be turned in the direction of the arrow, the needle will retain its original position, as it must always point to the north; and the point marked E will come under the end of the needle at A; that is, it would appear as if the box had been turned to the east, whereas it was in reality turned to the west. To save the trouble of always recollecting this, the east and west points are reversed, when, if the box be turned in the direction of the arrow, the point W, which would be where E now is in (Fig. 365), will come under A, showing that the box has been turned to the west. If the needle were attached to the card and revolved with it, as in the case of the prismatic compass, this reversing of the east and west points would not be necessary.

14. EMPLOYMENT OF THE SURVEYING COMPASS. The Surveying Compass is used for the same purpose as the Prismatic Compass, namely, for filling in the interior detail of a survey by means of bearings. The disadvantage of the former, when compared with the latter, is, that when looking through the sights in intersecting an object, the angle is not at once, in *one* operation, presented to the eye, but must be read off afterwards with the naked eye, as pointed out by the needle; that is, in *two* operations.

15. DESCRIPTION OF THE FIELD-BOOK. (Figs. 379, 381.) The *field-book* (*see* Fig. 381) for a compass and chain survey is very similar to that for a survey made by the chain only, as all the distances that are chained as well as all the offsets, stations, etc., are entered in exactly the same way, but in addition the bearings taken in the field have to be entered. The bearing to the next forward station is called the *forward bearing*, and that to the back station is called the *back bearing*. The forward bearing should be written in the centre column immediately above the station of observation, and the other bearings to the right or left of the centre column, according as their directions are to the right or left of the

chain line; this will prevent any mistake in plotting them.* It is only ***necessary*** to take bearings to the forward stations, but in practice it is advisable to take also the back bearings, as an indispensable check on the other work; the mean of the forward and back bearings should be taken as the correct bearing of the line; for instance, in (Fig. 379) at A, the forward bearing is $312^\circ\ 50'$, and that at B, the back bearing, is $132^\circ\ 20'$; their difference is $30'$ the half of which, namely, $15'$, must be subtracted from $312^\circ\ 50'$, which equals $312^\circ\ 50' - 15' = 312^\circ\ 35'$, and added to $132^\circ\ 20'$, which equals $132^\circ\ 20' + 15' = 132^\circ\ 35'$, to give the correct bearing of the line.

16. SPECIMEN OF FIELD-BOOK. (Fig. 381.) In the Figure is shown a specimen of the field-book for a Compass Survey, in which both the angular and linear measurements are entered.

17. METHOD OF SURVEYING WITH THE COMPASS. (Figs. 380—381.) Fig. 380, which is the plot of the field-book in (Fig. 381), represents a portion of country bounded by a road which is to be surveyed with a prismatic compass.† Fix upon a starting-point ⊙ A at some corner of the road, set up the compass there, and send a man with a flag-staff to ⊙B, as far down the road as can be seen, and let it be placed in such a position that a long forward shot may be obtained the next time. Now take the bearing of ⊙B, and proceed to chain from A to B, taking offsets to the sides of the road and any remarkable objects, as in a chain survey. Having arrived at ⊙B, the compass must be again set up, and a flag-staff having been sent to ⊙C, its bearing must be obtained; then the line BC is to be measured, and so on. Bearings must also be taken to any conspicuous objects at a considerable distance from the chain line, and that are out of reach of an offset, either from the stations themselves, or from points on the chain line, something in the same way as by oblique offsets (*see* Chap. IX, Art. 23); readings from two determined points are sufficient to fix it, but a third should be taken as a check. For a proof of the angular work of a circuit of traverse survey made with the compass (*see* Chap. XIV, Art. 9, 2nd condition, and Art. 15).

18. PLOTTING THE SURVEY. (Figs. 380, 381, 386.) In order to plot the survey from the notes in the field-book of Fig. 381, first fix on a convenient spot on the paper for the starting-point A (*see* Fig. 386), that is, so that the survey may be contained in the paper, and as nearly in the middle as possible, draw a line through it to represent the magnetic meridian, place the protractor, if it is a semi-circular one, to the *left* of this line, as the angle required to be set off is greater than 180°, the edge coinciding with it, and the centre at the point A; now, with a pencil, mark the required angle and draw a line through this point and A; this will represent the direction of the first bearing, that of ⊙B; on this lay off the length AB from the scale on which the map is to be projected, and through B draw the line BN parallel to AN to represent another meridian, and placing the protractor as before, lay off the angle SBC; set off BC to the required length, and proceed with each angle in the same way until the end of the line FA should coincide with the starting-point A. If an angle less than 180° is required to be set off, the protractor, if a semi-circular one, must be placed to the ***right*** of the meridian, as at C, and the outer row of figures marked on it used. Having completed the circuit and found it to be correct, the offsets can now be plotted in the same manner as in the chain survey (*see* Fig. 380).

19. TO FIND BACK BEARINGS. (Fig. 396.) When the forward bearing is given and it is required to know the back bearing, if it is ***less*** than 180°, ***add*** 180° to it, and if ***more*** than 180°, ***subtract*** 180° from it, for the back bearing.

Example: At O the forward bearings of B and A are respectively $74^\circ\ 15'$ and $301^\circ\ 30'$; required the back bearings of O at B and A.

At B, back bearing of O $= 74^\circ\ 15' + 180^\circ = 254^\circ\ 15'$;
At A, ,, ,, $= 301^\circ\ 30' - 180^\circ = 121^\circ\ 30'$.

OBSERVATIONS.

* There is, also, another method of recording these bearings. The bearings taken at the station of observation to the forward flag and objects situated on the left and right of the chain line may be entered in some convenient place in the right or left marginal column of the field-book, under the heading "Bearings at——"; so also bearings taken at intermediate stations on the chain may be similarly entered.

† In executing such a survey, the area to be surveyed must always be kept to the *left* of the observer when going round the land that is thus enclosed.

20. EXAMPLES IN PLOTTING. (Figs. 382, 390, 395). (1) Plot a survey, from the Notes in Fig. 410, on a scale of 100 links = 1 inch.

(2) Draw the plan of a forest, from the field-notes in (Fig. 390), on a scale of 500 links = 1 inch.

(3) Plot a survey, having given the field-book for the same in (Fig. 395), on a scale of 50 links = 1 inch.

21. PROOF OF THE ANGULAR WORK OF A CIRCUIT. (Figs. 335, 345.) If a circuit or traverse* survey is made with the compass, that is, if from a starting-point A (*see* Fig. 335) we proceed along several stations successively measuring the distances and obser ng the bearings till we come back to the starting-point, then the angular work of the polygon thus formed can be proved by the following theorem: *The sum of the inward angles of a polygon is equal to twice as many right angles (or 90°) as the figure has sides, less four right angles (or 360°).*†

Thus, from the entries in the following table:-

Station of Observation.		Observed bearing of Forward Station.		Measured distance, Links.
⊙ A	...	... 281° 40′	...	... 1,238
⊙ B	...	... 107 05	...	... 2,055
⊙ C	...	... 210 15	..	... 2,385
⊙ D	...	... 72 30	...	... 1,740
⊙ E	...	... 134 40	...	... 1,449
⊙ F	...	... 27 45	...	... 3,036
⊙ G	...	... 326 55	...	... 2,490
⊙ H	...	... 239 25	...	.. 913
⊙ A				

the angular work of the circuit is proved as follows:—

Bearings.	Inward angles.
At ⊙ A 281° 40′ − 59° 25′ (239° 25′ − 180°)‡	= 222° 15′
⊙ B 107° 05′ − 101° 40′ (281° 40′ − 180°)	= 05° 25′
⊙ C 210° 15′ + 360° 00′ 347° 05′ (107° 05′ + 180°)	= 223° 10′
⊙ D 72° 30′ − 30° 15′ (210° 05′ − 180°)	= 42° 15′
⊙ E 134° 40′ + 360° 00′ 252° 30′ (72° 30′ + 180°)	= 242° 10′
⊙ F 27° 45′ + 360° 00′ 314° 40′ (134° 40′ + 180)	= 73° 05′
⊙ G 326° 55′ − 207° 45′ (27° 45′ + 180°)	= 119° 10′
⊙ H 239° 25′ − 146° 55′ (326° 55′ − 180°)	= 92° 30′
∴ Sum of inward angles	= 1,080° 00′

Also, by rule, sum of inward angles = 2 × 8 × 90° − 360° = 1,080°.
Hence the angular work of the survey is correct.

22. SECTION PAPER. (Fig. 474.) For a compass survey the bearings and distances instead of being recorded in a field-book, may be at once plotted in the field as the survey is proceeded with. In the latter case the surveyor must provide himself with a common drawing board mounted with paper, ruled with parallel lines at short distances apart, running north and south, or east and west, or both, by which the protractor can at once be placed in position, and the bearings laid off without the trouble of drawing a meredian through every station. The section paper (*see* Fig. 474) used in the Indian and Burma Cadastral surveys is well adapted for this purpose, being ruled with faint blue lines into one inch squares, which do not interfere with the details, nor detract from the appearance of a plan. The advantage of a field-book is, that the plan can at any future time be plotted on any required scale. It is much more convenient, however, to plot the measurements on a drawing board in the field, on a scale adapted to the size of the board,

OBSERVATIONS.

* For a definition of the term *traverse*, *see* Chap. XIV, Art. 1.

† For the method of applying the angular corrections, if any, in a traverse, *see* Chap. XIV, Art. 10. So also with respect to the measured distances. If after the angular work has been proved the plot of the survey does not "close," it may be corrected (by Dr. Bowditch's method) as follows: Let ABCDE (*see* Fig. 345) be the boundary lines plotted according to the given bearings and distances, and suppose that the last course comes to E instead of ending at A, as it should. Suppose, also, that there is no reason to suspect any single great error, and that no one of the lines was measured over very rough ground, or was specially uncertain in its direction when observed, the inaccuracy must then be distributed among all the lines *in proportion to their length.* Each point in the figure B, C, D, E, must be moved in a direction parallel to EA by a certain distance which is obtained thus: multiply the distance EA by the distance AB, and divide by the sum of all the courses. The quotient will be the distance BB′. To get CC′, multiply EA by AB + BC, and divide the product by the same sum of all the courses. To get DD′, multiply EA by AB + BC + CD, and divide as before. So for any course, multiply by the sum of the lengths of that course and of all those preceding it, and divide as before. Join the points thus obtained, and the closed polygon AB′ C′ D′ A will thus be formed, and will be the most *probable* plot of the given survey.

‡ The back bearings are obtained by the rule in Art. 19.

as the survey is proceeded with, and thus be saved the labour of entering them first in a field-book and then plotting them at some future time; besides, the advantage of plotting directly in the field is that one can be sure that no details are omitted.

23. PLOTTING BY CO-ORDINATES. Although compass surveys, whether triangulation or traverse, are usually plotted with a protractor, they may be protracted by the method of rectangular co-ordinates, by which all risk of laying down incorrect bearings and distances is obviated. The magnetic north line is drawn through the starting-point, and an east and west line at right angles thereto, through the same point. The calculations of the latitudes and depatures are effected with the aid of Traverse Tables (*see* Table XXXIII, "Calculating Tables," 5th edition). The algebraic sum of the several sets of resolved distances should in any closed traverse be equal to zero, and this affords a verification of the whole before plotting it, and enables it to be plotted correctly without alterations. For information on the subject of calculation and plotting by the method of co-ordinates, or Gale's Traverse System (*see* Chap. XIV).

24.* MANNER OF FINDING THE MERIDIAN AND VARIATION OF THE NEEDLE. (Fig. 442.) The true meridian or true north and south line of any station can only be found, with accuracy, by astronomical observations, but there are several methods of approximating to it, which are quite accurate enough for most practical purposes; of these three are given below.

METHOD NO. 1. BY THE SUN'S SHADOW. Proceed as directed in Chap. X, Art. 36. The bisector is the *true* North and South line; if, now, a compass be set up on the line and the bearing of the north end of the line read, this bearing, if it is *less* than 180°, will be the variation of the compass *west*; and if it is *over* 180°, the difference between it and 360° will give the variation of the compass *east*.

METHOD NO. 2. BY A MERIDIONAL BEARING OF THE SUN.† For this method the surveyor must provide himself with a reliable watch which gives the *local* time *accurately*. To this local time, *at noon* (or gunfire), apply the equation of time (given in Table XX of the "Calculating Tables," 5th edition) with the *reverse* sign, the result will be the *mean* time shown by the watch at the instant the sun will be on the meridian. Set up the compass truly horizontal, and observe the bearing of the sun's centre at the instant of time aforesaid, when the direction of the sight-vane will be that of the meridian, from which the variation of the needle, *east* or *west*, can be deduced by the following rule:—

If sun is S., and bearing in defect of 180°, Variation E. = 180° —bearing;
" " " " " " " excess " " " W. = bearing — 180°;
" " " N., " " " defect " 360° " E. = 360° —bearing;
" " " " " " " excess " " " W. = bearing --0°.

Example: At Moulmein, on the 13th October, the sun was observed on the meridian with a Prismatic Compass, and the variation of the needle determined therefrom as follows:—

Mean time of watch when Sun was to cross meridian = 0 h. 0 m. + 14 m.
= 0 h. 14 m. P.M.,
The bearing of the Sun, S., observed at 14 m. past noon, watch time, was 177° 25′,
∴ by rule, Variation E. = 180° — 177° 25′ = 2° 35′.‡

METHOD NO. 3. BY THE POLE STAR. The Pole Star, *Polaris*, reaches the true meridian when it is in the same vertical plane with the star ε *Ursæ Majoris* (*Alioth*) at the root of the tail of the Great Bear (*see* Fig. 442). This circumstance occurs twice during the 24 hours (more precisely 23 h. 56 m.), but as stars cannot be seen in the daytime, only that opportunity which presents itself at night can be taken advantage of for finding the vertical position of the pole star on the

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OBSERVATIONS.

* The astronomical terms used in this Article are defined in Chapter XVI.

† The north and south line may be *roughly* found by means of a watch. Thus, if the hour hand is pointed to the sun, the south will be exactly midway between the hour and the figure XII on the dial. Suppose, for instance, it is 4 o'clock. Point the hand indicating IV to the sun and II on the watch is exactly south. If it is 8 o'clock point the hand indicating VIII to the sun and the figure X on the dial is due south.

‡ The variation of the needle at Moulmein, accurately observed with the theodolite in 1880, was found to be 2° 38′ E.

† This method, though a fair approximation, is absolutely correct only at the time of the *solstices* (*vide* Chapter XVI, Art 23, *seq*); on the other hand, the largest error that would be engendered in this respect is at the time of the *equinoxes* (*vide* Chapter XVI, Art 22, *seq*.).

meridian, which is thus ascertained: Suspend a plummet in a convenient place, free from currents of air; let the observer place himself behind the thread and wait until the moment when the stars *Polaris* and ε *Ursæ Majoris* are hidden by the thread; at the same time a small lantern is placed about 100 yards off in the line of the pole star by an Assistant, who is directed to move to the right or left, as necessary, by means of certain previously arranged signals. Next have a picket driven into the ground underneath the light, then the line joining the observer's position and the picket is the meridian line, the bearing of which can be read at leisure the following morning.

If a theodolite is at hand, the meridian may be found immediately, thus: Let the vertical wire of the diaphram of the telescope be taken for the plumb line along which the two stars are to be seen; set the instrument to zero of magnetic north. If the bearing of the pole star be then read, at the instant of time when the two stars are seen on the vertical wire, it will show the variation of the compass.

The pole star, which is about 1¼ degrees* distant from the pole, is easily identified by the stars called the "Pointers" in the above named constellation, which always point almost directly towards the pole.

The time at which *Polaris* (or the North Star) passes the meridian *above* the Pole for every tenth day (civil) in the year, is given in the following table in common clock time.† The *upper* transit (*see* Fig. 442) is the most convenient, since at the other transit the ε *Ursæ Majoris* is too high to be conveniently observed:—

	Month.	1st day (civil).	11th day (civil).	21st day (civil).
Times of Polaris *(or North Star) crossing the Meridian.*		H. M.	H. M.	H. M.
	January ...	06 36 P.M.	5 57 P.M.	5 17 P.M.
	February ...	04 34 "	3 54 "	3 15 "
	March ...	02 43 "	2 04 "	1 24 "
	April ...	00 41 "	0 02 "	11 23 A.M.
	May ...	10 43 A.M.	10 04 A.M.	9 25 "
	June ...	08 42 "	8 02 "	7 23 "
	July ...	06 44 "	6 05 "	5 25 "
	August ...	04 43 "	4 04 "	3 25 "
	September ...	02 42 "	2 02 "	1 23 "
	October ...	00 44 "	0 05 "	11 21 P.M.
	November ...	10 38 P.M.	9 59 P.M.	9 19 "
	December ...	08 40 "	8 01 "	7 21 "

To find the time of the star's passage across the meridian for other days than those given in the Table, take from it the time for the day most nearly preceding that desired, and subtract from this time 4 minutes for each day from the date of the day in the Table to that of the desired day.

Polaris passes the meridian later every year by about 22 seconds. In 1908 it will pass the meridian about 4 minutes later, and in 1918, 8 minutes later than in 1898, the year for which the preceding Table has been calculated.

The times at which *Polaris* passes the meridian *below* the Pole, in its *lower* transit (*see* Fig. 442), can be found by adding 11 h. 58 m. to the time of the upper transit, or by subtracting that interval from it.

USEFUL PROBLEMS.

25. PROBLEM I. *To survey a field, by setting up the compass at* ONE *station.* (Fig. 368.)

OBSERVATIONS.

* The *mean* distance of *Polaris* (α *Ursæ Minoris*) from the Pole, in 1898, is 1° 14′. This distance is, by the effect of Precession, yearly decreasing by a variable amount (at present about 19″) till the year 2120, when it will be reduced to under half a degree, after which it will begin to increase.

† To calculate the time of *Polaris* passing the meridian at its *upper* culmination: Find in the Nautical Almanac (under the head of "Apparent Places of Stars") the right ascension of the star, and from it (increased by 24 hours, if necessary, to render the subtraction possible) subtract the right ascension of the sun at mean noon, *i.e.*, the sidereal time at mean noon (found in Table II of the Nautical Almanac) for the given day (omitting the consideration of the correction, + or –, for difference in longitude, which in no case exceeds 2′—*see* Chap. XVI, Art. 50). Convert the remainder, which is sidereal time, into mean solar time (by Table XXXII, "Calculating Tables," 5th edition), and the result will be the required mean solar time of the *upper* passage of the star across the meridian, in *astronomical* reckoning, the astronmoical day beginning at noon of the common civil day of the same date.

Place the compass at any point O, near the middle of the field, and having placed marks at every corner of the field, measure the angles COD, DOE, EOA, AOB, and BOC; also measure with a chain the distances OC, OD, OE, OA, and OB; and with this data a plan of the survey can easily be formed.—Or, the instrument may be planted at one of the corners, as B, from which the others are visible; and the following measurements made: the angles CBD, DBE, and EBA, and the distances BC, BD, BE, and BA.

26. PROBLEM II. *To survey a field, by setting up the compass at* TWO *stations.* (Fig. 367.)

Select two convenient points x and y, within the field, a short distance apart, from which all the corners of the field are visible, and measure the distance as a *base-line* with a chain. Place the compass at x and measure the angles yxA, yxB, yxC, yxD, yxE, yxF, and yxG; again place the compass at y, and measure the angles Ayx, Byx, Cyx, Dyx, Eyx, Fyx, and Gyx. The distance xy, and the angles being plotted, the intersections of the dotted lines will give the corners of the field, which being connected will represent the field surveyed.

27. PROBLEM III. *To find one's place in a Survey.* (Fig. 396.)

Set up the compass at O and take the bearing of a fixed point A, which is, say, 301° 030′; again take the bearing of another fixed point B, which is, say, 74° 15′. But at O the bearings of A and B being got, the *back-bearings* of O at A and B can be deduced by the rule in Art. 19, as follows: bearing of O at A = 301° 30′ — 180° 00′ = 121° 30′; and at B = 74° 15′ + 180° 00′ = 254° 15′. Now the points A and B being known, they can be plotted on paper and with the two deduced back-bearings the point O can be laid down; thus, draw the magnetic meridian through the points A and B, adjust the protractor at A and lay off 121° 30′ S. E., the bearing of O at A; similarly adjust the protractor at B and lay off 74° 15′ S. W., the bearing of O at B; then the intersection of these two lines will give the position of the required station O. Observe that the nearer the two bearings meet at a right angle, the more accurately will the required station be determined; also when it is possible to take a bearing to a third known point, it should be done as a check on the work.

28. PROBLEM IV. *To draw a Mariner's Compass.* (Fig. 391.)

With the north, east, south and west lines of a given point, draw therefrom lines to the right making the following angles: 11° 15′, 22° 30′, 33° 45′, 45° 00′, 56° 15′, 67° 30′, 78° 45′ and 90° 00′, also writing the letters which denote their respective directions, as in the figure, N standing for north, E for east, W for west and *b* for the word *by*.

TO FIND CLOCK TIME.

29. TO FIND TIME. By any of the methods given in Chap. X, Art. 24, obtain the true meridian line on, say, a piece of paper pasted on a board. Somewhere on this north and south line fix the point of a lead pencil which is held perpendicular to the paper. At noon the shadow of the pencil will fall on the meridian line, giving the *apparent* time, to which if the equation of time given in Table XX of the "Calculating Tables," 5th edition, is applied, the *mean* or clock time will be found to within a few minutes, sufficiently accurate for practical purposes. Instead of using a pencil or other similar contrivance, it would perhaps be safer to suspend a small plummet attached to a piece of thread from above the paper so that its point may be exactly over the drawn N and S line; thus the shadow would be better defined and more precise, and the perpendicularity of the string ensured beyond doubt.

DIALLING.

30. 'DIALLING' DEFINED. *Dialling* is the art of constructing sun-dials.

31. 'SUN-DIAL' DESCRIBED. A *sun-dial* is a surface, generally a plane, on which a system of lines is drawn in such a manner that the coincidence of the shadow of a straight rod or edge with any of them, points out the hour of the day in apparent time. The straight rod or edge is called the *stile* or *gnomon* of the dial, and the system of lines are called *hour lines.* When the stile is the edge of a plate, the latter is called a *plate stile.* The plane of the plate stile is generally

placed perpendicularly to the plane of the dial, and its intersection with the plane of the dial is called the *substile*. The inclination of the stile to the plane of the dial—that is, to the substile—is called the *elevation* of the stile. The stile is always placed parallel to the earth's axis, and the hour lines are merely the intersections with the surface of the dial of planes passing through the stile, which, with the plane of the meridian, are inclined to one another at an angle of 15° in succession. When the plane of the dial is horizontal, it is called a *horizontal dial;* when it is vertical, it is called a *vertical* or an *erect dial;* and when the dial is both vertical and perpendicular to the meridian, it is called a *prime vertical dial;* but the construction of only the first and third of the 3 kinds, as being the ones in general use, has been described in this Chapter. In a horizontal dial, the *elevation* of the stile is equal to the *latitude* of the place. The most accurate method of constructing dials embraces the principles of spherical trigonometry, with which however the student need not be familiar to follow the working given below.

32.* TO CONSTRUCT A HORIZONTAL DIAL. (Figs. 398, 399, 406.) Let SNT (*see* Fig. 406) be the plane of the dial, extended to cut the celestial sphere, P the pole, SPN the plane of the meridian, and CPT the plane of an hour circle. CP is the direction of the stile, and CN the direction of the substile. Then PN is the latitude, and T′CT the hour line corresponding to the meridian, which gives the hours of the same name in the forenoon and afternoon—as, for instance, five o'clock in the evening and morning; also SN is the hour line of twelve.

Let $l =$ PN, the latitude of the place,
$h =$ angle TPN, the hour angle in degrees,
and $t =$ NT, the distance in degrees of the hour line from N;
then in the spherical triangle PNT, having the right angle N,
$\sin. l = \cot. h. \tan. t,$
$\therefore \tan. t = \sin. l. \tan. h,$
or $\log. \tan. t = \log. \sin. l + \log. \tan. h - 10.$

For any given latitude $= l$, the above equation will give the values of t when $h = 15°, 30°, 45°$, etc.

Example : Suppose a horizontal dial is required for Moulmein, latitude 16° 29′. The angular distances of the hour lines in succession from the hour line of noon will be—

For 11 A.M. or 1 P.M., $t = 4° 21'$
$(\tan. t = \sin. 16° 29' \times \tan. 15° = \tan. 4° 21')$
" 10 " " 2 " $t = 9° 18'$
$(\tan. t = \sin. 16° 29' \times \tan. 30° = \tan. 9° 18')$
" 9 " " 3 " $t = 15° 50'$
$(\tan. t = \sin. 16° 29' \times \tan. 45° = \tan. 15° 50')$
" 8 " " 4 " $t = 26° 10'$
$(\tan. t = \sin. 16° 29' \times \tan. 60° = \tan. 26° 10')$
" 7 " " 5 " $t = 46° 38'$
$(\tan. t = \sin. 16° 29' \times \tan. 75° = \tan. 46° 38')$
" 6 " " 6 " $t = 90°$.

And in case it is desired to show every 15 minute line, it will only be necessary to take $h = 3° 45', 7° 30', 11° 15', 15° 00'$, etc., and the value of t similarly found to the above from the equation before given. Hence, for a horizontal dial at Moulmein, the angular distance to show every 15ᵐ line will be as follows :—

A. M.	*t.*	P. M.	A. M.	*t.*	P. M.	A. M.	*t.*	P. M.
H. M.	° ′	H. M.	H. M.	° ′	H. M.	H. M.	° ′	H. M.
11 45	1 4	0 15	9 45	10 44	2 15	7 45	29 55	4 15
30	2 8	30	30	12 17	30	30	34 25	30
15	3 14	45	15	13 58	45	15	39 53	45
00	4 21	1 00	00	15 50	3 00	00	46 38	5 00
10 45	5 30	15	8 45	17 38	15	6 45	54 58	15
30	6 42	30	30	20 17	30	30	65 07	30
15	7 58	45	15	23 00	45	15	76 00	45
10 00	9 18	2 00	8 00	26 10	4 00	6 00	90 00	6 00

OBSERVATION.

* The astronomical terms employed in this and the two succeeding articles are explained in Chapter XVI.

Next, on a brass disc of convenient diameter, mark off a diameter to represent the north and south line, and on each side of it mark off the hour lines at the estimated angles, *i.e.*, 4° 21′, 9° 18′, 15° 50′, 26° 10′, 46° 38′, and 90°, as in (Fig. 399). To this disc affix the gnomon, also of brass, the slant side of which must make an angle of 16° 29′ with the plane of the disc. Now, by means of a spirit level, carefully level in all directions some place, not liable to be shaken, so as to receive the metal disc. Then, having found the meridian by the method in (Art. 26), carefully make the N and S line of the disc coincide with it, and then finally secure the disc to its stand, as seen in (Fig. 398).

33. TO CONSTRUCT A PRIME VERTICAL DIAL. (Fig. 161.) Let TNZ be the plane of the dial produced to cut the celestial sphere, P the pole of an opposite name from the latitude, NPZ the plane of the meridian, CPT the plane of an hour circle, and CP the direction of the stile. Then PN is the co-latitude, and TCT′ the hour line corresponding to that meridian, which gives the hours of the same name at T and T′ in the forenoon and afternoon.

Using the same notation as in the preceding problem, excepting c for the co-latitude PN, instead of l, it appears from the triangle PNT that—

$$\sin. c = \cot. h. \tan. t,$$

$$\therefore \tan. t = \frac{\sin. c}{\cot. h.} = \cos. l. \tan. h.$$

Example: Find the angular distances, for a vertical dial, of the hour lines in succession from the hour line of noon, for a place in the latitude of 31° 33′.

For 1 P.M. or 11 A.M., $t = 12^\circ\ 52'$.
2 " " 10 " $t = 26\ \ 12$.
3 " " 9 " $t = 40\ \ 27$.
4 " " 8 " $t = 55\ \ 54$.
5 " " 7 " $t = 72\ \ 33$.

34. BRIEF RULES FOR THE CONSTRUCTION OF HORIZONTAL AND VERTICAL DIALS. (Figs. 188, 190.)

Horizontal sun-dial, in north latitude. (*See* Fig. 188.)
MIH = Horizon of place.
NXM = Latitude of place.
The gnomon should be fixed in the plane of the meridian.
Rule for finding the hour lines:
If MI represent n hours from noon,
tan. MXI = sin. lat. × tan. ($n \times 15^\circ$). (15° corresponds to 1 hour.)
Vertical sun-dial, facing the south. (*See* Fig. 190)
ZIQ = a vertical circle facing the south,
SXQ = co-latitude of place.
If QI represents n hours from noon,
tan. QXI = cos. lat. × tan ($n \times 15^\circ$).

35. TRAVELLING SUN-DIALS. In the case of travelling sun-dials, which are numerous and of various constructions, some are intended for universal usage and others for particular countries. They are instruments with which sun or apparent time can be obtained, but only approximately. They usually carry a magnetic needle fitted inside a box, with which the N. and S. line of a dial is capable of being adjusted to the true meridian. In setting the instrument for use, the N. and S. line of the disc is brought to correspond with the true north line, by making the magnetic needle point to the number of degrees on the right or left of zero, according to the number of degrees the variation of the needle is E. or W. at that particular place.

36. TO FIND THE MERIDIAN IN WHICH TO SET THE GNOMON OF A SUN-DIAL. (Fig. 404.) Fix a *vertical* rod, about 7 inches high, at the southern edge of the horizontal surface which is to form the sun-dial. Between the hours of 8 to 10 A.M. mark positions of the shadow of top of rod—in the figure these positions are *a*, *b*, *c*. With the bottom of the vertical rod (*r*) as centre, and the

distances *ra*, *rb*, *rc* as radii, describe arcs. In the afternoon, between the hours of 2 and 4, observe passage of shadow of top of rod across the arcs at *c′*, *b′*, *a′*. The meridian will be in the line bisecting *aa′*, *bb′*, *cc′*.

37. TO CONVERT SUN-DIAL TIME INTO CLOCK TIME. In order to convert the apparent or sun-dial time, as indicated by the shadow of the gnomon of a sun-dial, into mean or clock time, apply to it the equation of time found in (Table XX of the "Calculating Tables," 5th edition) with the proper sign.

CHAPTER XI.

Plane-table Surveying.

1. USE OF THE PLANE-TABLE. The *Plane-table* is an instrument used for filling in the interior details of a survey, after the boundaries and principal chain lines which have been surveyed by compass and chain, or by the theodolite and chain, have been plotted on paper. It should never be used for purposes of extension, or any external work where great accuracy is required.

2. CONSTRUCTION OF THE PLANE-TABLE. (Fig. 402.) The *Plane-table* * consists of an ordinary drawing board of well-seasoned oak, teak, or toon wood, about an inch thick, and is of various dimensions; the one that is 30 inches by 24 inches is generally worked with, being as large as can conveniently be used. Deal and other soft woods are not recommended for plane-tables, as they imbibe moisture quickly and expand across the grain. The surface of the table is smooth and level. There are two wooden bars about 2 inches in width screwed on to the under part of the table, running across the grain of the wood, to prevent its warping. It has a screw in the centre of the table, underneath, by which it can be fixed tightly upon a tripod stand, or moved horizontally in any required direction. On one side of some of the tables is a small projecting ledge with a groove cut in it to hold a rectangular compass-box, about 4 inches long, 1 inch broad, and ¾ inch deep, and which has a glass cover and contains a magnetic needle swinging on a pivot at its centre. The box is so made to fit in as to lie in the direction of the edge of the table to which it is fixed. In other plane-tables this projecting ledge does not exist, it being considered more convenient to detach the compass-box, and place it, whenever required, on the margin of the table in the proper direction.

3. DESCRIPTION OF THE INDEX-RULE. (Fig. 402.) The plane-table has also an *index-rule*, about the length of its diagonal, made of ebony or other heavy and well-seasoned wood, 1½ inches wide and ⅓ of an inch thick. Both edges of the index-rule are bevelled, and on each end a *sight-vane*, like that in the prismatic compass, about five inches long, is fixed perpendicularly, by means of screws, to the index-rule.† On the eye sight-vane is a fine slit in the centre running along its length; the slit on the object sight-vane is ½ inch in width, along the centre of which is fixed a fine straight wire. To make an intersection, the eye *slit*, the vertical *wire*, and the *object* must appear in the *one* line.

4. TO TEST THE PERPENDICULARITY OF THE SIGHT-VANE. Point the index-rule on some small and well-defined object, at the distance of 200 or 300 yards. Intersect the object with the upper or lower portion of the sights, and moving the eye up and down, see if the *slit*, the *wire*, and the *object* still remain in the same line; if they do not, the wire or even the sight must be altered until they do. It is not often that this alteration is necessary.

5. ADVANTAGE OF DIRECTLY PLOTTING WORK ON A PLANE-TABLE. On a plane-table the surveyor plots off direct from nature the angles subtended at his eye by the various objects he may desire to lay down or work from, and as his index-rule is of full length, his plotting is as accurate as it well can be. He

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OBSERVATIONS.

* The plane-table is not a Goniometer, or Angle-measurer, like the Compass, etc., but a Gonigraph or Angle-drawer.

† Some Index rules have the sight-vanes only hinged on, to admit of their being folded down when not in use; but the ones with fixed sights are preferable to these.

therefore saves much time and avoids all the errors and mistakes to which he would be liable if he observed his angles with a suitable instrument first and then plotted them with a protractor.

6. DIRECTIONS FOR MOUNTING THE PAPER. Before commencing operations with a plane-table, the first thing to be done is to mount the board with paper, which must be fresh, thick, and good; in fact, such as is used for drawing on (*see* Chap. VII, Art. 18). First cut the paper to $\frac{1}{8}$ of an inch less than the size of the board all round, then take a wet sponge and thoroughly damp the *reverse* side of the paper. Take a strip of long-cloth or muslin 3 inches in width, having first washed out the starch and dried it, and paste one inch of it with fine paste* all round the board, allowing the other two inches to overlap it; then paste the edge of the paper on the pasted strip of cloth, and smooth it out with a dry cloth, pasting the overlapping edges of the cloth with stiff paste underneath the board; allow the paper and table to dry thoroughly in the shade for 2 or 3 days, and they will be ready for use.

7. PROJECTING THE POINTS OF THE SURVEY. (Fig. 405.) After the table has been mounted with paper, proceed to project on it the points of the survey, the positions of which have been already fixed either by a theodolite or a compass, and join the points by red lines; also draw the scale of the map in a spare corner, and plot the magnetic north line of the survey parallel to that edge of the paper next which is the compass box, and the table will be ready for use.

8. TO PLANT THE TABLE IN POSITION. (Fig. 405.) Place the table on its stand, without tightening the screw (below), make it as level as possible with the hand, and turn it slowly round until the magnetic needle points to the north; then clamp it †; the north line of the survey being parallel to the needle, it is evident that all the other lines of the survey on the paper must be parallel to their original directions on the ground. Remove the table to any other place on the ground, and repeat the operation; since the magnetic needle always points to the magnetic north, it is in its new position, parallel, in practice, to itself in the first position, and therefore all the edges of the table and all the lines of the survey are in positions exactly parallel to those they occupied before. Upon this fact depends the method of using the plane-table for survey purposes. Practically, the table ought never to be set up by the compass alone, but sights should be taken to different fixed points in addition, as a check upon the accuracy of the needle, and of its being truly parallel to the magnetic north line, as plotted on the paper. It should be observed that in placing the plane-table over any station, that spot on the table representing such station, and not the centre of the table, should be over the station on the ground; it may be so placed by dropping a pebble from the corresponding point on the underside of the table. Now, having fixed the table at a convenient part of the ground, make a point on the paper to represent that part of the ground. Run a fine pin through this point into the table, against which apply *one* edge ‡ of the index-rule, moving it round until some remarkable object, or mark set up for that purpose, is perceived. Then draw a pencil line from the station point, along *this* edge of the index-rule. Again, set the sights to another mark or object, and draw a pencil line, as before, and so proceed until as many lines of direction as are necessary from this station are obtained. Then measure, by means of the chain or perambulator (*see* Chap. IX, Arts. 56—60), the distances from the station to as many objects as may be necessary, taking at the same time the offsets to the required corners or crooked parts of hedges, and setting off all the measures upon their respective lines upon the tab'e. Now, remove the table

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OBSERVATIONS.

* The best and cleanest paste is very thick rice water.

† This method of planting the plane table in posit'on in a survey by means of the magnetic needle is also known as *orienting*.

‡ *Either* of the two edges may be employed, since there would be no appreciable difference in the direction of an object situated at a distance of above 20 feet from the table. For large-scale surveys the plane-table generally employed is $30'' \times 24''$, and the index-rule $30'' \times 2''$. The difference in the direction of an object 20 feet distant with such a rule, therefore, would be represented at 12 inches from its position on the map by $2'' \times 12'' \div 20' = 0{\cdot}1$ inch, by the property of similar figures. And as 20 feet on the map for the largest scale surveys usually undertaken, viz., $64'' \doteq 1$ mile and $50' \doteq 1$ inch, would be shown by distances of $20' \times \frac{1''}{\dot{9}\dot{9}0} \doteq 0{\cdot}24$ inch, and $20' \times \frac{1''}{600} \doteq 0{\cdot}4$ inch, respectively, the difference in direction referred to would be practically *nil.*

to some other station, whose distance from the foregoing was previously measured; then lay down the objects which appear from thence, and so continue until the work is finished, measuring only such lines as are necessary, and determining as many objects and points as possible by intersecting lines of direction drawn from different stations.

9. MAGNETIC NEEDLE TO BE POWERFUL. The magnetic needle should play freely, be strongly magnetized, be very sensitive, and yet settle quickly.*

10. PLANE-TABLE USED CHIEFLY IN CONNECTION WITH THEODOLITE SURVEYS. (Fig. 334.) As neither the surveying compass nor the prismatic compass have telescopes attached to them, it is evident that an object cannot be seen at a greater distance with them than with the plane-table, and hence the plane-table is seldom, if ever, used to fill in the details of a compass survey, as the positions of objects can be fixed by offsets, but it is very useful for filling in a survey made with the theodolite. Thus, in the Figure, the thick lines and the thin lines by the river bank represent the survey lines from station to station of a theodolite traverse, which have been plotted on paper duly mounted on the plane-table; the alternate dot and stroke lines are the principal chain measurements made and plotted on the paper, which divide the ground into convenient blocks; the thick dotted lines are the secondary chain measurements made and plotted on the paper, which run close to such boundaries and objects as are to be fixed; the thin dotted lines are offset measurements made from both principal and secondary lines to boundaries and objects for their determination—these lines are never drawn on the plan, but are shown in the Figure as explanatory.

11. NUMBERING OF FIELDS AND OTHER ITEMS IN A PLAN. (Figs. 323, 507, 508.) In surveys † made on a large scale, of 16″ = 1 mile, or 32″ = 1 mile as is the Cadastral Survey of India and Burma, every cultivated field, waste land, jungle, grazing ground, house, tank, well, etc., shown on the Plan is numbered for convenience of reference, and the numbers entered in an Area Statement form with the corresponding area of each, in acres. And in order that any item of the plan (which probably contains hundreds and thousands of items) of which the number is given may be found on immediate reference to the plan itself, it is clear that the numbering of these items must be executed in some systematic order to enable this to be possible. Hence, in the Cadastral Survey it is laid down that the numbers on a plan (which may consist of one, two, three, or more sheets) should invariably commence at the N. W. or upper left-hand corner of the plan, following the course shown by the arrow in (Fig. 507), until it ends at the S. E. or lower right-hand corner of the plan. (Figs. 323, 508) show how the numbers of the fields are to be entered, recollecting that in numbering, no items are to be skipped, but are to join on to each other in serial order; if, by mistake, any item has been omitted, it must be numbered with a *fractional* number composed for the numerator of the item number adjoining it, and for the denominator of the number following the last number entered on the plan, as, for example, $\frac{24}{30}$ in (Fig. 508), 24 being the number of the adjoining item, and 29 the last number entered on the plan; if there was, besides, another fractional number to be entered, as $\frac{11}{31}$, its denominator would be 31; and so on, for others.

12. SURVEYING WITH THE PLANE-TABLE AND CHAIN ONLY. (Fig. 401.) The plane-table can be used to make a survey independently of any other instrument except the chain. If the table be set up and turned round till the needle points to the north, and a line be drawn on the paper parallel to the needle, this will of course be a meridian line. If, now, a flag be put into the ground, in

OBSERVATIONS.

* A good compass needle is a great aid to rapid plane-tabling, inasmuch as it enables the surveyor to set his plane-table so correctly that the intersections of the rays to the surrounding points give him at once his true position, without the necessity of a second approximation. If the needle is found to be weak, it can be re-magnetized by the process described in Chap. X, Art. 3. When not in use, the needle should be thrown off its pivot by the lever provided for this purpose: this should always be done whenever it is taken off the plane-table.

† It also happens that subsequent to survey, changes take place in the land, when original items are split up into two or more parts; in such cases the additional parts receive fractional numbers having the number of the item of which it originally formed part for numerator, and for denominator the number succeeding the last number entered on the plan.

any position of the land to be surveyed, and a point on the meridian line be taken to represent the point on the ground over which the table is standing, the position of the flag on the paper can be found. Place a pin in the assumed point in the meridian line, and place one edge of the Index-rule against it, and turn it round till the sights coincide with the flag; draw a line along this edge of the index-rule, and measure, with a chain, the distance of the flag on the ground, and lay down by scale the same distance on the paper; this will give the position of the flag. The positions of other objects can be found in the same way. Offsets can be taken as in a chain survey, and also by obtaining the directions with the index-rule, to a place, from a known point, and the distance measured with a chain and set off on the paper. The advantage of a plane-table and chain survey over a chain or compass survey is that the work is self-plotted on the board, as surveyed, and no field-book is necessary to be kept up.

13. ADVANTAGEOUS CONDITIONS OF COUNTRY FOR THE PROSECUTION OF PLANE-TABLE SURVEYS. A hilly country offers the fairest field for the practice of plane-table surveys where the stations of a theodolite survey are given, and the more rugged the surface, the greater will be the relative advantages and facilities this system possesses over the methods of actual measurement. On the other hand, in flat lands the plane-table works at a disadvantage, while the traverse system is facilitated.

USEFUL PROBLEMS.

14. PROBLEM I. *To obtain the position of a plane-table within a survey, and to fix points therefrom.* (Figs. 164, 401.)

Suppose the plane-table to have been set up (or *oriented*) by means of a magnetic needle somewhere within the boundaries fixed by the pegs driven into the ground in the course of a compass or theodolite survey (*see* Fig. 401), the first thing to be done is to find the point on the paper corresponding with the point over which the table has been set up. To do this: (1) set up flags on 3 or 4 stations on the ground that can easily be seen from the position of the table; (2) put a pin in the point of the paper corresponding to the first of those flags; (3) place the edge of the index-rule against this pin, and turn the ruler slowly round till the slit in one sight-vane and the wire in the other coincide with the flag; (4) draw a pencil line along the edge of the ruler. The point required must evidently be on this line. (5) Repeat this operation for flag No. 2, and the point required must evidently be upon the line drawn this time, and therefore it must be the point of intersection of these two pencil lines. If the flags were exactly at the distance apart they were supposed to be, if the needle pointed exactly to the north, if there were no mistake in observing the flags or in the plotting of the survey, this would be all that was required; but there are so many sources of error, that it is always better to repeat the above operation to one or two more flags.* Next, suppose the table had been set up, as aforesaid, in the vicinity of a rectangular tank *abcd*, which it was desired to show on the plan. Set up a flag at each corner of the tank; put a pin in the point on the table, which has just been found to correspond with the point, ⊙ 1, on the ground over which the table has been set; put the edge of the index-rule against this pin, and turn it round until the flag at *a* is seen through the slit in one sight-vane, coinciding with the wire in the other; draw a pencil line along the ruler, and the corner of the tank must be somewhere on that line. In a similar way draw lines to the other corners of the tank. Remove the table to another side of the tank, as ⊙ 2; find the point on the table, as before, corresponding with the point on the ground where it has been set up; draw lines to the four corners of the tank, as before; and the intersection of these lines with the former lines will determine the required positions of the four corners of the tank. In a similar way the positions of any other required object may be obtained.

OBSERVATION.

* The above process is termed *interpolation*, which is the act of cutting in one's position from three or more known fixed points; thus, in (Fig. 164), A, B, C, are three known fixed points represented on the plane-table by the letters *a*, *b*, *c*. Having set up the table with respect to the magnetic north line, draw in with the sight-rule, the directions *Ccp*, *Bbp*, *Aap*, meeting in *p*; then *p* is the interpolated point denoting the position of the table on the paper.

15. PROBLEM II. *From any two points to fix a series of others.* (Fig. 400.)

Select two stations, M and N, some distance apart, from which the series of objects are visible. Draw the scale of the plan, whatever that is to be, in a corner of the paper. Measure the distance between M and N, and prick a point in a convenient part of the paper to denote one of the stations, as M. Set up the instrument at M, put a pin at the point on the paper representing that station, and place one edge of the index-rule against it. Turn the table round until the zero of the compass points north, and in that position clamp the table firm. Now keeping the edge of the index-rule against the pin at M, turn it round so as to direct the sights to the other station N, and along this edge draw a pencil line on the paper to represent the direction of N. With a pair of compasses take the measured distance between M and N off the protracted scale, and lay it off on this pencil line from M, thus fixing N, the other station of the base line. Again move the index-rule round the point M as a centre, direct the sights to the object A, and draw a line from the point M along the edge of the rule. In the same manner direct the sights and draw the lines in succession, in the direction of B, C, D, E, and F, from the point M. Having done this, move the instrument to station N and turn the table round till the zero of the compass points north; lay one edge of the index-rule against the pencil line NM, and the sights should show the direction of M. Now fix a pin in the point N, lay one edge of the index-rule against it, and direct the sights in succession to the objects A, B, C, D, E, and F, drawing lines from the point N in their several directions, as before, and the intersections of these lines with those drawn from the point M will be their respective positions on the plan.

16. PROBLEM III. *To survey a piece of land in the form of a trapezium by setting up the plane-table at two of the opposite angles only.* (Fig. 165.)

Set up flags at B, C and D; plant the table at A by means of a compass, with due respect to the N and S line. Make a point on some convenient part of the plane-table paper to represent A. Apply one edge of the index-rule at A and direct its sight until the vertical wire in it intersects the flag at B; draw a pencil line along the edge of the rule from A in the direction of B. Now proceed to chain the line AB on the ground, and having determined its length, with a scale mark off the exact length AB on the paper. Next direct the rule towards C and D, and in like manner measure the distances on the ground and plot the same on the paper, so that the relative positions and lengths of the lines AB, AC and AD are obtained. Again, move the table to B and adjust it as before, taking care to check the line AB in reference to the position of the table at B by directing the index-rule back upon A. Now proceed to directt he rule towards the flags at C and D, and measure the lines BC, BD, upon the ground, and having plotted them with the scale, the survey is completed, and the accuracy of the work will be proved if the lengths BC, BD are found by measurement to coincide exactly with the points C and D formed by measuring the lines AB, AC, and AD.

17. PROBLEM IV. *To survey with the plane-table, from a station within the field, with or without a compass.* (Fig. 403.)

Plant the table at a point O somewhere within the field; if it has a magnetic needle, adjust it so that its zero may point north, otherwise no matter; fix a convenient point on the paper to represent O; bring one edge of the index-rule against a pin fixed in this point, and direct the sights to A, drawing a line in that direction; measure OA, and from the scale already fixed on, lay down its length on the pencil line, and the point A will be determined; similarly, determine the points B, C, D, E, and F; join them as shown in the figure, and the plan of the field will be completed.

18. PROBLEM V. *To traverse with the plane-table, with or without a compass.* (Figs. 405, 407.)

Set up the instrument at A (*see* Fig. 405), with the compass if it has one, otherwise do so roughly with the hand, so as to have the edges of the board approximately in the directions N, S, E, and W, respectively; then screw the board tight. Put up flags at all the stations. Make a mark in some convenient part of the paper to represent station A, put a pin into this mark and apply one edge of the index-rule against it, and turn the ruler round till the sights are in a direct line with the flag at B; then draw a line from A in the direction of B; measure the distance between A and B with a chain or perambulator, and plot it from the scale of the map, thus determining B; similarly, get the direction of G and fix it on the paper. Again remove the table to station B; put a pin in this point, apply one edge of the index-rule against it and lay this edge along the pencil line BA, and turn the board round till the sights are in a direct line with the flag at A; screw the board tight, and the instrument is ready to work with; it being borne in mind that if the compass was used, it would act only as a check on the work. Now turn the index-rule round till the sights are directed to the flag at C, and draw a pencil line indicating its direction; measure the distance between B and C, and lay it off from the scale, as before; thus C will have become determined. In this manner proceed from station to station till the starting-point A is arrived at.

While carrying on a traverse, offsets can be taken to the bends of roads, corners of houses, tanks, etc.; besides any remarkable objects can be fixed by their directions being taken from two or more stations, such as the pagoda in the figure, which has been fixed by intersections from the stations A, B, and C. It should be remembered that in plotting generally, when a point has been fixed by intersections from 3 or more stations, if the lines drawn from them do not exactly meet in a point, but form a little triangle as *xyz* in (Fig. 407), it is usual to prick a hole in the centre of this little triangle to denote the point intersected; that is to say, if the error is not very considerable and the point is not of sufficient importance to re-do the work.

19. PROBLEM VI. *To fix a plane-table in position at an unknown point x, by means of three points A, B, C, whose positions are laid down on the plane-table, and represented by a, b, c, respectively.* (Figs. 447—449.)

Fix a pin in the point *b* (*see* Fig. 475) on the plane-table, and placing the index-rule against it and the point *a*, with the object and sight towards *a*; turn the table about, until the point A is intersected; then clamp the table in this position, turn the index-rule and intersect the point C with the edge of the ruler still against the pin at *b*, and draw the line *bm*; now remove the pin to the point *a*, and unclamp the table; place the index-rule against the pin at *a* and the point *b*, and turn about the table until the point B (*see* Fig. 448) is intersected; clamp the table again and having intersected the point C as before, draw the line *an* through the intersection *p* of the lines *an* and *bm*; draw the line *cpq* passing through the point *c*; and placing the edge of the index-rule against this line, unclamp the table once more, and turn it about until the point *c* (*see* Fig. 449) is intersected; now clamp the table, and it will be in position, and the unknown point *x* will be situated on the line *cpq*; to find this point it is merely necessary to intersect either of the points, as A, and draw the line A*ax*, and the accuracy of the operation is tested by intersecting the other point B and drawing the line B*bx*, which should intersect the line A*ax* on the line *cpq*, thus giving the position of *x* on this line. When the point *c*, with regard to the point *x*, is situated on the other side of the line AB, or below it, the lines *an* and *bm* will intersect on the opposite side of the line *ab* to that on which *c* is; and if the point *x* be situated within the triangle ABC, these lines *an* and *bm* will diverge instead of converge, in which case they must be prolonged in the opposite direction until they intersect at the point *p*. The accuracy of the result depends on the length of the line *cp*.*

CONTOURING.

20. CONTOURING DEFINED. *Contouring* means tracing on the surface of the ground horizontal lines, with a fixed vertical height, and is, in fact, another species

OBSERVATION.

*The above problem for fixing an unknown station, by observations thereat, to three known ones, is only applicable to reconnoisance, and totally inadmissible for accurate surveying.

of levelling. The positions of these horizontal lines are surveyed and protracted on a map, in the same way as any other boundary lines. Contour lines give a perfect delineation of the ground, but the process is a tedious and expensive one.

21. WHERE EMPLOYED. Contour lines are either traced round isolated features of ground for designing plans for drainage, buildings, or other engineering purposes, when the correct features and levels of the country are required to be seen at a glance, or over a whole tract of country in connection with some extensive survey.

22. PLANS. (Figs. 362, 359, 498). (Fig. 498) represents the contoured top of a hill, the figure below an elevation, parallel to the line AB. Then each of the lines in the plan would be seen in elevation as follows:—*abcd* as *ac* ; *efgh* as *eg* ; *kmn* as *kn* ; and the top as *p* ; the vertical heights between *a* and *e*, *e* and *k* ; being all equal, say, 1, 5, 10, or more feet apart as circumstances require. It will be seen that the steeper the hill is, the nearer the contour lines approach each other in the plan. (Fig. 362) is an example of an isolated hill, illustrating the principles of contouring; the section in any other direction as CD can be given at once. (Fig. 359) is an example of a supposed more extensive district, and that other contours are interpolated *by the eye*, which can be done with great accuracy where there is so much fixed data.

23. HILL SHADING. (Fig. 490.) As above stated, ground can be represented by means of contour lines on its surface, at any given vertical intervals apart ; but these lines only furnish information regarding the surface of ground represented, at the exact points where the line or contour passes ; hence irregularities of ground which take place between any two successive contours, of which the contours cannot possibly give any information as they do not pass through them, are shown by what is termed " hill shading." Another object of hill shading is to represent the surface of ground by means of shading, in such a conventional manner as shall without the assistance of contours give every information concerning it. By the latter method of hill shading, or *eye contours*, a general representation of the ground is only obtainable, while by means of surveyed contours distinct accuracy may be relied upon. The conventional method is obtained by a standard —termed a " Scale of Shade "—set of marks or lines of different thickness, and differently spaced, according to the particular slope of the portion of ground to be represented. The Figure shows the scale of the shade adopted in the Topographical Survey of India for filling in the features of ground between contours already fixed, for a scale of 6 inches to the mile, but it can be made use of for any other scale by altering the horizontal interval over which the different hachures extend for a given extent of the corresponding slopes. It also illustrates on one side the base contour lines and on the other side the same filled in by the method of hill shading. For further particulars on the subject of shading, *see* (Chap. VII, Arts. 9—16).

24. SURVEY OF HILLS. All prominent peaks of high land falling within the area for survey should be first fixed either by triangulation or sight-vane, and the requisite connections made by short traverses run wherever necessary, from one known point to another, on the plane-table Measurements should be made up the slopes of hills, wherever required, and the contour lines drawn in for the scale of vertical intervals adopted for the survey,* the minor details being sketched in, based on the principal measurements. For all ordinary purposes, it will be found convenient to determine the positions of the contour lines by short horizontal chain measurements, aided by a 7½-links or 5-feet rod, for surveys with 5, 10, 15, 20 or 25 feet vertical interval standards. The crests and feet of the hill ranges and their principal spurs should invariably be laid down in the first instance before the contour lines are attempted.

25. METHOD OF CONTOURING BY SURVEYED CONTOURS. The method of tracing contours is thus performed. Drive long pickets, one at the top and another at the bottom of such slopes as best define the ground, particularly the ridge lines and water-courses, as they always have the most gradual and even

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OBSERVATION.

* The contour lines for 16-inch maps are usually assumed at 1 in 10, that is, with 10 feet vertical intervals. Suitable contour intervals are 50 feet for 1-inch scale maps, and for other scales proportionately, e.g. 25 feet for 2-inch maps, 100 feet for ½-inch maps, etc.; in other words, the contour interval, *in feet*, would be $\frac{1}{50}$th of the number of inches to a mile in the scale.

Scanned by

slope. Run a line of levels down each, by means of a levelling instrument or a theodolite (for the former, *see* Chap. XV), fixing pegs at the point where each contour line crosses it. Measure the *horizontal* distances between the pegs; next run an auxiliary line of levels from a point in the line down one spur to some point similarly placed on the next spur, and so on connect the whole work. The reduced levels down each spur having been thus connected, the relative heights and horizontal distances of the contour pegs obtained, the work can be plotted on paper and the irregular features of ground between the several contour lines sketched in as before described.

26. MOUNTAINOUS FEATURES HOW DRAWN. (Figs. 496, 497.) Mountainous features may be drawn: 1*st*, by surveyed contours; 2*nd*, by eye contours, a modification of the first; 3*rd*, by vertical hatching; 4*th*, by shading with the brush in Indian ink, or some other suitable tint. (Fig. 496) gives an idea of the mode of delineating ground by surveyed contours, but is so expensive that in no country is it adopted. Eye contours, sometimes called the horizontal system of sketching, is an eye modification of contouring. The style is easy, light, effective, and affords great opportunity for artistic skill and is that adopted by the Topographical Survey Department of India. The vertical system, or hatching, starts by considering the course a volume of water, equally disseminated from the top of a hill, would naturally pursue in running down its sides, *see* (Fig. 497). In this system the steepest sides have the shorter bases, and the different slopes are shaded darker or lighter according as they are steep or gentle. In shading with a brush the same principle of light and shade is observed.

27. THE "WATER-LEVEL" USED FOR CONTOURING. The "water-level" described in (Chap. XV, Art. 44) is an instrument well adapted for contouring where a rapid delineation of country is an object frequently of greater importance than accuracy, its best recommendation being the facility with which it can be made and requiring no adjustment when using it.

28. THE CLINOMETER.* (Fig. 509.)

(*a*) *Hill shading.* In hill shading constant vertical intervals cannot be used when the horizontal hachures are intended to serve the double purpose of shade lines and contour lines; but a system of vertical intervals increasing with the angle of slope may be introduced, if accompanied with sufficient variations in the thickness of the shade lines, to indicate the value of the vertical interval which each line is intended to represent. Two tables are given specifying the vertical intervals in feet, the number of shade-contour lines in an inch, the thickness of these lines, and the breadth of the white spaces between them, for five governing angles of slope, namely, 5°, 15°, 25°, 35°, and 45°; these two tables differ merely in that the second gives greater vertical intervals and breadths of white spaces than the first, and is thus more easy to carry out in practice.

On comparing the clinometric shade-scale, shown in the Figure, with some of the latest and best specimens of hill shading on the 1-inch topographical maps, it appears that the gradation in the depth of shade for the several governing angles of slope is well suited for the purpose of conveying a correct general idea of the actual magnitudes of the angles of slope; and that a more faithful and intelligible delineation of the ground will be produced by draftsmen of average ability if they adopted this scale of shade, than if they invariably followed their individual fancies.

The plane-tabler should invariably lay down the crests and feet of the hill ranges and their principal spurs, in the first instance; he should then draw the shade lines, as indicated by the scale, measuring the angle of slope with the clinometer, whenever possible, otherwise estimating it by the eye.

(*b*) *Delineation of broken ground.* Much difficulty is usually experienced in distinguishing between the lower hill ranges and the broken ground which is ordinarily met with at their feet, and which also occurs on the banks of most of the principal rivers and water-courses in India and Burma. The custom is very generally followed of delineating such ground by horizontal hachures; thus in many cases it is probable that none but the surveyors themselves could point out which of their hachures indicate the broken ground, and which the actual hills. It is

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OBSERVATION.

* Extracts from a Circular Order of the Survey of India.

desirable therefore to make a marked distinction between these two predominant features of the ground which shall readily catch even an untutored eye, by employing short vertical hachures—of a length corresponding to the base, and a thickness corresponding to the angle of slope—to indicate the banks of ravines and broken ground generally, and even to show isolated hills which cover so small a space on the ground that they cannot be delineated on the 1-inch scale by less than 3 or 4 horizontal shade-lines.

It is not desirable to lay down a definite height above the general ground-level at which the vertical system should pass into the horizontal; that is a matter which must be left to the discretion of the surveyor. Obviously the long slopes of gently undulating ground should be indicated, not by vertical touches, but by horizontal hachures, the finest that can possibly be drawn, their distances apart being regulated by those of the slopes from 0° to 10° in the scale of shade given in the Figure. But it may be laid down as a broad rule that vertical touches should be employed for the minor features of the ground, horizontal lines for the major features and slopes up to 50° or 60°, and strong vertical hachures for all scarps of hill and mountain ranges which approach the perpendicular. Thus vertical touches and hachures may be employed for the two extremes of the scale of shade, and horizontal lines of varying thickness for all that is intermediate between these extremes.

(*c*) *Determination of approximate heights.* However skilfully a map may be shaded, it should still be supplied with a sufficiency of determinations of height to enable the more important differences of level, and alternations of ground relief, to be readily ascertained with fairly approximate accuracy from the numerical data on the map, instead of being estimated from an examination of the variations in the depth of shade. In fact a map which has many heights, but is badly shaded, may be much more useful than one which has few heights and is well shaded. Such determinations of height may be of much value even when they are not exact, but only approximate; if the errors are not more than 1 or 2 per cent. of the range between the highest and lowest points on the map, the determinations will give a fairly exact idea of the actual configuration of the ground, which may be of much service for engineering and military purposes, provided that the precaution is taken of indicating that they are approximate only, by so printing them on the map as to distinguish them from the more exact values.

With a view therefore to enable a number of approximate heights to be readily determined by the plane-tablers, a scale of 5,280 feet to 1 inch, and a table of tangents, have been drawn on the clinometric shade-scale. Thus the plane-tabler, having measured at any station the angle of elevation or depression of any contiguous point of which the height has already been determined by triangulation (with a theodolite), has only to measure the distance in feet of his station from the point in question off the plane-table, and multiply that distance by the tangent of the angle of elevation or depression to determine the difference of level and then deduce the height of his station. As the angle cannot be measured within much less than 15 minutes of the truth by the clinometer, and the error in height corresponding to that angle is nearly $= 0{\cdot}005 \times$ distance in feet, much accuracy is not to be expected; but when the points are not far off and two or three are observed and the mean is taken, the result would probably not be more than 50 to 100 feet in error, and in a hilly country it is much better to have a value of height with no greater error than this than to have no value at all.

Plane-tablers should calculate their heights on the spot, as the values may frequently be serviceable, aiding them to delineate the features of the ground with due attention to the relief. But they should also enter the values of the observed angles and adopted distances in their note-books, in order that the calculations may be subsequently checked and verified. The stations whose heights are thus determined should be shown on the maps by a dot only, and the heights should be given in smaller figures than usual, with the letters *ap.* written after them, to indicate that they are approximate.

In forest-clad tracts of country the points of the Trigonometrical Survey of India are rarely seen. Still, wherever the ground is hilly or undulating and presents many marked variations of level, a knowledge of these variations is a desideratum of importance, and surveyors are undoubtedly expected to supply it. Much may probably be done in this way by clinometer readings at plane-table stations so selected as to be mutually visible in succession, namely, the 2nd from the 1st, the height of which has been fixed by observations to surrounding trigonometrical stations, the 3rd from the 2nd, and so on. At each station it will be necessary to place a man with a flag-staff, or to build a cairn of stones, for observation at the next station, but this can be readily managed. As a matter of precaution, however, all such lines of (what may be called) clinometer levels should be checked by being made to close at a plane-table station of which the height is well fixed by observations to surrounding trigonometrical points.

The points for which subsidiary heights are desirable are the following :—

(1) Junctions of rivers and streams.
(2) Junctions of roads.
(3) Crests of passes through hills and mountains.
(4) Ferries.
(5) Open and level or cultivated plateaux in hill tracts.
(6) Dâk bungalows.

CHAPTER XII.

The Pocket Sextant.

1. USE OF THE POCKET SEXTANT. The *Pocket Sextant* is a reflecting instrument for measuring the angle between two objects in either a horizontal or vertical plane, the reflected image of one object being brought to coincide with the other seen direct. It is generally employed in the filling in of plans. It is very portable, being a circular box 3 inches in diameter and 1½ inches deep, requires no other support than the hand, is easily adjusted, and can be used on the tops of trees and places where other instruments cannot be carried, during all weathers.

2. ITS DESCRIPTION. (Fig. 451.) The pocket sextant is represented in the Figure. I is the *index-arm*, carrying a *vernier* V at one end, which enables minutes to be read ; L is the *limb* graduated to half-degrees, which is numbered to 140° only, as the sextant will not measure an angle greater than about 125° ; the index-arm is moved by a milled-head screw X ; *a* and *b* are the key-holes for adjusting the horizontal glass, into each of which the adjusting key Y, removable at pleasure, fits ; T is the telescope which can be pushed into the box when not in use. Two mirrors (not seen in the Figure) are placed inside ; the larger one, or *index-mirror*, is fixed to, and moves with, the index ; the smaller one, called the *horizon glass*, has only half of its surface silvered. Between these are placed two dark glasses for intercepting the sun's rays, which sink out of the way by raising two little levers at *l*. The telescope is also provided with a dark glass for observing the sun, which can be fitted on to its eye-end or removed at will.

3. THE ADJUSTMENTS. The instrument is in adjustment when, the index being at zero, the index-mirror and horizon-glass are parallel to each other. To test this, unscrew the telescope clean out of its box, set the index at zero, apply your eye at the eye-hole and look through the unsilvered part of the horizon-glass at some well-defined object, as the corner of a house or a flagstaff, *over half a mile distant*. If the reflected image of the selected object in the horizon-glass coincides with the object seen direct, the adjustment is perfect ; otherwise, the coincidence must be effected by moving the index-arm and inserting the adjusting key in the key-hole on the *top* of the instrument, when it is required to move the image up or down, and in the key-hole at the *side*, when the image is required to be moved to the right or left, and turning it the screw or unscrew motion, as necessary. Another manner of satisfying oneself as to the adjustments is by setting the index to zero, placing the dark glass before the eye-end of the telescope, and looking through it at the sun, of which, if only one perfect orb is seen, the instrument is in exact adjustment ; but if the reflected image project beyond the other, the coincidence must be effected by the adjusting key, as before described.

4. TO MEASURE HORIZONTAL ANGLES. To obtain the angle subtended by two objects in, or nearly in, the same *horizontal plane*, hold the sextant

horizontally in the *left hand,* look through the telescope, and by turning the milled-head X with the right hand, bring the reflected image of the right hand object into coincidence with the direct image of the left hand object, and the reading of the vernier will give the horizontal angle between the two objects to the nearest minute.

5. TO MEASURE VERTICAL ANGLES. To obtain the angle subtended by two objects in, or nearly in, the same *vertical plane,* hold the sextant vertically in the *right hand,* and by turning the milled-head X with the left hand, bring down the image of the upper object till it exactly coincides with the direct image of the lower object, and the reading of the vernier will give the vertical angle between the two objects to the nearest minute.

6. THE LID AND DARK GLASSES. When the sextant is to be used, the *lid,* D, of the box is taken off and screwed to the bottom, where it forms a convenient handle. Previous to observing horizontal or vertical angles, the levers of the *dark glasses* should be raised. In observations for altitude, when the sun is the object observed, the dark glasses would come into use, but when a star is taken no shade will be required.

7. PARALLAX. The *parallax* of the instrument is the angle subtended by the point of vision and centre of the index-mirror, occasioned by the necessity of placing the eye of the observer on one side of the index-mirror. There would be no parallax if we could look from the middle of the index-mirror. Parallax is perceptible when the object observed is near—within half a mile—hence, the necessity of observing only to objects *over half a mile* distant; but at a quarter of a mile this error is very trifling, and hence objects situated at that distance from the observer may be observed to.

8. OBJECTIONS TO SEXTANT. The chief and indeed only objection to the sextant as a surveying instrument, arises from the angles taken with it not being always like those measured by the theodolite and compass, *horizontal* ones, but *inclined* ones. It is used rather for surveying than astronomical purposes, though, in conjunction with an artificial horizon, it becomes valuable for obtaining solar time, or the latitude.

9. ARTIFICIAL HORIZON. (Fig. 210.) The *artificial horizon* is a reflector, the surface of which is perfectly horizontal; it is used in combination with the sextant for observing altitudes. The one in most common use, as well as the best, consists of a small shallow wooden trough containing pure, clean quicksilver * which reflects the image of a celestial body. This is protected from the action of the wind by a roof, the two sloping sides (whose surfaces must be absolutely parallel) of which are made of plate glass accurately ground to true planes. The whole is packed in a small box with a vessel containing quicksilver ready for use.

10. † METHOD OF OBSERVING FOR ALTITUDE. In observing the altitude of the sun or a star with a sextant and an artificial horizon, the *two* images are brought into contact on the surface of the mercury, and the angle observed is *double* the angle of elevation above the sensible horizon. The observer has to place himself in a situation to see the reflected image of the sun, or other body, in the fluid; and he has then to bring down the image, as reflected from the index-glass, until it reaches its reflection in the fluid. The altitude will then be *half* the number of degrees, indicated by the graduated arc, subject to certain corrections for semi-diameter and parallax (for the sun), and for refraction (for the sun or star).‡ When the *lower* limbs of the sun are in apparent contact, the image reflected from the index-glass appears uppermost; when the *upper* limb, of course the reverse.§

OBSERVATIONS.

* Oil, tar, or treacle would answer the purpose in place of mercury, but would not give so clear an image.

† The astronomical terms made use of in this Article are explained in Chapter XVI.

‡ The corrections for semi-diameter, parallax, and refraction are respectively given in Tables XL, XXXIX and XXXVIII of the "Calculating Tables" ([illegible]th edition).

§ When the contact is formed at the *lower* limb, the images will separate shortly after the contact has been made, if the altitude be *increasing*; but if the altitude be *decreasing*, they will begin to overlap. When the contact is formed at the *upper* limb, the reverse takes place. If the observer is in doubt as to which limb he has been observing, he should watch the object for a short time after he has made the observation.

EXAMPLE:

Observed angle	=	116°	29′	20″
Apparent altitude of sun's lower limb	=	58	14	40
Sun's semi-diameter * ...	=	+	16	18
,, parallax *	=	+	00	05
Refraction *	=	—	00	36
True altitude of sun's centre ...	=	57	58	53

11. PRINCIPLE OF CONSTRUCTION OF SEXTANT. (Fig. 357.) The principle of the construction of the sextant may be gathered from the following demonstration :—Let ABC represent a sextant, having an index AG (to which is attached a mirror at A) moveable about A as a centre, and denoting the angle it has moved through on the arc BC : also let the half-silvered (or horizon) glass *ab* be fixed parallel to AC ; now a ray of light SA, from a celestial object S impinging against the mirror A, is reflected off at an equal angle, and striking the half-silvered glass at D, is again reflected to E, where the eye likewise receives, through the transparent part of that glass, a direct ray from the horizon. Then the altitude SAH is equal to double the angle CAG measured upon the limb BC of the instrument. For, the reflected angle BAG (or DAF) = the incident angle SAI, and the reflected angle *b*DE = the incident angle *a*DA = DAE = DEA, because *ab* is parallel to AC. Now, HAI = DFA = (FAE + FEA), and DAE being equal to DEA, it follows that HAI = (DAE + FAE). From HAI and (DAE + FAE) take the equal angles SAI and DAF, and there remains SAH = 2 FAE or ∠GAC ; or, in other words, the angle of elevation SAH is equal to double the angle of inclination of the two mirrors, DGA being equal to GAC. Hence the arc on the limb BC, although only the sixth part of a circle, is divided as if it were 120° on account of its double being required as the measure of CAB, and it is generally extended to 140°.

12. PRINCIPLE OF ARTIFICIAL HORIZON. (Fig. 358.) The following explanation will illustrate the principle of the artificial horizon, and the method of observing altitudes with it :— Let AB represent the surface of the quicksilver contained in a wooden trough whose plane is continued to C ; DEF the roof, in which are fixed two plates of glass DE and EF, whose surfaces are plane and parallel to each other, and ⊙ the sun at S, whose altitude is required. Now the ray SH, proceeding from the sun's lower limb to the surface of the quicksilver, will be reflected thence to the eye in the direction of HG, and the upper limb of the sun's image, reflected from the quicksilver, will appear in the line GH, continued to R ; and it is a well-known principle in optics that the angle of incidence, SHA or SHC, is equal to the angle of reflection GHB ; and as the angle AHR or CHR is the opposite angle of GHB, it is therefore equal to it, and to the angle SHC, the altitude of the sun's lower limb above the horizontal plane ; so that, supposing the angle SHR, measured by a sextant, to be 80°, the altitude of the sun's lower limb will then be 40°, subject to the corrections mentioned in Art. 10.

CHAPTER XIII.

Theodolite Surveying.

1. HORIZONTAL CIRCLE DEFINED. The *horizontal circle* is a circle whose plane is a tangent to the earth at the place of the spectator.

2. HORIZONTAL ANGLE DEFINED. (Fig. 393.) A *horizontal angle* is the angle formed by the inclination of two straight lines in the plane of the horizontal circle, or in one parallel thereto, which meet in its centre, as ACB.

3. VERTICAL CIRCLE DEFINED. A *vertical circle* is a great circle of the sphere which passes through the zenith and nadir of a place, and is consequently at right angles to the horizontal circle.

OBSERVATION.

* The correction for semi-diameter is + or — according as the *lower* or *upper* limb of the sun has been observed. The correction for parallax is always *additive*, and that for refraction *substractive*. An observation of a star requires no correction for either semi-diameter or parallax.

4. VERTICAL ANGLE DEFINED. (Fig. 394.) A *vertical angle* is the angle formed by the inclination of two straight lines in the plane of the vertical circle which meet in its centre, as ACB.

5. ANGLE OF ELEVATION DEFINED. (Fig. 408.) An *angle of elevation* is an angle which an object makes with the horizon *above* it at the place of the observer, as ACB.

6. ANGLE OF DEPRESSION DEFINED. (Fig. 409.) An *angle of depression* is an angle which an object makes with the horizon *below* it at the place of the observer, as ACB.

7. THEODOLITE DEFINED. The *theodolite* is a most important surveying instrument used for measuring horizontal and vertical angles.

8. HOW THEODOLITES ARE CHARACTERIZED. Theodolites are of various constructions and dimensions and are characterized according to the diameters of their horizontal circles; for instance, a theodolite the diameter of whose horizontal circle is 5 inches, is called a 5-inch theodolite, and so on.

9. THE THEODOLITES IN GENERAL USE. The theodolites general'y used are the 5, 7, 8, 12 and 14-inch theodolites, depending entirely upon the degree of accuracy required, the readings of the graduated circles of which are read by means of *verniers*.[*]

10. VERNIER DEFINED. The *vernier* is a contrivance invented by a person named Peter Vernier,[†] and is attached to the graduated limb of an instrument for the purpose of measuring aliquot parts of the smallest spaces into which the instrument is divided; it obtains this object by measuring the differences between the divisions of two approximating scales, one of which is fixed and called the *primary scale*, the other moveable and called the *Vernier*.[‡]

11. THE PRINCIPLE OF THE VERNIER. (Figs. 411, 415, 416.) Let it be supposed that the limb or graduated circle of the theodolite is divided to degrees and subdivided to half degrees or 30′, as is usually the case with a 5-inch theodolite, such a one as T. Cooke and Son's No. 112, which forms the special subject of this paper. Then any number of minutes less than 30′ (the number of minutes in half a degree) is measured by the vernier scales which move along its edge. The length of a vernier scale is made equal to 29 of the divisions on the limb or primary scale, and it is divided into 30 equal parts (*see* Fig. 411); there is consequently a difference in length of 1 minute between the scale on the limb and the vernier scale. Since

$$\begin{array}{llllll} 30 \text{ vernier divisions} & = & 29 \text{ half degrees} & = & 870 \text{ minutes,} \\ 1 \quad ,, \qquad ,, & = & \frac{29}{30} \quad ,, \qquad ,, & = & 29 \quad ,, \end{array}$$

therefore an angle is, by means of the vernier, capable of being read off to a single minute, although the limb is graduated to only half degrees. In different instruments verniers of various constructions are adapted to them; for instance, where the limb is divided to 10′ and it is desired to read an angle to 10″ by means of the vernier (*see* Fig. 415), or where the limb is divided to 20′ and it is desired to read an angle to 20″ by means of the vernier (*see* Fig. 416), in both cases 59 divisions of the limb are equal to 60 divisions of the vernier. To take another instance: where the limb is divided to 20′ and it is desired

OBSERVATIONS.

[*] On many of the larger surveying and astronomical instruments, where the angle is desired to be read to a greater degree of minuteness than is attainable with the aid of the vernier, the vernier is superseded by the reading microscope. It may not therefore be out of place here to briefly describe (without the aid of a diagram) this appliance, though vernier instruments, from the fact of their being in general use for ordinary survey operations, are what are chiefly dealt with in this chapter. It, then, consists of a microscope of moderate power with a system of cross-wires at the common focus of the objective and eye-piece. The wires are fitted in a frame which can be made to travel across the field of view by a micrometer screw. For recording the position of the wires there is a *comb* in the field of view, one tooth of which corresponds to a turn of the micrometer screw, and the parts of a turn are read by graduations on the head of the screw. If the circle to be read is graduated into spaces of 5′, it is usual to so arrange the machine that five turns of the screw carry the wires over the space of one division. One turn of the screw will therefore move the wire through a space of one minute, and as the screw head is divided into 60 parts, the seconds on it can be easily read while the decimals may be estimated. In taking a reading, the moveable wire is set exactly on the division of the scale next the zero of the comb; then the number of teeth from zero give the number of minutes, and the reading of the screw head gives the number of seconds and decimals.

[†] Peter Vernier lived in France about the year 1630.

[‡] In some instances it is the contrary, as in the vertical circles of theodolites, where the vernier is fixed and the primary scale moveable.

to read an angle to 30″ by means of the vernier, 39 divisions on the limb will be equal to 40 divisions on the vernier, and so on for other instruments, depending upon the manner their limbs are graduated and the number of minutes or seconds the angles are sought to be read.* Generally, if a be the arc measure of one division on the limb, and the united distance of n such divisions be taken off on the vernier, then *one division on limb—one division on vernier* $= \frac{a}{n+1}$ *in arc.*

12. HORIZONTAL LIMB GRADUATIONS. (Fig. 410.) The horizontal limb of every theodolite is a complete circle graduated from 0 to 360 degrees, and, conceiving oneself to be standing within this circle, they would appear increasing from left to right. In a 5″ theodolite the graduations are at every half degree. Every tenth degree is marked by figures, every fifth degree by a dot, every degree by a long line, and every half degree by a short line. The 0° is termed the *zero* of the limb.

13. VERNIER GRADUATIONS. There are 30 divisions on the vernier beginning at the arrow. Every tenth minute is marked by figures, every fifth minute by a dot, and every minute by a short line. The arrow is termed the zero of the vernier.

14. ERRORS OF INSTRUMENT, EYE, AND OBSERVATION. Since there is nothing perfect in this world, the graduations on the limb of a theodolite are not perfect, and although apparently so, yet, in fact, are not so. These errors, however trifling, are known as *errors of instrument.* Similarly the eye of man is not perfect, hence in reading an angle on the limb we are liable to errors, however small, and such are known as *errors of eye.* So also as regards the intersections of objects with the instrument; several causes tend to throw the intersections out, however little, and these errors are technically termed *errors of observation.*

15. ERRORS OF INSTRUMENT, EYE, AND OBSERVATION ELIMINATED. (Figs. 410, 414.) Errors of instrument, eye, and observation are done away with, so far as is in the power of man, to a certain extent by angles being read on different parts of the limb, on both faces (as hereinafter explained), and a repetition of observations, and a mean of all taken for the most probable correct value. To illustrate the principle of taking means: if the length of a compound was measured once and found to be 356·6 feet, and when measured a second time was 354·8 feet, it is evident the both measurements cannot be correct, hence a mean of the two, or $\frac{356{\cdot}6 + 354{\cdot}8}{2} = 355{\cdot}7$ feet, is the more probable correct value; and the oftener the distance is measured, and the mean of all taken, so much more likely is the result to be the true distance. Consequently an angle is read on three parts of the limb at each observation by three verniers, marked A, B, and C, each 120° apart from the other, and the mean thereof taken for the correct angle. The degrees of the vernier A only are read, but the minutes and seconds of A, B, and C. If the graduations of the limb and the verniers were faultless, vernier A would read 0° 00′ 00″, vernier B 120° 00′ 00″, and vernier C 240° 00′ 00″ (*see* Fig. 410); but as faults there must be, small differences do exist, as in (Fig. 414), where the readings of the verniers are as follows:—

Vernier A = 167° 51′ 00″
„ B = 52 00
„ C = 51 30

The mean of which therefore = 167 51 30

16. THE INDEX. (Fig. 411.) The arrow on the vernier is called the *index*, and points to the reading of the angle on the limb; for instance, in the Figure it points *exactly* to 360°, therefore the reading is 0°.

OBSERVATION.

* In general to read to the nth part of a scale division in surveying and astronomical instruments, n divisions of the vernier must equal $n - 1$ divisions on the limb. The scale and vernier divisions, it may be noted are read in similar, and not contrary, directions.

17. TO OBTAIN THE READING ON THE LIMB. (Figs. 411—414.) If the index should not cut any of the divisions, that is, any of the lines representing degrees and half degrees of the limb, then make a note of the number of degrees it has passed, and look along the vernier and observe which of its lines coincides (only one being able to do so) with one of the lines on the limb; and the number of divisions on the vernier scale, from zero to where such coincidence occurs, are so many minutes to be added to the degrees and half degrees already noted down, for the required reading. If none of the lines of the vernier and limb coincide, the number of seconds or half of a vernier division, namely, 30″, must be estimated by the eye on that portion of the vernier where two adjacent lines on it very nearly coincide with two corresponding lines on the limb; *vide* Vernier C in (Fig. 414). In (Fig. 411) the index points to zero of the limb; the reading therefore is 0° 00′ 00″; again, suppose the index of the vernier to be in the situation shown in (Fig. 412), or a little beyond 40′, then the reading would be 40 degrees and some minutes, and these some minutes will be found by running the eye from the arrow along the divisions of the vernier to see which one of them coincides with a division on the limb, the 10th line is so found, hence the reading will be 40° *plus* 10′ or 40° 10′. In (Fig. 413) the index is beyond the 30′ division of the limb and the tenth line of the vernier coincides with a line of the limb, the reading therefore will be 40° 30′ *plus* 10′ or 40° 40′.

18. EXAMPLES IN READING HORIZONTAL VERNIERS. (Figs. 415, 416.)

1. Required the reading of the vernier in (Fig. 415), where the limb is divided to 10′ and the vernier to 10″. *Ans.*: 68° 25′ 40″.

2. Find the reading of the vernier in (Fig. 416), where the limb is divided to 20′ and the vernier to 20″. *Ans.*: 320° 46′ 20″.

29. VERTICAL LIMB GRADUATIONS. (Figs. 417, 418, 420.) The graduations of the vertical limbs of theodolites vary in description just as their horizontal limbs; those of Cooke and Son's 5″ are graduated to half degrees. In some theodolites, as transits, the vertical circles are complete, but in the generality of them they are only segments of the circle (*see* Fig. 417), these being found sufficient for all practical purposes. The method of numbering the vertical arcs differs from the horizontal; each of the two segments has its zero in the middle, from which, both towards the top and bottom, the degrees are reckoned. The segments are attached to the telescope and move with it. Every tenth degree on the limb is numbered, every fifth degree denoted by a dot, every degree by a long line, and every half degree or 30 minutes by a short line. There is a long horizontal arm which carries a vernier at either end for the purpose of reading the vertical angles on the two segments, to the nearest minute. These verniers are called A and B (or G and H), and are in every way similarly constructed to the horizontal verniers, except that a difference exists between their methods of numbering and that they are 180° apart. In both cases the numerals will be found marked at every tenth minute, but in the case of the vertical verniers there appear two sets of numbers (*see* Fig. 420), such that the sum of the two numerals at any given line is always equal to 30. The internal numbers or those nearest the transit axis, are intended for reading depressions on face left, and elevations on face right; and the external numbers or those farthest from the transit axis, are intended for reading elevations on face left, and depressions on face right. But the observer may readily decide for himself by this Rule, the truth of which is apparent. In any given position of the vertical circle, adopt that set of numbers on the vernier which *increase* in the direction *from* the zero on the limb *towards* the zero on the vernier. Thus let the instrument be pointing to an elevation (*see* Fig. 418). Then 0° on the limb at the object end of the telescope will be above the zero on the vernier B, and the direction from the former towards the latter is a downward direction; whence that set of numbers should be adopted which increases numerically in a downward progress, namely, the external numbers. So also with vernier A, the external numbers must be employed. Thus vernier B = 7° 13′ 00″ and vernier A = 7° 12′ 00″.

20. DIRECTION OF HORIZON. The arm carrying the vertical verniers is a fixture, and does not move with the telescope like the segments. The zeros or

indices of the vertical verniers A and B show the direction of the horizon when the instrument is level.

21. ELEVATION AND DEPRESSION. When the zero of the vertical limb to the object end of the telescope is *above* the index of its vernier, the angle formed by the direction of the telescope and the horizon is an angle of *elevation;* and when the zero is *below* the index, the angle formed is an angle of *depression.*

22. VERTICAL INDICES. (Fig. 418.) The indices of the vertical verniers point out the number of degrees of the angle on the limb, and are reckoned *from* the zero on the limb; in other ways they are read like the horizontal verniers; for instance, in the Figure the reading on the limb as denoted by the index of vernier B is 7° and that on the vernier 13′; therefore the angle is E. 7° 13′; and so vernier A is 7° *plus* 12′ or 7° 12′. In recording the readings of the two verniers, the degrees of A are only to be put down and the minutes and seconds of both A and B, the mean of which is taken for the correct angle.

23. EXAMPLES IN READING VERTICAL VERNIERS. (Figs. 418, 419, 421, 422.)

1. Read the verniers A and B on face left of the instrument, (i) in (Fig. 418), and (ii) in (Fig. 419).

Ans. : (i) A = E. 7° 12′ 00″; B = E. 7° 13′ 00″.
(ii) A = D. 8° 40′ 00″; B = D. 8° 39′ 00″.

2. Read A and B on the face right, (i) in (Fig. 421), and (ii) in (Fig. 422).

Ans.: (i) A = E. 4° 25′ 00″; B = E. 4° 23′ 00″.
(ii) A = D. 16° 55′ 00″; B = D. 16° 56′ 30″.

24. REMARKS ON CONTENTS OF THEODOLITE BOX. Suppose T. Cooke and Son's 5-inch Theodolite No. 112 is in our possession. The instrument is in a box 11″ × 9″ × 10″, packed for travelling. The first thing to be done by the Surveyor is to open the box and carefully examine the position of the contents, making a note of the side the *eye-end* and *object-end* of the telescope and the *face* of the instrument are, by writing with pencil on the inside of the box, E, O, and F on the respective sides of the eye-end, object-end, and face. By attending to this precaution accident to the instrument is avoided when putting it back into its case hereafter. Besides the instrument the following will usually be found in the box:

(1) *Triangular Plate* of the stand.
(2) *Magnetic Needle* and a *Mill-headed screw* for fixing it to the telescope.
(3) *Plummet.*
(4) *Direct Eye-piece.*
(5) Two *dark glasses.**
(6) Two *Turn-screws* and a *pin.*

Of these No. 4 is rarely used, for by reason of its four lenses, less light is refracted to the eye than with the *Inverting Eye-piece,* which contains only two lenses and is found attached to the telescope.

25. CAUTION AGAINST USING FORCE. Never employ violent force in the manipulation of a theodolite. When a screw works stiff, or a slide cannot be moved, when the theodolite itself will not settle into its appointed recesses for travelling, or the lid of the box does not close, and in every other similar case, be patient and let your movements be gentle. The great secret in handling an instrument is to learn to acquire a delicate touch of the hand.

26. THE STAND. (Fig. 450.) Every theodolite possesses a three-legged stand upon which it is placed for observation. Stands are either moveable or fixed, and about 5 feet in length. The stands of small theodolites are generally moveable, as the pattern in the Figure.

27. LIFTING THE INSTRUMENT ON TO ITS STAND. (Fig. 461.) Remove the triangular plate and fix it upon the head of the stand, pushing its upper plate *a* back with the thumb to receive the feet of the theodolite. Draw out the slides, if any, which hold the instrument down to its box, catch the "*Ys*" firmly by both hands and lift the theodolite on to its stand, inserting the heads of the foot-screws which are turned downwards into the holes of the lower triangular

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OBSERVATION.

* In all observations of the sun, care should be taken to cover the eye-piece with a dark glass of sufficient density to protect the eyes from injury.

plate, and push the upper triangular plate back to its original position with the thumb. The foot-screws have a flange (or shoulder) upon them, so that, when they rest upon the triangular plate fixed upon the head of the stand, the upper triangular plate locks over the flange, and being acted upon by a spring *b*, retains the whole instrument firmly upon the top of the stand and thus renders it secure against falling over from ordinary knocks. When placing the theodolite back in its box, see that the threads of the foot-screws are in their accustomed place, which the process of levelling has perhaps slightly displaced, otherwise the instrument may be found not to settle down in its proper position in the box.

28. VARIOUS PARTS OF A THEODOLITE NAMED. (Figs. 462, 463.) The Theodolite (*see* Fig. 462) rests upon three *Foot-screws*, F, which work in a *Tribrach*, T. The *Boss* of this tribrach is pierced with a *Female Axis*, in which the *Lower Vertical* (or *Azimuthal*) *Male Axis* works, and attached to the boss of the tribrach is an arm carrying the *Lower Horizontal Clamp*, C, and *Lower Horizontal Tangent Screw*, S. Next is seen another arm projecting from the boss of the *Upper Female Vertical Axis* which carries the *Upper Horizontal Clamp*, C′, and *Upper Horizontal Tangent Screw* (not seen in the Figure). This last mentioned axis encloses the *Upper Male Vertical Axis*. Then comes the *Horizontal Plate*, P, of the instrument, which bears the *Graduations* marked on silver, the silver circle being termed the *Limb*. The graduations again are read by means of three *Horizontal Verniers*, V, attached to a thin circular plate called the *Vernier Plate*, P′, and the three moveable *Horizontal Lenses*, L, above them. So, in succession, the *Azimuthal Level*, I, which is fixed to the vernier plate, is arrived at; from thence we come to the *Ys*, Y, within which the *Pivots* of the *Transit Axis* work. The *Cleats*, *c*, covering the pivots, prevent the latter from rising out of their Ys. Between the pivots lies the *Telescope*,* *t*, carrying with it two segments or *Arcs*, A, of the *Vertical Circle* denominated the *Face* of the instrument. The vertical arcs also bear their graduations marked on silver; the silver segments, as before, being termed the limb. These graduations are read by means of two *Vertical Verniers*, V′, on either end of a horizontal arm, upon which latter is fixed the *Vertical Level*, I′, the whole being fixed in position by means of a couple of antagonistic screws termed *Clips*, *c*′, which work against a *Shoulder* between them. As in the case of the azimuthal verniers, the vertical verniers are read by means of two *Vertical Lenses*, L′. Another arm projecting beyond vernier A carries a *Vertical Clamp* C″, and *Vertical Tangent Screw*, S″. In the telescope are the *Eye-piece*, † E, the *Object Glass*, O, and its *Cap*, *c*″, as also the large *Milled-head*, which carrying a *Pinion* and working in a *Rack* attached to the *Sliding Tube* of the telescope, affords the means of nice adjustment for *Focus*. On the telescope the *Magnetic Needle*, N, is fitted on. If the eye-piece and the tube within which it fits were removed, there would be exposed to view a *Diaphragm* (*see* Fig. 463) carrying three *Wires*, whereby a point in the *Field* of the telescope becomes defined. The diaphragm is held by means of four screws whose *Capstan-heads* appear round the sliding tube of the telescope. These four screws are termed the *Collimating Screws*, *s*.

OBSERVATIONS.

* The magnifying power of a telescope = the focal length of the object glass ÷ the equivalent focal length of the eye-piece. The equivalent focal length is twice the product of the focal lengths of the eye lens and field lens divided by their sum.

† The eye-piece contains two lenses, known as the eye lens and the field lens. Half the sum of these small lenses is the distance at which they are fixed from each other. In a *positive* eye-piece the focus is just beyond the field lens, and in a *negative* eye-piece the focus is between the two lenses. By the use of compound eye-pieces *spherical* and *chromatic* aberration are got rid of. The field lens or object glass of a *refracting* telescope forms the image of a distant object, and the eye lens or eye-piece, simple or compound, magnifies this image. In a *reflecting* telescope a concave mirror receives light from the distant object, and reflects it so that the rays converge to a focus and form an image which is received by the eye-piece. A terrestrial telescope consists of two such astronomical telescopes which give an inverted image, the latter inverting the inverted image of the former and so giving an upright image. The *equatorial* telescope is an ordinary telescope mounted in such a way that it can be easily directed to any part of the heavens. The *polar* axis is parallel at an angle equal to the latitude of the place. The telescope can be moved round the polar axis in a plane which is parallel to the earth's equator, and this motion is said to be motion in *right ascension*; it can also be moved up and down in a plane at right angles to the earth's equator, and this motion is called motion in *declination*. The *transit* telescope is mounted on a horizontal axis, so as to be capable of moving in the meridian only. It is used to determine the exact moment at which celestial bodies cross the meridian, that is, when they are in a true north or south position. It is also used for determining the declination of celestial objects, that is, how far in angular measures these bodies are from the celestial equator.

29. THE FOOT-SCREWS. The foot-screws are for the purpose of levelling the theodolite, and should admit of being turned easily by the hand; when this is not the case, the slit made vertically across the female screw should be opened as required, so as to keep the threads in close action with the foot-screw.

30. THE AXES. The axes should be cleaned with pure olive oil and a soft cloth, once a year—by an experienced hand.

31. THE LIMBS AND VERNIERS. The limbs and verniers should be brushed free of dust and grit, with the edge, not against it, by means of a soft camel's hair brush, every time before use; and once a year they should be very carefully brushed by an experienced hand, next washed with a little soap and water, and finally rubbed over with a cautiously prepared mixture of lamp-black (made by holding an oiled plate over the flame of a wick dipped in oil, the more smoke the better) and oil. In these processes the tip of the finger alone should be employed.

32. THE CLAMPS AND TANGENT SCREWS. The clamps and tangent screws give the instrument a slow motion—which is thus effected. The clamp is tightened, in the direction an ordinary screw is driven home, and the tangent screw turned by the thumb and forefinger to the left or right as required. The horizontal tangent screws give the telescope a horizontal motion, and the vertical tangent screw a vertical motion. The lower horizontal clamp and tangent screw are used in connection with the horizontal plate, the upper horizontal clamp and tangent screw in connection with the vernier plate, and the vertical clamp and tangent screw in connection with the vertical limb.

33. THE LENSES. The lenses have a spherical glass at either end and magnify an object. They are used to read the divisions on the verniers and limbs, and are to be adjusted with the screw or unscrew motion, as required, for the eye of each observer, since the sights of observers vary in power, some being weak and others strong.

34. THE AZIMUTHAL LEVEL. The azimuthal level is a hollow tube of glass fitted into a metal case which is fixed by two capstan-headed screws to the vernier plate. It is filled with spirits of wine, a little space with air being allowed to exist in the tube, which forms itself into a bubble. When this bubble is in the centre of its tube, the level is horizontal. If the bubble is to either end, that end to which it is, is the higher; just as may be seen in a full brandy bottle, if held horizontally, the bubble of air in it will come to the centre, and if raised to the side of the stopper the bubble will proceed in that direction. The glass of the level tube has divisions cut on it from the centre, to either side so as to guide the eye in bringing the bubble to the middle of the tube. The bubble contracts with heat and expands with cold, but this fact must not alarm the observer, as a very great change in the bubble's length does not mainfest itself all at once. The azimuthal level is used to level the instrument horizontally by means of the three foot-screws.

35. THE VERTICAL LEVEL. The vertical level is in every way similar to the azimuthal level, and therefore needs no description. It is used to level the instrument, by means of the clips, vertically, that is, to make the telescope when set to zero of the vertical verniers, parallel to the horizon.

36. THE PIVOTS AND YS. The pivots of the transit axis and the Ys should always be wiped clean before placing the former within the latter, for the interposition of grit must necessarily wear them away.

37. THE CAP. The cap of the object glass must always be both put on and taken off by the screw (not the unscrew) motion, otherwise the object glass is likely to be displaced in its cell.

38. THE TELESCOPE. The telescope is used for viewing objects to be observed, which it magnifies and makes appear as if they were situated quite near to the observer.

39. THE EYE-PIECE. There are two eye-pieces for the telescope; one, the inverted eye-piece, has two lenses, and the other, the direct eye-piece, has four. The latter, which shows objects direct, is usually found in the box, the former, which shows objects inverted, is fitted in the telescope and should be the one used, as, when seen through it, objects appear more distinct. If the object and eye glasses are very dim and clouded, and cannot be cleaned with external

wiping, select a dry day and clean them externally and internally with spirits of wine and the use of a soft handkerchief. It is, however, objectionable to unscrew the object glass, since doing so deranges the line of collimation; so also as to the eye-piece, since by removing it air and damp get inside the telescope and break the wires of the diaphragm.

40. THE COMPASS BOX. A compass box is fitted on to the telescope, which can be removed and put on at pleasure. It carries a magnetic needle and is for the purpose of determining the bearings of points; when not in use, the needle should always be thrown off its centre by means of the small lever provided for this purpose.

41. DIFFERENCE OF HYPOTENUSE AND BASE. On the reverse side of the vertical arcs are certain graduations marked on brass as *Difference of Hypotenuse and Base*, which are the number of links to be deducted from each chain, for various angles of inclination, in order to reduce distances measured on the ground rising or falling at these angles, to the corresponding horizontal distances; in other words, when the index of this graduation reads the number 2, it is signified that the hypotenusal length at that inclination *minus* its corresponding horizontal length, is equal to 2 links per chain.

42. DIVISION OF INSTRUMENT INTO TWO PARTS. The instrument may be considered to consist of two parts, the lower or horizontal portion from the foot-screws to the Ys and the upper or vertical portion from the Ys upwards.

43. THE ADJUSTMENTS. The adjustments of the theodolite are of two kinds, namely: *temporary* and *permanent*. The former are necessary every time the instrument is set up for use; the latter are only necessary when the instrument has newly come into the hands of the Surveyor, or when he has reason to think the instrument has been shaken by some means, and the adjustments so displaced.

44. THE TEMPORARY ADJUSTMENTS. The temporary adjustments are—

(1) *Centering.*
(2) *Obtaining distinct vision of Wires.*
(3) *Eliminating Parallax*, commonly called "finding focus."
(4) *Levelling*, that is, making the vertical axis perpendicular to the horizon.
(5) *Vertical Levelling*, that is, making the zero of altitude parallel to the horizon.
(6) *Setting to Zero.*

45. CENTERING. The theodolite being placed on its stand over the station from which the observations are to be taken, suspend the plummet by means of a thin cord from the hook at the bottom of the instrument till, by shifting the legs of the stand, the plummet hangs over the station mark, and the theodolite is at the same time roughly level.

46. OBTAINING DISTINCT VISION OF WIRES. Having removed the cap of the telescope, direct it to the sky, apply your eye to the eye-end and push in or draw out the eye-piece, always by the screw motion, until the wires are distinctly visible.

47. ELIMINATING PARALLAX. Turn the telescope in the direction of any object, look through the eye-end, and by means of the milled-head draw the telescope out till the object is nicely seen. Then direct the telescope to some well-defined object, as the top of a pagoda or corner of a house, about the same distance away as are the points of observation; clamp, that is, tighten, the upper horizontal clamp, and intersect the object by bringing it in the centre of the wires by means of the upper horizontal tangent screw. Now imagine the wires to be fixed, and move your head laterally as far as the field will admit, while your eye is intently watching the intersection of the object by the wires. According as the object appears to move, or otherwise, so it is said that there is, or is not, *parallax*. In the former case parallax must be destroyed by turning the milled-head inward or outward, till by continuing to shake the eye, as before, the object ceases to move.

48. LEVELLING. Place the horizontal level in a direction parallel to two of the foot-screws, and by their motion, turning them both inwards or both

outwards equally, according as the bubble is required to go to the right or left, bring it to the centre of its tube; then turn the instrument a quarter round, so as to bring the bubble over the third foot-screw; and turn it to the right or left, until the bubble also becomes stationary in the middle in this position. In performing this last operation the level will perhaps be thrown out on moving the instrument back to the first position, in which case it must again be levelled by means of the first two foot-screws, as before described. When the bubble remains stationary in the middle during a whole revolution, the instrument is said to be level.

49. VERTICAL LEVELLING. By means of the two clips bring the bubble of the vertical level to the centre of its tube, turning them outwards or inwards according as it is desired to make the bubble come to the left or right. The clips should always press tightly the shoulder against which they work.

50. SETTING TO ZERO. Setting to zero means that any given point shall read a given reading on the limb, generally 0°, or nearly so. For this purpose, first level the instrument, then set vernier A to the given reading, or 0°, by clamping the upper horizontal clamp and making the arrow head of the vernier A correspond exactly with the division at 360° (or zero) by means of the upper horizontal tangent screw; next unclamp the lower horizontal clamp and turn the telescope by the horizontal plate, until the given object is nearly on the wire. Now clamp the lower horizontal clamp and intersect with its tangent screw, and the zero will be set. In setting to zero, vernier A should always be made to read 2 or 3 minutes over 0°, so that in re-observing the point the reading may not be less than 360°. The point to which zero is set is called the *zero point.* Zero may be set to the magnetic north by having made vernier A read 0°, as before, and turning the telescope, to which the compass box is attached, by the horizontal plate, until the needle points to its own zero. Once the horizontal plate is set to zero point, it must remain untouched throughout the observations on that zero.

51. INSTRUMENT READY FOR OBSERVATION. After the temporary adjustments, as above described, have been completed, observations can be begun with the instrument.

52. THE PERMANENT ADJUSTMENTS. The permanent adjustments are—

(1) *Setting the Vernier Lenses.*
(2) *Collimating.*
(3) *Making the Vertical and Horizontal Wires respectively Vertical and Horizontal.*
(4) *Setting the Horizontal Level parallel to the Horizontal Plate,* to indicate the verticality of the vertical axis.
(5) *Setting the Vertical Level parallel to the zero of Altitude of the Vertical Arcs,* to indicate the horizontality of the line of collimation.

53. SETTING THE VERNIER LENSES. Raise or lower each lens in turn until distinct vision of the vernier is obtained.

54. COLLIMATING. The *line of collimation* is the imaginary straight line joining the intersection of the wires and the optical centre of the object glass. The *effect* of collimation error * is destroyed by observing vertical angles on both faces of the instrument, and taking the mean thereof. To collimate, *i.e.*, to remove the *presence* of collimation error, go through the temporary adjustments of the theodolite, next direct the telescope to some well-defined object, intersect it by the wires, and read vernier A, which is, let us suppose, 37° 46′ 30″; then lift the telescope out of its Ys and turn its verniers through a semi-circle, then change pivots, replacing the telescope, which should still point in the direction of the object, level the vertical level, direct the telescope once more to the object, intersect it by the wires, and read vernier A, which this time is, say, 37° 50′ 30″; the difference 4′ 00″ is the deviation, half of which must be corrected by the two horizontal collimating screws at the eye-end of the telescope, and the other half by the tangent screw—done thus :—Add half the deviation or 2′ to 37° 46′ 30″ = 37° 48′ 30″ (which is the same thing as taking the mean of the two readings 37° 46′ 30″

OBSERVATION.

* The *error of collimation* is the deviation of the optical axis of the telescope from perpendicularity to the line joining the pivots.

and 37° 50′ 30″), apply your eye at vernier A, and by means of the upper horizontal tangent screw set it to read that quantity; now apply your eye to the telescope and the object will be discovered to have gone left or right of the intersection: take the pin and alternately loosen one horizontal screw and tighten its opposite one, till the cross wires intersect the object as before, when the instrument will have been collimated. The process ought to be repeated till the adjustment is perfect.

55. MAKING THE VERTICAL AND HORIZONTAL WIRES RESPECTIVELY VERTICAL AND HORIZONTAL. First thoroughly level the vertical and transit axes. Then adjust by, say, the vertical wire, thus: intersect any small well-defined object, and see if it continues bisected along the wire when the telescope is moved in the vertical plane. If otherwise, make the diaphragm revolve by means of the 4 collimating screws until this condition be secured. Similarly an object bisected by the horizontal wire should continue bisected when the instrument is turned azimuthally. In more recent instruments this trouble is avoided, as the diaphragm is constructed to slide exactly horizontally along two even edges, and thus only two collimating screws will be seen, one to the left and the other to the right hand side.

56. SETTING THE HORIZONTAL LEVEL. Level the instrument carefully as explained in (Art. 48). Now turn the telescope so as to place the horizontal level parallel to two foot-screws, and by their motion bring the bubble to the centre of its run. Turn the telescope half round and if the bubble continues in the centre, the level is correct; but if not, it will show that the bubble tube is not parallel to the horizontal plate. To remove this error, the bubble must be brought to the middle, half by the foot-screws and half by the capstan-headed adjusting screws at the ends of the level. Reverse the level again end for end, thus bringing it to its first position: any deviation from the centre must be corrected, as before, half by the foot-screws and half by the adjusting screws aforesaid. Repeat the process until the level will bear reversing without displacing the bubble from the centre. During the above operation the level should occasionally be turned a quarter round, when, if the bubble leaves the centre, it should be brought back to it by the third foot-screw.

57. SETTING THE VERTICAL LEVEL. Level the horizontal and vertical levels thoroughly, direct the telescope to some well-defined object, intersect it and read off the vertical angle, which is, say, E. 0° 25′ 0″; next change face, level the vertical level, and re-observe the object which this time reads, say, E. 0° 30′ 00″. Take the mean of the two observations, which in this case would be E. 0° 27′ 30″, and set the vernier to read that quantity; then intersect the object, *by using the clips*, which procedure will necessarily have displaced the bubble. Now by means of the two adjusting screws at either end of the level tube bring the bubble to the centre, and the vertical level will be set. The process ought to be repeated once or more often till satisfactory.

58. CAUTION AS TO PERMANENT ADJUSTMENTS. With the exception of setting the vernier lenses, the permanent adjustments must not be attempted by an inexperienced hand or the instrument may become worse than useless.

59. STABILITY. The ground on which the theodolite stand is fixed should be firm; and when composed of straw and earth or sand, the way to remedy the evil is to drive three ordinary tent pegs nearly flush with the ground and rest the stand on them; this will secure the isolation required.

60. TO WIRE THE DIAPHRAGM. (Fig. 423.) The diaphrgam will be found nicked to receive one horizontal and two diagonal wires cutting each other in nearly a common point. It often happens that the wires break, and hence the necessity of a Surveyor being possessed of the knowledge of replacing them. Well, then, to wire the diaphragm, remove the four small screws which hold the eye-end of the telescope to the sliding tube. Draw out this end together with the eye-piece, and remove the diaphragm, keeping the screws by carefully, so that they may not hereafter change places. The wires employed should be either silk fibres or spider's lines; the former being the stronger of the two, are to be preferred. In wiring, clean the surface of the diaphragm and the nicks on it with spirits of wine, and lay it on some slightly raised surface as an upturned tumbler. Now

select *one* silk fibre, append a small ball of wax about half the size of a pea to each end, lift up the wire on a pair of compasses opened out a couple of inches, and near its points fit the fibre gently into two opposite nicks and remove the compasses, letting the balls of wax hang free to tighten the fibre. When the other two wires are similarly placed, fix them to the diaphragm with tiny drops of laudanum, or any other adhesive liquid. When dry cut off the ends with the wax balls, carefully replace the diaphragm in the telescope, so as not to hurt the wires and fix on the screws in their former places. Wiring should be done on a bright sunny day or else the wires will be found jagged and useless.

61. HOW TO CHANGE FACE. The face of the instrument *invariably stands over vernier A,* a condition which is *always* to be secured when replacing the pivots in their Ys. To *change face* signifies to turn the telescope 180° vertically, that is, on its transit axis. Suppose the instrument to stand at "*face left,*" which implies that, standing at the eye-end of the instrument, its face falls to the observer's left hand. Now let it be required to change face. The manipulation is as follows:—

Open the cleats and release the two clips, the vertical clamp being free to avoid strains from accidental jerks. Raise the telescope out of its Ys by catching the thicker part of either end with the forefinger and thumb. Hold it by its back arm, remembering that the object-end should never be held uppermost, or any dust within the telescope will fall on to the wires. Now unclamp the vertical clamp and turn the verniers 180°, whereby they will change places; clamp, change the telescope end for end in the hand by bringing the level upward. Catch the thicker part by the forefinger and thumb as before, replace it in its Ys, *keeping the face immediately over vernier A,* and make the released clamps pinch. The object and eye-ends of the telescope will thus have exchanged positions, and if, as before, the observer stand at the latter end, the face will now fall on his right hand. Before this process was gone through, the instrument was said to be at "*face left;*" it now reads "*face right.*" It is not possible to change face with the compass box fitted on to the telescope, since doing so would bring the box into contact with the nut of the vertical axis—hence it must be removed before a change of face is attempted.

62. METHOD OF OBSERVING HORIZONTAL ANGLES. (Figs. 452, 456.) Let it be required to observe the angle at the station S (*see* Fig. 452) between two objects A and B. Set up the instrument on its stand exactly over the point S by means of the plummet, and put it through its temporary adjustments. Now having the instrument at face left, clamp the lower horizontal clamp firmly with the horizontal plate in any position, or by first having set it to zero point, as may be required; next, direct the telescope to the object A, moving it till the cross-wires and object nearly coincide; then clamp the upper horizontal clamp and by its tangent screw make the intersection of the wires nicely bisect the object; now read off the degrees, minutes and seconds of vernier A and the minutes and seconds only of verniers B and C, and work out their mean for the reading of the object, thus:—

Vernier A = 127° 46′ 30″
„ B = 46 00
„ C = 47 30

Mean reading = 127 46 40

Again unclamp the upper horizontal clamp and turn the telescope to the object B, clamp it, bisect the object as before, and read off the verniers as before, which are as follows:—

Vernier A = 193° 04′ 00″
„ B = 05 00
„ C = 06 30

Mean reading = 193 05 10

Now take the difference of the two means for the required angle, thus: 193° 05′ 10″ — 127° 46′ 40″ = 65° 18′ 30″, the observed angle at S between A and B *on* *face left only*. Again, to observe the same angle on face right. Change face, and bring the telescope round to B, intersect and read as before:

Vernier A = $13^\circ\ 03'\ 30''$
„ B = $05\ 00$
„ C = $06\ 00$

Mean reading = $13\ 04\ 50$

Turn the telescope to A and intersect and read as above:

Vernier A = $307^\circ\ 46'\ 00''$
„ B = $45\ 30$
„ C = $47\ 00$

Mean reading = $307\ 46\ 10$

Now, take the difference of these two means for the required angle, thus:— $13^\circ\ 04'\ 50''$ ($360^\circ + 13^\circ\ 04'\ 50'' = 373^\circ\ 04'\ 50''$) $- 307^\circ\ 46'\ 10'' = 65^\circ\ 18'\ 40''$, the observed angle at S between A and B *on face right*. Finally take the mean of the angle observed on face left and face right for the true angle, which $= 65^\circ\ 18'\ 35''$. Angles should always be observed from left to right on one face and from right to left on the opposite face; thus in (Fig. 456), if it be required to observe the angles at S between A and B, B and C, C and D, and D and E; on one of the faces, as face left, begin with intersecting A, then B, C, D and E, in succession. Angles are sometimes repeated when great accuracy is wanted, which is by going through the process of observing a second, third and fourth time, as required, and then for the true angle, extracting the means of all the observations of each separate angle. To obtain an angle always subtract the reading of the left hand object, which is the lesser, from the reading of the right hand object, which is the greater; when the reading of the latter is less, add 360° to it to make it greater, as done in this very article to $13^\circ\ 04'\ 50''$. The observer must always conceive himself standing *at* the station of observation, with his face *towards* the angle required to be measured; for instance if at S (*see* Fig. 452) the smaller angle ASB was to be measured, then A would lie to his left and B to his right, and consequently the reading of A would be subtracted from the reading of B for the required angle. Again, if the larger angle ASB was to be measured, then the observer would have to turn his face *towards* the required angle, when B would lie to his left and A to this right, and by the rule, the reading of B must be taken from the reading of A for the angle.

Thus let A read 20° and B read 70°,

Reading of B = 70°	Reading of A = 380° (= $360^\circ + 20^\circ$)
„ „ A = 20	„ „ B = 70
Smaller angle = 50°	Larger angle = 310°

Always recollect that in measuring azimuthal angles the vertical clamp should be free.

63. HORIZONTAL ANGLE BOOK. The field-book in which the horizontal angles as observed in the field are recorded is termed the *Horizontal Angle Book*, a specimen of which is given further on. It is usually 10 inches broad by 8 inches long, opening out as the field-book for a chain survey, the entries being made from the top to the bottom begun at the first page. At each station of observation, before commencing work, is written in, at the heading, (1) the name of the station, (2) date of observation, (3) the number and size of the instrument employed together with the maker's name, and (4) a brief description of the station to enable it to be got at easily in the event of future reference having to be made to it. The 1st column, "Objects," is to write in the names of the objects observed to; in the second column, "Face," *L* or *R* is entered, denoting the left or right face of the instrument on which the observations were taken; the 3rd, 4th and 5th columns, "Verniers," are for the readings of the verniers A, B and C to be respectively recorded; the 6th column, "Means," is for the mean of the three vernier readings; the 7th column, "Angles," is for the entry of the deduced angles; and the 8th column, "Remarks," is intended for any remarks the observer

may think necessary to enter. Means and angles are always worked out on the spot immediately after the observations are taken, so that any glaring errors may at once be checked and rectified. After concluding the observations at a station, some space ought to be left before commencing the entries at the next station. This space is intended for the drawing up of an "Abstract of angles" taken at every station, for convenience of reference and subsequent calculation, in which the general mean of each successive angle is entered. The form for the abstract of angles has to be ruled out by the Surveyor according to the pattern in the specimen of a horizontal angle book below given:—

Horizontal Angles taken at *Kadobia Village* Station on the *20th October 1880*, with *Cooke and Son's 7″* Theodolite No. *112*.

DESCRIPTION OF STATION: *The station is situated in the centre of a paddy-field 86 feet from the bank of the Gyne river eastward, and 52 feet north of a tamarind tree. The village of Kadobia is about 500 yards south of the station.*

OBSERVER: *A. R.*—

Objects.	Face.	VERNIERS A.	VERNIERS B.	VERNIERS C.	Means.	Angles.	Remarks.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″	
A Flag	L	0 02 30	03 00	01 30	0 02 20		
B Peepul tree ...	...	67 53 00	53 30	52 00	67 52 50	67 50 30	*Near Pagoda.*
C Flag	...	193 17 00	16 30	17 30	193 17 00	125 24 10	
C Flag	R	13 16 00	17 00	16 00	13 16 20	125 24 20	
B Peepul tree ...	...	247 51 00	52 30	52 30	247 52 00	67 49 40	*Near Pagoda.*
A Flag	...	180 02 00	03 30	01 30	180 02 20		

Abstract of Angles.

Objects.	Angles.
	° ′ ″
A Flag and *B Peepul tree*	67 50 5
B Peepul tree and *C Flag*	125 24 15

64. METHOD OF OBSERVING VERTICAL ANGLES. Set up the theodolite over the station of observation, go through the temporary adjustments, having both the horizontal and vertical levels truly level. Then direct the telescope to the object whose angle of elevation or depression is sought, intersecting it with the horizontal wire by clamping the vertical clamp and using the vertical tangent screw; next read off the vertical verniers A and B and obtain their means for the vertical angle; this is done on face left. Again loosen the vertical clamp and change face, re-level the upper level, and intersectthe object as before; read off the verniers A and B, and take their mean for the vertical angle on face right. Then take the mean of both face left and face right for the true vertical angle. Recollect that in entering vertical angles, E and D are made to represent angles of elevation and depression respectively. To illustrate the above method of observing:—

Face left.	*Face right.*
Vernier A = E. 2° 18′ 30″	Vernier A = E. 2° 16′ 00″
„ B = E. 20 00	„ B = E. 15 30
Mean of verniers = E. 2 19 15	Mean of verniers = E. 2 15 45

∴ Mean of both faces = E. 2° 17′ 30″.

Always bear in mind that in measuring vertical angles the upper horizontal clamp should be free.

65. VERTICAL ANGLE BOOK. The vertical angles as observed in the field are recorded in a book similar in size to the horizontal angle book, called the

Vertical Angle Book. At each station of observation, before commencing operations, is written in at the heading—

(1) The name of the station,
(2) Date of observation, and
(3) The number and size of the instrument employed together with the maker's name.

The 1st column, "Objects," is to write in the names of the objects which are observed.

In the 2nd column, "*Face*," L or R, is entered, signifying the left or right face of the instrument on which the observations were taken.

The 3rd and 4th columns, "Verniers," are for the readings of the verniers A and B to be respectively recorded.

The 5th column, "Means of one Face," is for the mean of the two vernier readings.

In the 6th column, "Means of both Face," the mean of face left and face right is to be written.

In the 7th and 8th columns, "Heights of eye and object," the height of the instrument and the height of the flag or other object observed to from the ground is recorded.

And the 8th column, "Remarks," is intended for the entry of any remarks the observer may think necessary. Before each of the vernier readings and in the means, the letters E and D must be entered, showing whether the angle is an elevation or depression. In vertical observations, there is no difference of verniers taken to get the angle, as is the case with horizontal observations, but the means are themselves the angles of elevation or depression. The means are always to be worked out on the spot, so that any grave errors may be there and then checked and rectified. After the observations at a station are concluded, an "Abstract of Angles" has to be drawn up, as in the horizontal angle book. In extracting the mean of two vernier readings, if one is an elevation and the other a depression, which usually happens when the station of observation and that observed to are nearly alike in height, take the *difference* of the two readings and halve it for the required mean, which will be an elevation or depression according as which was the greater reading; the same procedure is adopted when a mean of the mean of both faces of an elevation and depression is obtained. The following is a specimen of a vertical angle book:—

Vertical Angles taken at *Kamalataung Hill* Station on the *28th October 1880*, with *Cooke and Son's 7″* Theodolite No. *112*.

OBSERVER : *P.N.*—

Objects.	Face.	VERNIERS		MEANS.		HEIGHT OF		Remarks
		A.	B.	One Face.	Both Faces	Eye.	Object.	
		° ′ ″	′ ″	° ′ ″	° ′ ″	Feet.	Feet.	
A Flag ...	L	E. 3 15 30	E. 14 30	E. 3 15 0	E. 3 15 23	5	0	*Bottom.*
" " ...	R	E. 3 16 30	E. 15 00	E. 3 15 45				
B Peepul tree ...	R	D. 5 48 00	D. 47 00	D. 5 47 30	D. 5 48 23	5	0	*Bottom.*
" " ...	L	D. 5 49 30	D. 49 00	D. 5 49 15				
C Staff ...	L	E 0 02 30	D. 0 30	E. 0 01 00	E. 0 00 08	5	11	*Top.*
" " ...	R	D. 0 00 30	D. 1 00	D. 0 00 45				

Abstract of Angles.

Objects.	Angles.	HEIGHT OF	
		Eye.	Object.
	° ′ ″	Feet.	Feet.
A Flag	E. 3 15 23	5	0
B Peepul tree	D. 5 48 23	5	0
C Staff	E. 0 00 08	5	11

66. TIME FOR OBSERVING. The best time for observing horizontal angles is during the cool of the morning and evening, from about sunrise to 10 A.M. and from 4 P.M. to sunset; and for vertical angles between 1 and 3 P.M., when there is least refraction, that is to say, during those hours when the object appears in its real position and does not rise from it as at other times of the day. For ordinary work the above rule need not be strictly adhered to.

67. THEODOLITE SURVEYING. The theodolite is used where great accuracy is an object and a large tract of country to be surveyed. Running a series of minor triangles through a country or covering it with a net work of small triangles of 1 or more mile sides, as necessary, is termed *triangulating*; simply surveying roads, rivers, boundaries and such like, proceeding from one point to another by a circuitous route, measuring along lines of various lengths and at different bearings, is termed *traversing*. While triangulating or traversing, any objects of importance may be observed to and fixed from two or more stations, such as pagodas, conspicuous trees, temples, etc. Afterwards the interior detail of a theodolite survey may be filled in by means of a plane-table or a chain survey, as explained in the chapters on those heads.

68. BASE LINE. (Fig. 453.) The first thing to be considered in triangulating is the careful measurement of what is termed a *Base Line*, such as AB, for example, in Fig. 453, to give the exact distance apart of two stations from which a series may be run, for it is clear if in a triangle one side and its three angles are known, the remaining two sides can be calculated, and having now got these two sides, other triangles can similarly be built on them and their sides calculated, and so on to any extent.* A nice level piece of ground should be chosen for the purpose of measuring a base line, which should be about 1,000 feet long for triangles of about 1 mile sides. The chain must be compared with a standard bar before commencing operations. Now set up the theodolite at one end of the base line, send a staff with the vane set to the height of the instrument in the direction of the other end of the base line, where a change of inclination takes place in the ground, and observe the angle of elevation or depression; then carefully measure the distance both forwards and backwards between these two points. Again remove the theodolite to the place where the staff was, sending the staff forward in the direction of the other end of the base line to where another change in the inclination of the ground occurs, and measure the vertical angle and distance as before, not omitting to observe the *back* vertical angle to the first end of the base line as a check. Proceed in this manner until the other end of the base line is arrived at. The following form shows the method of entering the above measurements:—

Station.	Chain Measurements.		Angles of inclination.			Remarks.	
	Forward and back.	Means.	Forward and back. A.	Forward and back. B.	Means.		
	Feet.	Feet.	° ′ ″	′ ″	° ′ ″		Feet.
1	206·84 206·90	206·87	E. 0 03 00 D. 0 03 30	E. 3 30 D. 4 30	E. 0 03 38	East end of base.	Length of chain before measurement of base = 99·83 Length of chain after measurement of base = 100·09 Mean = 100·06
2	427·28 427·11	427·20	E. 4 07 00 D 4 08 00	E. 8 00 D. 8 30	E. 4 07 53		
3	183·57 183·55	183·56	D. 1 01 30 E. 1 00 00	D. 0 30 E. 0 00	D. 1 00 30		
4	249·98 250·14	250·06	E. 0 26 00 D. 0 27 30	B. 25 30 D. 26 00	E 0 26 15	West end of base.	

OBSERVATION.

* The degree of accuracy required in a base line is determined by the ratio its length bears to the extent of the survey, and by the least amount of error which is acceptable in the extent of that survey. In geodetic operations, for instance, which are conducted on the principles of spherical trigonometry, the base line is measured with scientific accuracy by means of what is known as "compensation bars"; but in ordinary surveying, such as is carried on by the rules of plane trigonometry very much simpler and less expensive methods are of course adopted.

The following is the form adopted for the calculation of the horizontal distance of the base line, deduced from the [illegible]:

Stations.	Measured distances, in feet.	Inclination of ground.	Logarithmic computations.		Reduced horizontal distances, in feet.	Remarks.
		° ′ ″				
1	206·87	E. 0 03 38	L. cos. L. [illegible] Log. [illegible]	10·000000 [illegible] [illegible]	206·87	Length of [illegible] Base Line [illegible] Cap. IX, Art. 7[illegible]
2	427·20	E. 4 07 53	L. cos. Log. Log. [illegible]	9·998870 [illegible] [illegible]	[illegible]·09	
3	183·56	D. 1 00 30	L. cos. Log. Log. [illegible]	[illegible] [illegible] [illegible]	[illegible]	
4	250·06	E. 0 26 15	L. cos. Log. Log. [illegible]	[illegible] [illegible] [illegible]	[illegible]	
					[illegible]	

69. TRIANGULATING. (Fig. 457.) After the base line measurements have been completed, proceed with the triangulation, measuring the angles at every station, and fixing by intersection from two or more of them any conspicuous objects that may be met with; for instance in (Fig. 457) set up the theodolite at A and measure the angle CAB; next go to B and measure the angles ABC, CBD, taking an intersection to the pagoda G; again move to C and measure the three angles ECD, DCB, BCA; then go to D and measure the three angles BDC, CDE, EDF, taking an intersection to the pagoda G; then go to E and measure the two angles FED, DEC; and lastly go to F and measure the angle DFE, also taking an intersection to the pagoda G.

70. WELL-FORMED TRIANGLES. The nearer a triangle approaches the equilateral, that is, the closer each of its angles is to 60°, the better. Properly speaking the magnitude of every angle ought to be limited to between 30° and 90°; for intersections of solitary objects [illegible] need not be strictly adhered to, but it must always be recollected that the nearer the angles are to 60° the more reliable will be the result of the observation.

71. SIGNALS. The best description of signal is a bamboo about 16 feet long with a red and white flag on the top. It is a good plan to intersect signals as near the ground as possible.

72. REDUCTION OF ANGLES OF A TRIANGLE. In some triangles only two angles are observed, and in others all three. In the former case the two angles are added together, and the sum deducted from 180° (or π), for the third or supplemental angle, since the three angles of any triangle are together equal to two right angles or 180°. In the latter case, the sum of the three angles is compared with 180° (or π), and if any difference exist between the two sums, it is apportioned equally amongst the three angles, and the angles so corrected are employed in the computation of the triangles.

73. To find the sides *a* and *b* and also the area of any triangle ABC, having given its three angles A, B, C, and a side *c*.*† (Fig. 132.)

OBSERVATIONS.

* For the computation of terrestrial latitudes, longitudes and azimuths of the stations of a triangulation, *see* Table LXX (A), with the Explanation [illegible] "Calculating Tables," [illegible] edition.

† In ordinary triangulation spherical excess is not considered, but in geodetic triangles of the highest type, the correction for sphericity has to be allowed for; by the rule spherical excess in seconds = area of △ $\times \frac{\text{cosec. } 1''}{2r^2}$, where r is the mean radius of the earth in the same terms as the sides, or $e = bc.\ \text{sin. A} \times \frac{\text{cosec. } 1''}{2r^2}$ (i) or $\frac{a^2 \text{ sin. B. sin C}}{\text{sin. A}} \times \frac{\text{cosec. } 1''}{2r^2}$ (ii). Spherical excess is distinct from errors due to observation. The triangle is first solved by adjusting the total error, as observed, equally among the three angles, reducing their sum to 180°, and calculating the sides, etc. Thus, for the triangle of which the measurements are given in Example 2 of Art. 71, *sequitur*, assuming the side *c* at [illegible] feet whereby the sides *a* and *b* would be respectively 110,943 feet and 101,[illegible] feet, the spherical excess would be computed as follows:

Rule: *For either of the other sides*, multiply together the given side, the cosecant of the angle opposed to it, and the sine of the angle opposite the side required; *for the area*, multiply together the two found sides, and the sine of the angle included by them, and take half their product. In other words:

$$a = c \times \text{cosec. C} \times \text{sin. A},$$
$$b = c \times \text{cosec. C} \times \text{sin. B},$$
$$\text{area} = \tfrac{1}{2}\, ab.\ \text{sin. C}.$$

Example: The angles A, B, C, of a triangleABC, are respectively 13° 27′, 85° 48′, 80° 45′, and the side c is 1,626 feet; find the remaining sides a and b, also the area.

* From the table of the Logarithms of numbers (*see* Table XXXI, "Calculating Tables," 5th edition) find the logarithms of the sides a, b, and c; and from the table of Logarithmic sines (*see* Table XXXII, "Calculating Tables," 5th edition) find the logarithms of the cosecant of the angle A and of the sine of the angles A, B, and C. Then proceed to work out the Rule above given by logarithms; thus: —

$$a = c \times \text{cosec. C} \times \text{sin. A}$$
$$\therefore \log. a = \log. c + \text{Log. cosec. C} - 10 + \text{Log. sin. A} - 10$$
$$= \log.\ 1{,}626 + \text{Log. cosec. } 80^\circ\ 45' + \text{Log. sin. } 13^\circ\ 27' - 20$$
$$= 3{\cdot}211120 + 10{\cdot}005684 + 9{\cdot}366604 - 20$$
$$= 2{\cdot}583408$$
$$\therefore a = 383{\cdot}2 \text{ ft. (the No. corresponding to the log. } 2{\cdot}583408\text{, from the Tables)};$$

$$\text{again } b = c \times \text{cosec. C} \times \text{sin. B}$$
$$\therefore \log. b = \log. c + \text{Log. cosec. C} - 10 + \text{Log. sin. B} - 10$$
$$= \log.\ 1626 + \text{Log. cosec. } 80^\circ\ 45' + \text{Log. sin. } 85^\circ 48' - 20$$
$$= 3{\cdot}211120 + 10{\cdot}005684 + 9{\cdot}998832 - 20$$
$$= 3{\cdot}215636$$
$$\therefore b = 1{,}643{\cdot}0 \text{ ft. (the No. corresponding to the log. } 3{\cdot}215636\text{, from the Tables)};$$

$$\text{also } 2 \text{ area} = ab.\ \text{sin. C}$$
$$\therefore \log.\ 2 \text{ area} = \log. a + \log. b + \text{Log. sin. C} - 10$$
$$= 2{\cdot}583408 + 3{\cdot}215636 + 9{\cdot}994316 - 10$$
$$= 5{\cdot}793360$$
$$\therefore 2 \text{ area} = 621{,}384 \text{ (the No. corresponding to the log. } 5{\cdot}793360\text{, from the Tables)},$$
$$\text{and } \therefore \text{area} = 310{,}602 \text{ sq. ft.}$$

† To get acres, *add* 5·360912 (the complement of the constant log. for feet,

OBSERVATIONS.

Constant log. $\left(\dfrac{\text{cosec. } 1''}{2r^2}\right)$ = Log cosec. 1″ + A. C. log 2 + 2 × log r = 5·314425 + $\bar{1}$·698970 + 2 × $\bar{8}$·680100 = 0·373595.

(i).

Log. b … … = 5·005 584
Log. c … … = 5·004 706
Log. sin. A … … = $\bar{1}$·043 714
Const. log … … = 0·373 595

∴ e = 2″·62; log. e … = 0·417 599

(ii).

Log. a × 2 … .. = 0·135 946
Log sin. B … … = $\bar{1}$·831 325
Log. sin. C … … = $\bar{1}$·970 447
Log. cosec. A … … = 0·056 286
Const. log. … … = 0·373 595

∴ e = 2″·62; log. e … = 0·417 599

---

* The student must be familiar with a little Trigonometry, as explained in (Chap. III), and with Logarithms, as explained in (Chap. IV), or he will be unable to follow the working of the above example. The knowledge of both is essential to every surveyor, as with their aid work is greatly facilitated. To obtain the areas of ordinary fields, where such can be got by the simple rules of Mensuration or by plotting the observed angles geometrically on paper with a protractor and applying the tale-square thereon, logarithms need not be employed. It is only where the survey is any-what extensive, or the area is required correctly and for which purpose the angles have been observed with a theodolite, that logarithms become essential.

† As the number corresponding to the logarithm cannot be carried with accuracy to more places than the logarithm itself, it is well to obtain the area in acres or other large unit of measurement; and since 43,560 square feet = 1 acre, and 4·639088 is the log. of 43,560, from the Tables, and to divide by a quantity is the same thing as *subtracting* its logarithm or *adding* its log.'s complement, if then 5·360912 the complement of 4·639088 (termed the "*constant log*") is *added* to 5·793360, and the corresponding number of the result 1·154272 be got, this corresponding number will be double the area *in acres*. It is only where the figures are large that such a plan should be followed, or where all the areas are taken in acres and decimals. If the measurements of the sides were not in feet but in links, then the *constant log*. would change to 5·000000, as that is the logarithm of 100,000, the number of square links that go to an acre, and its complement would also be 5·000000; and so on according to the standard of measurement employed by the surveyor.

4·639088) to 5·793360, and find the number corresponding to the result for double the area, as before.

log. 2 area = 5·793360 + 5·360912
= 1·154272

∴ 2 area = 14·265 (the No. corresponding to the log. 1·154272, from the Tables),

and ∴ area = 7·133 acres.*

74. FORM FOR CALCULATION OF TRIANGLES.† (Figs. 131, 132.) (The three angles and one side ‡ being known.)

Example 1. Take the Example in (Art. 73) (*see* Fig. 132).

Stations.		Observed angles.	Corrections.	Reduced angles for calculation.	Logarithmic computation.			Deduced sides, in feet.		Area computation.	
		° ′ ″	″	° ′ ″		3·211120	....	1,626·0	AB		
C	...	80 45 00	00	80 45 00	L. cosec.	10·005684	.....	...	..	L. sin. C ...	9·994316
A	...	13 27 00	00	13 27 00	L. sin.	9·366604	2·583408	383·2	BC	log. BC ...	2·583408
B	...	85 48 00	00	85 48 00	L. sin.	9·998832	3·215636	1,643·0	AC	log. AC ..	3·215636
										co. con. log.§	5·360912
		180 00 00	00	180 00 00							

∴ area = 7·133 acres 2 area = 14·265 = 1·154272

Example 2. Find the areas and sides of the successive triangles ABC, ACD, and DCE (*see* Fig. 131) in the form for calculation given above, having the following measured data to compute with :—

angle CAB = 61° 27′ 36″, angle ACD = 112° 14′ 03″,
„ ABC = 49 32 53, „ CDA = 38 45 32,
„ BCA = 69 00 24, „ EDC = 79 49 51,
„ DAC = 28 59 58, „ DCE = 85 38 00,
and the base AB = 12,436·7 links.

Stations.		Observed angles.	Corrections.	Reduced angles for calculation.	Logarithmic computation.			Deduced sides, in links.		Area computation.	
		° ′ ″	″	° ′ ″		4·094706	....	12,436·7	AB		
C	...	69 00 24	− 18	69 00 06	L. cosec.	10·029843	.....	..	...	L. sin. C ...	9·970157
A	...	61 27 35	− 18	61 27 18	L. sin.	9·943714	4·067973	11,694·3	BC	log. BC ...	4·067973
B	...	49 32 53	− 17	49 32 36	L. sin.	9·881325	4·005584	10,129·4	AC	log. AC ..	4·005584
										co. con. log.§	5·000000
		180 00 53	− 53	180 00 00							

∴ area = 552·949 acres. 2 area = 1,105·897 = 3·043714‖

OBSERVATIONS.

* The above example can also be worked by obtaining from a Table of Natural Sines (found in most Log. Books), the natural sines of the angles A, B, and C, and proceeding with the simple multiplication as required by the rule; but as the method almost invariably involves much labour and time, it has been deemed advisable not to introduce the system here, nor therefore has a Table of Natural Sines been introduced into the "Calculating Tables" ([illegible]th edition).

† AB is the base of the triangle. The system of arrangement set out in this form should invariably be followed, otherwise errors will be bound to creep in.

‡ If this is in miles, to find the area in acres: Since there are 640 acres in one square mile, the area must be multiplied by 640 to reduce it to acres; in other words, to the log. of the area in square miles add the constant log 2·806180, which is the log. of 640, for the log. of the area in acres.

§ When the measurements are in { feet, co. con. log. = 5·360912.
{ links, „ „ „ = 5·000000.

‖ The reason for rejecting 20 from the index, is because the *tabular* Log. sine of C is taken, which is too much by 10 (*see* Chap. IV, Art. 15); and because the complement of the constant log. 4·639088 is $\bar{5}$·360912, to which 10 is added making it 5·360912. A total of 20 therefore has to be substracted from the sum in the area computation. This is done for convenience sake, and to avoid confusion with regard to negative indices.

Stations.	[illegible]	Log. of the computation.			Deduced sides, in links.		Area computation.	
			4[illegible]	....	10,12[illegible]4	AC		
D		L. [illegible]	[illegible]	.....	.....	...	L. sin. D ..	9·706623
A		L. [illegible]	[illegible]	[illegible]1·54	7,[illegible]4[illegible]3	CD	log. CD ...	3·894554
C		[illegible]	[illegible]	[illegible]3[illegible]8	1[illegible],[illegible]7[illegible]	AD	log. AD ...	4·1753[illegible]8
							co. con. log.	5·000000
					2 area = 733·405 =			2·[illegible]66[illegible]80
			[illegible]	....	7,[illegible]4·3	DC		
E		L. [illegible]	[illegible]	.....	......	..	L. sin. E ...	9·[illegible]9[illegible]643
D		[illegible]	[illegible]	[illegible]4[illegible]3 29	[illegible]7,63·1	CE	log. CE ...	4·880298
C		[illegible]	[illegible]7	4·[illegible]44	31,163·4	DE	log. DE ...	4·4[illegible]3644
							co. con. log.*	5·000000
					2 area = 2,4[illegible]6·1[illegible]8 =			3·3[illegible]1321

75. TO F[illegible]ND [illegible] SID[illegible] [illegible] THE [illegible] B, C, AND THE AREA OF THE TRIANGLE ABC, HAVING GIVEN THE SIDES *b*, *c*, AND THE INCLUDED ANGLE A. (Fig. 13[illegible])

Rule: Multiply [illegible] difference of the two given sides, and the cotangent of half the [illegible] their product by the sum of the two given sides for the tangent of half the [illegible] of the two required angles; then half the difference of [illegible] added to half their sum will give the greater of them, and deducted from half their sum will give the lesser of them. In other words:

$$\tan.\frac{B-C}{2}=\frac{b-c}{b+c}.\cot.\frac{A}{2};$$

$$\text{then } \frac{B+C}{2}+\frac{B-C}{2}=B,$$

$$\text{and } \frac{B+C}{2}-\frac{B-C}{2}=C.$$

Next, knowing the angles [illegible] B, C, and the sides *b* and *c*, proceed as in (Art. 73), to [illegible] the area of the triangle.

Example: The [illegible] *b* and *c* [illegible] ABC are respectively 1,643·0, and 383·2 feet, and [illegible] 80° 45′; find the side *a*, the angles B and C, and the [illegible].

From the [illegible] of [illegible] (*see* Table XXXI, "Calculating Tables," 5th [illegible]) find the [illegible] of the difference of the sides *b* and *c* and also that of [illegible] of Logarithmic sines (*see* Table XXXII, "Calculating Tables," [illegible]) find the logarithm of the cotangent of half the angle A. Then proceed to work out the Rule above by logarithms; thus:—

$$\tan.\frac{B-C}{2}=\frac{b-c}{b+c}.\cot.\frac{A}{2},$$

$$\therefore \text{Log. tan.}\frac{B-C}{2}=10+\log.(b-c)+\text{Log. cot.}\frac{A}{2}-10-\log.(b+c)$$

$$=\log.(b-c)+\text{Log. cot.}\frac{A}{2}+\text{co. log.}(b+c)$$

$$=\log.(1{,}643{\cdot}0-383{\cdot}2)+\text{Log. cot.}\frac{80^\circ\ 45'}{2}+\text{co. log.}(1{,}643{\cdot}0+383{\cdot}2)$$

$$=\log.1{,}259{\cdot}8+\text{Log. cot.}\ 40^\circ\ 22'\ 30''+\text{co. log.}\ 2{,}026{\cdot}2.$$

$$=3{\cdot}100303+10{\cdot}070420+6{\cdot}693318$$

$$=9{\cdot}864041$$

$$=\text{Log. tan.}\ 36^\circ\ 10'\ [illegible]9''$$

OBSERVATION.

---

* When the measurements are in { feet, co. con. log. = 5·360912.
links, „ „ „ = 5·000000.

$$\therefore \frac{B-C}{2} = 36^\circ\ 10'\ 29'';$$

$$\text{but } \frac{B+C}{2} = \frac{180^\circ - 80^\circ\ 45'}{2} = \frac{99^\circ\ 15'}{2} = 49^\circ\ 37'\ 30'',$$

$$\therefore B = 49^\circ\ 37'\ 30'' + 36^\circ\ 10'\ 29'' = 85^\circ\ 47'\ 59'',$$

$$\text{and } C = 49^\circ\ 37'\ 30'' - 36^\circ\ 10'\ 29'' = 13^\circ\ 27'\ 01''.*$$

Having now found the angles B and C, and knowing the sides b and c, the sides a and the area of the triangle can be found, as in (Art. 73), as follows :—

c = 1,626·0 feet ; area = 7·133 acres.

76. FORM FOR CALCULATION OF TRIANGLES.† (Figs. 132, 112.) (Two sides ‡ and the included angle being known.)

Example 1. Take the Example in (Art. 75). (*See* Fig. 132.)

Stations.		Data.	Formula. $\text{Tan.} \frac{B-C}{2} = \frac{b-c}{b+c} \cdot \text{cot.} \frac{A}{2}$.		Logarithmic computation.		Value of $\frac{B-C}{2}$.	Values of $\frac{B+C}{2}$.	Values of B and C.
							° ′ ″	° ′ ″	° ′ ″
C	....	b = 1,645·0	$b-c$ ... =	1,259·8	log.	3·100303	......	180 00 00	
A	......	c = 383·2	$b+c$ ... =	2,026·2	co. l. g.	6·693318	......	80 45 00	B = 85 47 59
B	......	A = 80° 45′ 00″	$\frac{A}{2}$ ... =	40° 22′ 30″	L. cot. ..	10·070420	.. ..	99 15 00	C = 13 27 01
					L. tan. ...	9·864041	36 10 29	49 37 30	

Stations.		Deduced angles.	Correction.	Angles for calculation.	Logarithmic computation.			Deduced sides, in feet.		Area computation.	
		° ′ ″	″	° ′ ″		2·583426	... ..	383·2	c		
C	......	13 27 01	00	13 27 01	L. cosec.	10·633387	... ..	......	...	L. sin. C ...	9·366613
A	......	80 45 00	00	80 45 00	L. sin.	9·994316	3·211129	1,626·0	a	log. a ...	3·211129
B	......	85 47 59	00	85 47 59	L. sin.	9·998832	3·215645	1,643·0	b	log. b ...	3·215645
		180 00 00	00	180 00 00						co. con. log. §	5·360912
					∴ area = 7·133 acres.					2 area = 14·266 =	1·154299

Example 2. Find the areas and sides of the series of triangles XVW, YXW and ZXY (*see* Fig. 112), in the calculation form given above, the following, measurements being known :—

base VW = 351·4 links,
side XV = 432·6 „
angle XVW = 36° 40′ 00″,
side WY = 277·7 links,
angle XWY = 40° 32′ 1″,
side YZ = 578·9 links,
angle XYZ = 83° 00′ 17″.

OBSERVATIONS.

* The apparent difference in excess of 1″ in the angle C, and 1″ deficiency in the angle B, when compared with the same angles given in the example to (Art. 73), is due to the logarithms not being carried to a greater number of places.

† In this Form b stands for the greater side.
B „ „ angle.
c „ lesser side
C „ „ angle.
a „ required side.
A „ „ angle.
and AB is the base of the triangle.

The system of arrangement set out in this form should invariably be followed, otherwise errors will be bound to creep in.

‡ If these are in miles, to find the area in acres : Since there are 640 acres in 1 square mile, the area must be multiplied by 640 to reduce it to acres ; in other words, to the log. of the area in square miles, add the constant log. 2·806180 which is the log. of 640, for the log. of the area in acres.

§ When the measurements are in {feet, co. con. log. = 5·360912.
{links, „ „ „ = 5·000000.

Stations.	Data.	FORMULA. Tan. $\frac{B-C}{2}=\frac{b-c}{b+c}$. cot. $\frac{A}{2}$.	Logarithmic computation.		Value of $\frac{B-C}{2}$.	Value of $\frac{B+C}{2}$.	Values of B and C.
					° ′ ″	° ′ ″	° ′ ″
X ...	VX = 432·6 ..	VX—VW = 81·2 ...	log.	1·909556	.....	180 00 00	
V ...	VW = 351·4 ...	VX—VW = 784·0 .	co. log.	7·105684	...	36 40 00	W = 89 01 26
	° ′ ″	° ′ ″					
W ...	V = 36 40 00	$\frac{V}{2}$ = 18 20 00	L. cot.	10·47[illegible]	.. ..	143 20 00	X = 51 18 34
			L. tan.	9·491135	17 21 26	71 40 00	

Stations.	Deduced angles.	Correction.	Angles for calculation.	Logarithmic computation.			Deduced sides, in links.		Area computation.	
	° ′ ″	″	° ′ ″		2·545802	......	351·4	VW		
X ...	54 18 34	00	54 18 34	L. cosec.	10·090[illegible]	......	......	.. ...	L. sin. X ...	9·909652
V ..	36 40 00	00	36 40 00	L. sin.	9·776090	2·412240	258·4	WX	log. WX ...	2·412240
W ...	89 01 26	00	89 01 26	L. sin.	9·999937	2·636087	432·6	VX	log. VW ...	2·636087
	180 00 00	00	180 00 00						co. con. log.* .	5·000000

∴ area = 0·454 acres. 2 area = 0·908 = 1·957979

Stations.	Data.	FORMULA. Tan. $\frac{B-C}{2}=\frac{b-c}{b+c}$. cot. $\frac{A}{2}$.	Logarithmic computation.		Value of $\frac{B-C}{2}$.	Value of $\frac{B+C}{2}$.	Values of B and C.
					° ′ ″	° ′ ″	° ′ ″
Y ...	WY = 277·7	WY—XW = 19·3	log.	1·285557	......	180 00 00	
X ...	XW = 258·4	WY—XW = 536·1	co. log.	7·270751	......	40 32 01	X = 75 18 07
	° ′ ″	° ′ ″					
W ...	W = 40 32 01	$\frac{W}{2}$ = 20 16 01	L. cot	10·432674	......	139 27 59	Y = 64 09 53
			L. tan.	8·988985	5 34 07	69 44 00	

Stations.	Deduced angles	Correction.	Angles for calculation.	Logarithmic computation.			Deduced sides, in links.		Area computation.	
	° ′ ″	″	° ′ ″		2·412272	......	258·4	XW		
Y ...	64 09 53	00	64 09 53	L. cosec.	10·045733	......	.. .	......	L. sin. Y ...	9·954267
X ...	75 18 07	—01	75 18 06	L. sin.	9·985550	2·443575	277·7	WY	log. WY ..	2·443576
W ...	40 32 01	00	40 32 01	L. sin.	9·812842	2·270867	186·	XY	log. XY ..	2·270867
	180 00 01	—01	180 00 00						co. con. log.* ..	5·000000

∴ area = 0·233 acres. 2 area = 0·466 = 1·668710

Stations.	Data.	FORMULA. Tan. $\frac{B-C}{2}=\frac{b-c}{b+c}$. cot. $\frac{A}{2}$.	Logarithmic computation.		Value of $\frac{B-C}{2}$.	Value of $\frac{B+C}{2}$.	Values of B and C.
					° ′ ″	° ′ ″	° ′ ″
Z ...	YZ = 578·9	YZ − XY = 392·3	log.	2·593618	......	180 00 00	
X ...	XY = 186·6	YZ + XY = 765·5	co. log.	7·116055	.....	83 00 17	X = 78 34 38
	° ′ ″	° ′ ″					
Y ...	Y = 83 00 17	$\frac{Y}{2}$ = 41 30 09	L. cot.	10·053154	.....	96 59 43	Z = 18 25 05
			L. tan.	9·762827	30 04 46	48 29 52	

OBSERVATION.

* When the measurements are in { feet, co. con. log. = 5·360912.
{ links, „ „ „ = 5·000000.

Stations.		Deduced angles.	Correction.	Angles for calculation.	Logarithmic computation.			Deduced sides, in links.		Area computation.	
		° ′ ″	″	° ′ ″		2·270912	......	186·6	XY		
Z	...	18 25 06	00	18 25 06	L. cosec. ...	10·500378	... ..	......	. ...	L. sin. Z ...	9·499622
X	.	78 34 38	−01	78 34 37	L. sin. ...	9·991311	2·762601	578·9	YZ	log. YZ ...	2·762601
Y	...	83 00 17	00	83 00 17	L. sin. ..	9·996755	2·768045	586·2	XZ	log. XZ ...	2·768045
		180 00 01	−01	180 00 00						co. con. log.* ..	5·000000
										2 area = 1·072 =	0·030268

∴ area = 0·536 acres.

∴ area of series = 0·454 + 0·233 = 0·536 = 1·223 acres

77. SATELLITE STATIONS. (Fig. 434.) It sometimes happens that the third station of a triangle which has been observed to from the two other stations is so situated that the theodolite cannot be set up at it and has to be placed a short distance from the mark, at what is known as a "satellite station." In such a case, set zero to the magnetic north and record the bearings of all three stations of the triangle referred to, and also carefully measure, with a measuring tape, the distance on the ground between the statellite station and the third station of the triangle, the necessary computation being left for afterwards. Thus, in (Fig. 434) let ABC be the triangle of which the angles ABC and BAC, and also the side AB are known, and let C and S respectively represent the positions of the third station and the satellite. It is required to obtain the angular correction, + or −, to be applied to ASB to reduce it to ACB; or which is the same thing as applying the corrections SAC and SBC to the bearings of the lines SA and SB to get the bearings of the lines CA and CB respectively, the difference between the last of which gives the reduced angle ACB, with which and the already given data of the △ ABC, the sides AC and BC may be correctly worked out in the usual way (*see* Chap. XIII, Art. 73). The position of the satellite may be any of the four- S, S′, S″, and S‴,—shown in the diagram, and it would therefore be necessary to make a rough plot of the work to know how the signs + or — are to be applied for the corrections.

Example: In the △ ABC, ∠ ABC, = 44° 35′ 55″, ∠ BAC = 120° 01′ 50″, and side AB = 1,9-0 feet; whence, approximately, AC = 5,084 feet and BC = 6,269 feet (by Chap. III, Art. 26, 1st case). Again, the bearings of the lines SA, SB, and SC were observed to be 110° 48′ 30″, 126° 06′ 30″, and 194° 08′ 30″; and the side SC measured at 36·17 feet. Now, in the △ ASC, the ∠ ASC = 194° 08′ 30″ − 110° 48′ 30″ = 83° 20′ 00″, and as the sides AC and SC are known, the ∠ SAC = 0° 24′ 17″ (by Chap. III, Art. 26, 2nd case), which applied with the sign — to the bearing of SA, gives the bearing CA = 110° 24′ 13″. Similarly, in the △ BSC, the ∠ BSC = 194° 08′ 30″ — 126° 06′ 30″ = 68° 02′ 00″, and as the sides BC and SC are known, the ∠ SBC = 0° 18′ 24″ (by Chap. III, Art. 26, 2nd case), which applied with the sign — to the bearing of SB, gives the bearing CB = 125° 48′ 06″. Lastly, the ∠ ACB = 125° 48′ 06″ — 110° 24′ 13″ = 15° 23′ 53″.

78. *Interpolation.*—(i). The following solution,† by a new method, of the problem to find the positions of two points by observations from them to two known points, will be found of especial use in trans-frontier reconnaissance, where a Surveyor is working in a valley with fixed but inaccessible points on either flank.

OBSERVATIONS.

* When the measurements are in { feet, co. con. log. = 5·360912. / links, „ „ „ = 5·000000

† From the "Auxiliary Tables" of the Survey of Ind a.

Let **A** and **B** be the two known points and **X** and **Y** the points whose positions are to be determined.

Fig. 1

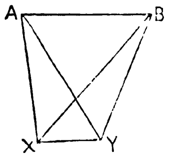
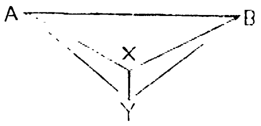


Fig. 2



From the triangles **B X Y** and **A X Y**,

$$\mathrm{BX} = \mathrm{XY}\ \sin.\ \mathrm{BYX}\ \mathrm{cosec.}\ \mathrm{XBY}\ ;$$
$$\mathrm{AX} = \mathrm{XY}\ \sin.\ \mathrm{AYX}\ \mathrm{cosec.}\ \mathrm{XAY}\ ;$$

therefore
$$\frac{\mathrm{BX}}{\mathrm{AX}} = \sin.\ \mathrm{BYX}\ \sin.\ \mathrm{XAY}\ \mathrm{cosec.}\ \mathrm{XBY}\ \mathrm{cosec.}\ \mathrm{AYX}.$$

Thus in the triangle **A B X** the ratio $\frac{\mathrm{BX}}{\mathrm{AX}}$ is determined, and the included angle **A X B** is given by observation and the side **A B** is known, so that the sides **A X** and **B X** may be determined as follows:—

Let
$$\tan.\ x = \frac{\mathrm{BX}}{\mathrm{AX}}.\ \sin.\ \mathrm{AXB}\ ;$$

then if **B N** is drawn perpendicular to **A X**,

A M parallel to **B N**,

and **B M** parallel to **A X**,

then
$$\frac{\mathrm{BX}}{\mathrm{AX}}.\ \sin.\ \mathrm{AXB} = \frac{\mathrm{BN}}{\mathrm{AX}}$$
$$= \frac{\mathrm{AM}}{\mathrm{AX}}$$
$$= \tan.\ \mathrm{AXM}\ ;$$

therefore the angle **A X M** $= x$.

Now
$$\sin.\ \mathrm{AXB}\ \sin.\ x\ \mathrm{cosec.}\ (\mathrm{AXB}-x) = \frac{\mathrm{BX}}{\mathrm{BM}}.\ \sin.\ \mathrm{AXB}$$
$$= \frac{\mathrm{BX}\ \sin\ \mathrm{AXB}}{\mathrm{AB}.\cos\ \mathrm{BAX}}$$

but
$$\frac{\mathrm{BX}}{\mathrm{AB}}.\ \sin.\ \mathrm{AXB} = \sin.\ \mathrm{BAX}\ ;$$

therefore
$$\tan.\ \mathrm{BAX} = \sin.\ \mathrm{AXB}\ \sin.\ x\ \mathrm{cosec.}\ (\mathrm{AXB}-x),$$

and
$$\mathrm{AX} = \mathrm{AB}\ \sin.\ \mathrm{ABX}\ \mathrm{cosec.}\ \mathrm{AXB},$$
$$\mathrm{BX} = \mathrm{AB}\ \sin.\ \mathrm{BAX}\ \mathrm{cosec}\ \mathrm{AXB},$$

and **X Y** is determined by the solution of the triangle **A X Y**.

The following is a specimen of the form in which the computation may be carried out.

	°	′	″		
B Y X . .	114	30	25	. . . log sin.	$\bar{1}$·958999
X A Y . .	24	02	25	. . . . „ sin.	$\bar{1}$·609998
X B Y .	19	20	17	. . . . „ cosec.	0·470987
A Y X . .	55	24	43	. . . . „ cosec.	0·084466
A X B . .	54	23	34	. . . . „ sin.	$\bar{1}$·910105
x . .	47	52	06	. . . . „ tan.	0·043555
A X B				„ sin.	$\bar{1}$·910105
x				„ sin.	$\bar{1}$·870173
A X B—x . .	6	31	28	. . . . „ cosec.	0·944518
B A X . .	79	19	39	. . . . „ tan.	0·724796
Sum of **B A X** and **A X B** . .	133	43	13		
1. **A B X** . .	46	16	47	. . . . „ sin.	$\bar{1}$·858972
2. **B A X**				„ sin.	$\bar{1}$·992422
3. **A X B**				„ cosec.	0·089895
4. **A B**				„	4·906188
Sum of 1, 3 and 4				„ **A X**	4·855055
„ 2, 3 and 4				„ **B X**	4·988505

(*ii*) An alternative method to the preceding is as follows :—

Take the same angles as in the example given in the first method, and *assume 1 foot as the value of* **X Y**, then find out the values of **B X** and **A X** in terms of it from the triangles **X B Y** and **X A Y**, thus

Log. **X Y**	= 0·000000	Log. **X Y**	= 0·000000
Log. sin. **B Y X**	= $\bar{1}$·958999	Log. sin **A Y X**	= $\bar{1}$·915534
Log. cosec. **X B Y**	= 0·479987	Log. cosec. **X A Y**	= 0·390002
Log. **B X**	0·438986	Log **A X**	= 0·305536

With these two values and the included angle **A X B** compute as below :—

Included Angle C	54° 23′ 34″
Log. sin. C	$\bar{1}$·910105
„ $(BC = a)$	0·438986
Co.log. $(AC = b)$	$\bar{1}$·694464
Sum = log. tan. x	0·043555
x	47° 52′ 06″
$(C - x)$ or $(x - C)$* = D	6° 31′ 28″
Log. sin. C	$\bar{1}$·910105
„ sin. x	$\bar{1}$·870173
„ cosec. D	0·944518
Sum = log. tan. y	0·724796
y or $(180 - y)$* = A	79° 19′ 39″
$180° - (A + C) = B$	46° 16′ 47″
Log. cosec. A	0·007578
„ sin. C	$\bar{1}$·910105
„ a	0·438986
Sum = log. $(AB = c)$	0·356669
Known value (see previous example)	4·906188
Difference	4·549519

Add this difference to the above log. values of **B X**, **A X** and **X Y** to get the correct log. values of these sides.

Assumed value :	**B X** 0·438986	**A X** = 0·305536	**X Y** 0·000000
	Diff. = 4·549519	Diff. = 4·549519	Diff. = 4·549519
Correct value :	**B X** = 4·988505	**A X** = 4·855055	**X Y** = 4·549519

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* This should be used when x is greater than C

79. PLOTTING. (Figs. 453, 457.) Before beginning the plot lay down in a corner of the paper the scale on which it is to be drawn; then in order to plot a series of triangles, as in (Fig. 457), measure the length of the base AB from off the scale and set it down in a convenient part of the paper with respect to its bearing; next obtain the calculated lengths of the sides AC and BC from the form for calculation of triangles, and by means of two pairs of compasses set off these distances, having first taken them off the scale; from the points A and B as centres form two arcs intersecting at C, and draw the lines AC and BC; again get the distances CD and BD and set off the point D, and so proceed until the whole series is completed. Such a plotting can be executed by means of a protractor, without having the trouble of computing out the sides of the triangle for the purpose, by setting out the angles, but such a method is at its best but rough. In plotting a traverse as in (Fig. 453), lay off the measured length AB as before with respect to its bearing, place the protractor at B and set off the angle ABC; lay off on BC its measured length, remove the protractor to C, and lay off the angle BCD, and so proceed till the starting-point A is reached.

80. EXAMPLES IN PLOTTING. (Figs. 453, 457.)

1. Plot on a scale of 1,000 feet to an inch, a series of triangles (*see* Fig. 457), proceeding northward, of which the following are the sides in feet, the base AB being approximately in an East and West direction: –

∆ *ABC.*	∆ *BCD.*	∆ *CDE.*	∆ *DEF.*	*Pagoda* (*to east*).
AB = 1,200	BC = 2,024	CD = 2,539	DE = 1,785	From B = 5,125
AC = 1,833	BD = 3,000	CE = 3,256	DF = 1,836	,, D = 3,100
BC = 2,024	CD = 2,539	DE = 1,785	EF = 1,994	,, F = 2,100

2. Plot on a scale of 500 links = 1 inch, the following traverse (*see* Fig. 453), whose dimensions are in links, the bearing of the line AB being given = 43° 30′:

Sides.	*Inward angles.*		*Intersections.*
AB = 2,000	BAH = 127° 25′	Tree —	At A angle with B = 51° 02′
BC = 1,365	CBA = 126 30	,,	,, H ,, ,, A = 44 36
CD = 1,218	DCB = 198 28	Temple —	,, D ,, ,, E = 50 30
DE = 2,885	EDC = 88 27	,,	,, E ,, ,, F = 94 07
EF = 1,996	FED = 147 34	Pagoda —	,, E ,, ,, F = 70 00
FG = 1,636	GFE = 70 26	,,	,, G ,, ,, H = 45 03
GH = 2,204	HGF = 211 38		
HA = 1,690	AHG = 109 30		

81. COMPUTATION OF HEIGHTS. By *computation of heights* is meant the deducing, by calculation, of the height of an unknown station above the mean height of the sea, technically called *mean sea level*, having already given the height of a known station above mean sea level, the vertical angle from it to the unknown station, and the horizontal distance between the two stations. Suppose A and B were the known and unknown stations respectively, then the relative height of B with reference to A would be thus found: multiply the horizontal distance between A and B by the tangent of the vertical angle at A of B, to which add the correction for curvature and refraction (obtained from Table XLI, "Calculating Tables," 5th edition). In the case of the vertical angle being an elevation add this relative height to, and if a depression subtract it from, the given height of A above mean sea level for a similar height of B. There is one other matter to be considered in finding the relative height of B, and that is, first find the height of the instrument (or eye) at A and the height, if any, of the signal (or object) at B, for both must evidently affect the height in question; the rule for obviating this difficulty therefore is, that—

in Depression the height of eye is subtractive —
,, Elevation ,, ,, ,, ,, ,, additive +
,, Depression ,, ,, ,, object is ,, +
,, Elevation ,, ,, ,, ,, ,, subtractive —

The following example with the form in which it is computed coming thereafter, will serve to illustrate the method of calculating heights:—

Example: (*a*) At A, a station 500·00 feet above mean sea level, the vertical angle of another station B is an elevation of 2° 17′ 30″, and the horizontal distance between them 3,568·79 feet, the height of the instrument at A being 4·76 feet, and the height of the signal at B being 14·83 feet; find the height of B above mean sea level. *Ans.*: 633·01 feet.

(*b*) Again take the above for a second example, with this difference, that instead of 2° 17′ 30″ being an elevation, it is a depression, and find the height of B above mean sea level. *Ans.*: 367·51 feet.

Stations.		Horizontal distances, in feet.	Vertical angles.	Computations.		Relative heights.	Reduced levels.	Remarks.
From	To							
					Feet.	Feet.	Feet.	
A	B	3,568·79	......	log. = 3·552522	..	...	500·00	
			E + 2° 17′ 30″	L. tan. 8·602258				
				log. sum 2·154780 =	+142·82			
				Correction for curvature and refraction. =	+ ·26			
				Height of eye .. =	+ 4·76			
				" object ... =	− 14·83	+133·01	633·01	
A	B	3,568·79	......	log. 3·552522	...	...	500·00	
			D − 2° 17′ 30″	L. tan. = 8·602258				
				log. sum = 2·154780	−142·82			
				Correction for curvature and refraction. =	+ ·26			
				Height of eye ...	− 4·76			
				" object ...	+ 14·83			
						−132·49	367·51	

When both a vertical angle is observed to a forward station and the corresponding angle to the back station, their mean is taken (the heights of eye and object not being considered if they are alike, since they counteract each other's effects) to obtain the vertical angle for computation; and when the height above mean sea level of a point is deduced from two or more other points independently, the mean of such results must be taken for the correct height of that point.

82.* THE TRANSIT INSTRUMENT. (Figs. 193, 431.) The *Transit* is a most important astronomical instrument, and is employed for determining the altitudes of, and times at which, celestial objects cross the meridian of a place. The larger instruments, such as the side-transit shown in Fig. 431, are used in permanent observatories, where they are permanently placed in the plane of the meridian, and with which the exact times of meridian passages and the meridian altitudes of stars are measured. The smaller or portable instruments are used in temporary observatories, chiefly for taking the time, but also for survey purposes in laying down the true direction of the meridian. Fig. 193 is a representation of a small, portable transit, and as it is very similar in construction and appearance to the theodolite, a full description of which appears in Article 1—61 of this Chapter, it is not considered necessary to re-describe it. The principal differences between the two classes of instrument are that in the case of the transit the vertical circle is a complete one, so that any altitude may be observed with it; the telescope revolves quite round the axis, thus obviating the necessity for lifting it out of its Ys, and the levels are larger and more sensitive on account of the delicacy of the operation of setting for zenith distances. The telescope, too, is provided with cross-wires, which are placed at the common focus of the object-glass and eye-piece, two of them being horizontal and five vertical. These wires are illuminated at night by means of a small lamp, which throws its light through the hollow axis on to a small reflecting mirror, which turns the light down the telescope on to the cross-wires. In other respects the instruments are practically the same, and the method of using them almost identical; so that all that need be done here is to give a set of simple instructions regarding the adjustments of the transit. The calculations for transit observations for the finding of time and altitude

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* The astronomical terms and calculations referred to in this and the two succeeding articles are described in Chapter XVI.

will be found in the explanation to Table LXIII of the "Calculating Tables" (5th edition), to which reference should therefore be made in connection with this subject.

83. ADJUSTMENTS OF THE TRANSIT. (Figs. 193, 431.) The adjustments of the Transit are five in number. For correct observation it is necessary—

(1) *That the wires and object be in focus.* [Unless this is so, a lateral movement of the observer's head will cause an apparent movement of the wires.] This is called the adjustment for *parallax.*

(2) *That the axis on which the telescope moves be horizontal.* This is the adjustment in level.

(3) *That the line of sight move in a vertical circle, perpendicular to the horizontal axis.*

(4) *That the centre wire be exactly in the optical axis of the telescope.* This is the adjustment of the line of collimation.

(5) *That the vertical circle described coincide with the plane of the meridian.* This is the adjustment for azimuth.

First adjustment. Parallax. Focus the wires accurately by means of the moveable tube which carries the eye-piece. Then focus to a star.

Second adjustment. To make the axis on which the telescope moves to be horizontal. Place upon the pivots the striding level* (which is a long spirit level, stretching from one trunnion to the other), and bring the air-bubble to the centre of its run by turning the footscrew which is under one of the pivots or standards. Turn the level, end for end, and if the bubble retains its middle position, the axis is horizontal; but if it does not, it must be brought back, half by the pivot-screw and half by the small vertical adjusting-screw at one end of the level. The operation must be repeated several times if need be till the result is satisfactory.

Third adjustment. To make the line of sight move in a vertical circle perpendicular to the horizontal axis. Turn the telescope on some distant well-defined terrestrial object, as a lamp †(referring). Elevate or depress the telescope to see whether the object still remains bisected in every part by the middle wire; if not, loosen the screws which hold the eye-end of the telescope in its place and turn the end round very carefully until the error is removed.

Fourth adjustment. To bring the centre wire exactly to the optical axis of the telescope. Lift up the whole instrument bodily from the Ys and reverse it, end to end; if the object (referring lamp) is still bisected by the centre wire, the collimation in azimuth is perfect; but, if not, move the centre of the cross-wires half-way towards the object by turning the small screws which hold the wire plate, and if this half-distance has been correctly estimated, the operation of the adjustment will be complete. Again, bisect the object (referring lamp—moving the lamp on the ground for the purpose) by the centre of the cross-wires, and repeat the operation till the object is bisected by the centre of the cross-wires in both positions of the instrument, and then the adjustment will be known to be perfect.

Fifth adjustment. To make the vertical circle, described by the telescope when moving on its horizontal axis, coincide with the plane of the meridian. Observe for this to the Pole Star (α *Ursæ Minoris*) either at its upper culmination

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* The value of a division of the level scale, in the absence of a specially constructed instrument known as the "bubble-tester" or "level-trier," may be readily determined by attaching the level tube to a vertical circle (or making it ride parallel to the telescope) reading to seconds and noting the readings of the verniers on the vertical arcs corresponding to a motion (of the circle and level together), by means of the tangent screw, which carries the bubble successively to one side and the other of the level. The difference of these readings in seconds divided by the number of divisions of the level that the bubble has moved will give the value of 1 division. Thus, suppose the ends O and L of a level so attached to a vertical circle and the verniers of the circle read as follows:

O	L.	Verniers (mean)
5·6	39·8	0° 08′ 10·0″
40·2	4·7	0 08 48·6
34·6	35·1	38·6

$$(\text{Mean})\ 34{\cdot}85\,d = 38{\cdot}6''$$
$$\therefore d = 1{\cdot}1076''$$

From 10 to 15 such observations should be made, at different times and under varying temperatures, with various lengths of run and on different parts of the vertical circle, the mean of all of which would give the most probable true value

† Placed on a stool or other raised movable object, such as a bench or table.

(when the star travels from East to West) or at its lower culmination (when the star travels from West to East), according to the time of the year. Calculate (1) the mean astronomical time (which begins at noon of the corresponding Civil Day) of the star's culmination (Upper or lower), and (2) the apparent altitude of the star. About 10 minutes before the calculated time fix the telescope in position according to the altitude and look out for the star's passage in the field of the glass. This star (*α Ursæ Minoris*) travels slowly. When the star is in the field of the glass bring it, if not already there, exactly between the two parallel horizontal wires by means of the vertical tangent screw, and then with the aid of the azimuthal screws (which should be brought well home), at the exact instant of the calculated time of transit, bring the star on to the centre wire (of the five vertical ones) precisely. The instrument will then be in the meridian.†

84. CALCULATIONS FOR TIME AND ALTITUDE. The following are the calculations for (i) Time and (ii) Altitude referred to in the 5th (and principal) adjustment (*see* Art. 83).

(i) *FOR TIME.*—At a place in λ. 16° 47′ 26″ N., L. 96° 07′ 59″ E., observations were taken to * *α Ursæ Minoris* (*Polaris*).

(*a*) *On the 30th December 1895*—

R.A. = 1 *h.* 20 *m.* 46·93 *s.*
Dec. = 88° 45′ 36·7″ N. — *h.* *m.* *s.*
R.A. of * + 24 hours ... = 25 20 46·93
R.A. of ⊙ (Sun), or sidereal time at mean noon = 18 34 42·38
Correction (constant) for Long. E. = 96·13305°λ. 0·6571 *s.* = — 1 *m.* 3·17 *s.*
} = − 18 33 39·21
Sidereal time of *'s culmination (upper) = 6 47 07·72 P.M.
∴ Mean astronl. time of *'s culmin. = 6 46 01·02 P.M.

(*b*) *On the 18th January 1896*—

R.A. = 1 *h.* 20 *m.* 29·15 *s.*
Dec. N. = 88° 45′ 38·26″. — *h.* *m.* *s.*
R A. of * + 24 hours ... = 25 20 29·15
h. *m.* *s.*
Sidereal time at mean noon = 19 49 36·99
Corrn. for Long. E. (constant) = − 00 01 3·17
} = − 19 48 33·82
Sidereal time of *'s upper culmination = 5 31 55·33 P.M.

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† In making observations of transits, the times may be noted by the *eye and ear* method, or a chronograph may be employed. In the former method, as the star enters the field the hours and minutes are written down, and the seconds are counted by listening to the beats of the clock (or chronometer). As the star crosses the wires the number of seconds counted is written down. It is possible in this way to reckon to the tenth of a second. In the second method, by which it is possible to subdivide a second of time with much greater accuracy than with the eye and ear alone, electric currents are employed. A metal cylinder is covered with cloth, over which a piece of paper can be stretched, and is made to revolve by means of a clock which is regulated by vanes. Below the cylinder and parallel to it is a screw carrying two electro-magnets, and this is made to rotate by the same clock which drives the cylinder. The armature of each electro-magnet is connected with a pricker for marking the cylinder. One of the electro-magnets is connected with the sidereal clock in such a way that a current is sent through it every second, and the other is connected with a key, by means of which the observer can send a current through it at any instant. All the connections being made and the clock started, the clock magnet traces a spiral of pricks on the paper at intervals of about half an inch, a double space being left at the end of each minute. When a transit observation is made, a series of extra pricks is made on the cylinder alongside the others. The time at which each of the pricks was made is then ascertained by first counting the number of whole seconds which have elapsed since the last double space, and then measuring the distance of the prick from the beginning of the last second which gives the fraction of a second. When the apparatus is started, the hour and minute is marked on the first double space. If an Assistant is employed for reckoning the times of transit as shown by the chronometer, shortly before the observation the observer should warn the Assistant to stand by the time-piece with the word "Ready"; as the star approaches the wire the word "Count" should be given; and when the star crosses the wire the word "Stop" should be made use of; in this way the exact chronometer time of transit would be obtained.

For greater accuracy, and to multiply the chances of observation, the transit is taken over several wires (in smaller instruments usually five) and the mean taken. The time taken for a star to cross the field of view depends upon its declination. The *equatorial interval* is the time taken for an equatorial star to transit from wire to wire, this being shorter than for any other star. The equatorial interval can be obtained from the interval for any star of known declination by multiplying the latter by the cosine of the declination.

		h.	*m.*	*s.*	
Corrn. (constant) for forward lower culmn. in the 2nd half of the astronl. day (18th Jany.), or 1st half of the corresponding Civil day (19th Jany.) = ½ (3 *m.* 56·56 *s*).	=	− 0	01	58·28	
Sidereal time of *'s lower culmn. ...	=	5	29	57·05	A.M.
∴ Mean astronl. time of *'s lower culmination.	=	5	29	3·00	A.M.

(ii) *FOR ALTITUDE.*—At a place in λ. 16° 47′ 26″, L. 96° 07′ 59″ E., observations were taken to * *α Ursæ Minoris* (*Polaris*).

(*a*) *On the 30th December 1895*—

Dec. N. = 88° 45′ 36·7″.

Apparent Altitude (reckoned from S.) of * = Co. lat. + N. Dec. + Refraction = (90° − 16° 47′ 26″) + 88° 45′ 36·7″ + Refraction = 161° 58′ 10·7″ + Refraction.

∴ App. Altitude (reckoned from N.) = 180° − 161° 58′ 10·7″ + Refraction = 18° 01′ 49·3″ + 2′ 57·3″ ... = 18° 04′ 46·6″ from N.

(*b*) *On the 18th January 1896*—

Dec. N. = 88° 45′ 38·26″.

Apparent Altitude (reckoned from S.) of * at upper culmination = 180° − (73° 12′ 34″ + 88° 45′ 38·3″) + 2′ 55·6″ = 18° 1′ 47·7″ + 2′ 55·5″ = 18° 04′ 43·2″ from N.

P.D. = 90° − Dec. N. = 90° − 88° 45′ 38·3″ = 1° 14′ 21·7″.

∴ Apparent Altitude (from N.) of * at lower culmination = {180° − (73° 12′ 34″ + 88° 45′ 38·3″) − 2 × (1° 14′ 21·7″)} + Refraction = (18° 01′ 47·7″ − 2° 28′ 43·4″) + 3′ 26·6″ ... ... = 15° 36′ 30·9″ from N.

CADASTRAL SURVEYING.

85. CADASTRAL SURVEYING EXPLAINED. As so much cadastral, or field-to-field, surveying is now-a-days carried on everywhere, it may not be out of place here to briefly describe the system as followed by regular survey parties. The surveys are all large-scale ones, generally 16″ = 1 mile,* but sometimes double or quadruple it, and even more than this. The 16″, however, is what we are particularly concerned with, although the methods apply equally to all except that the measurements have to be made with greater care the larger the scale of survey. For the purpose of this particular class of 16″ survey, the work in it is divided into two sections, namely, the *traverse* and the *cadastral.* The former concerns itself with the *theodolite* portion and supplies the necessary skeleton plots of the polygons laid out by it for the detail work that is to follow, while the latter consists of the *plane-tabling* of all the requisite topography of the country on the plotted sheets aforesaid. The out-door duties of both sections are performed in the field, or dry season, and the office work in the recess, or wet season of the year; the field-season being usually somewhat longer than the recess. To distinguish between the two classes of surveyors engaged on the work, those employed on the traversing are called "traverse" or "sub-surveyors" and those doing the plane-tablings are known as "detail" or "field-surveyors." These have their inspectors to check their work, while a responsible officer is placed in charge of each of the sections,

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* Or 5 chains (Gunter's) to 1 inch, or $\frac{1}{3{,}960}$.

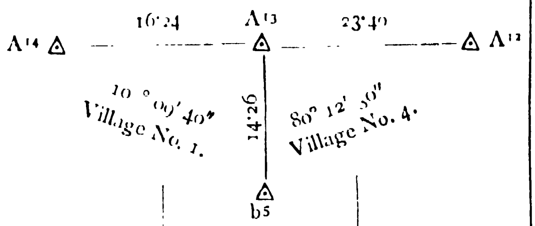
who, in their turn, are controlled by a third officer in complete command of the combined operations. Much of the present subject is duly treated of in previous chapters, more especially Chapters VII, IX, XI, XIII, XIV and XVI, so that the matter dealt with before will not now be repeated, but only that written upon which immediately relates to the particular class of surveying herein referred to. With these preliminary remarks, the following instructions on the prosecution of traverse and cadastral surveys will, it is hoped, prove useful. A large polygon, from 50 to at the outside 200 square miles in area, termed the "main circuit," is first enclosed by traverse. This is then subdivided into smaller polygons called "village traverses," of from 1 to 2 square miles in extent, of which plots are later on furnished for the detail survey. These plots are prepared on standard sheets of imperial paper having one-inch co-ordinate squares marked on them in faint blue lines, to facilitate the work of plotting.

TRAVERSE SECTION.

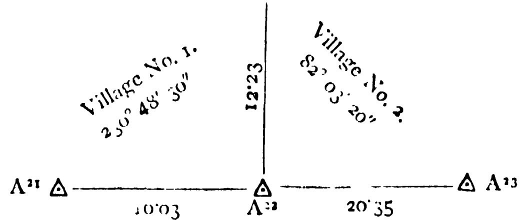
(i) THE FIELD SEASON.

86. MANNER OF OBSERVING WITH THE THEODOLITE. (Fig. 363.) Set up the theodolite over the station mark, placing the stand firmly in position; centre the instrument, remove the cap off the object glass; brush the limbs and verniers free of dust; obtain distinct vision of the wires in the eye-piece, eliminate parallax, level horizontally, then vertically, and finally, set zero to the magnetic north. The theodolite is now ready for use; if from any cause the level is found displaced during the observations, it should be re-levelled and the angles observed anew. It is a good practice to examine the level after the observations have been completed at a station, to satisfy yourself as to the value of the angles. Having had flags set up at the stations to be observed at, unclamp the upper horizontal clamp, turn your telescope to the back flag, and clamping it, work the flag gradually to the intersection of the wires with the upper horizontal tangent screw, record the readings of the three verniers in your field-book (given below), and loosen the upper clamp. Again turn the telescope from left to right, and similarly observe the forward flag, recording the readings of the verniers, as before, in your field-book. Now allow the upper clamp to be fastened, loosen the lower horizontal clamp and turn the telescope round again from left to right, by means of the lower horizontal plate, till it points to the back flag; clamp the lower clamp and with the lower horizontal tangent screw intersect the flag. The reading of the limb is yet what it was when the forward flag was observed; now unclamp the upper clamp, turn the telescope from left to right to the forward flag, clamp the upper clamp, observing to the flag as before, recording the vernier readings in your field-book. This is termed the "double angle." Take out the means of the three sets of the vernier readings and write them in the column to the right; then subtract the first from the second reading, and the second from the third reading for the interior angles between the two stations and enter them in the column to the left, as shown in the field-book below. When one reading (as in the case of the third reading of the field-book) is smaller than that which has to be subtracted from it, 360° has to be added to the former before the subtraction is made. If the two angles differ from each other by 1′, the process of observing the angle between the two stations must be repeated, once or oftener, as necessary. That is, the reading of the forward flag must be brought and set to the back flag as before, and the reading of the forward flag then re-observed for the third time, called the "treble angle," and so on. In the case of a trijunction station, a third flag would also have to be observed to; but here a single reading of vernier A, with the zero set to magnetic north, need only be read and recorded. When any conspicuous objects have also to be observed to, two observations to each must invariably be made, the observations being made *in serial order* from left to right commencing with the back station, first on the original zero set to the magnetic north, and then on the reading of the forward station set to the back flag. Having completed the observations at a station, remove the theodolite into its box and proceed to the next station of observation.

(Closed on M.C. No. 1). Mingalun Circle.



Village No. 1.	101-09-10	50-30	177-49-00	49-00	177-49-30 △A12	Village No. 4.
	101-09-40	40-30 31-30	255-18-30 70-40-00 335-30-00 △A13	41-00 30-30	76-40-20 335-30-40	
			9·42 14·20		= 14 28	
	138-30-00 138-30-40	61-00 30-30 58-30	292-56-30 154-30-00 16-00-30	60-30 30-30 00-00	293-00-30 154-30-20 15-59-40	
			6·00 9·06 6·03 9·08		= 9·09 = 9·14	
	127-11-30	05-30	83-05-00	05-30	83-05-20	
	127-12-50	53-30 20-00	315-54-00 247-20-00	54-00 21-00	315-53-50 247-20-20 Pagoda.	
	127-10-00	42-00 11-00 32-00	337-28-00 188-40-00 120-10-00 61-30-30 △b2	41-00 10-30 30-30	△d1 188-41-00 120-10-30 Pagoda. 61-31-00	
			9·64 14·64		= 14·62	Village No. 2.
	135-17-30	18-30	16-17-00	18-30	16-18-00	
	135-17-30	61-00 43-00	240-56-30 106-42-00 △b1	60-00 43-00	241-00-10 106-42-40	
	Long chain ... Short chain ...	... ...	8·08 12·23	...	= 12 24	
	230 48-20	09-00	158-09-30	10-00	158-09-30 △ A23	
	82-03-20 230-48-30	... 21-30 32-30	138-35-50 287-21-00 56-32-30 △b1	21-00 33-00	287-21-10 56-32-40	



(Started on M. C. No. 1). Launglaunglauk Circle

Survey of village No. 1, by M. P., 23rd December 1891.

87. INDEX PLOTS. Traverse-surveyors should by means of a protractor make rough angle plots of their work as they proceed, on the scale of 2″ = one mile; these serve as an index to the *kwins* in office and facilitate work.

88. SKELETON TRAVERSE. Traverse-surveyors make only a skeleton traverse, taking no offsets; the latter is left to the field-surveyors to do.

89. RESPONSIBILITY FOR CHAINING. Traverse-surveyors are held responsible for the chaining of their chainmen, and they should therefore see that the men understand their duties and perform them properly.

90. TESTING OF CHAINS. Surveyors are usually provided with three chains—one a test chain (Gunter's) 66 feet long, which is not to be used for measuring distances, and two, one a 66 feet Gunter's and another a 100 feet, for the double measurements required. Both these chains should invariably be tested, and if necessary rectified, with the test chain every morning before starting for the day's work. Once a month, when the surveyor visits the camp office, he should bring his test chain with him and have it tested with the standard bars kept there for the purpose.

91. INTERSECTED POINTS. While traversing, any objects of importance should be observed to and fixed from two or more stations (three at least, when possible), such as G. T. points, pagodas, conspicuous trees, etc. When the object observed is at a great height, as a hill, it will be necessary, in order to determine its altitude, to observe a vertical angle to it. If it is found that prominent points, which it is necessary to determine the positions of, cannot be seen from any two stations of the traverse, they may be fixed by a system of triangulation based on one of the traverse sides by observing the angles alone of the various triangles thus formed.

92. G. T. STATIONS. The main circuit surveyor should invariably connect all G. T. stations that are situated within reach of him.

93. THE TRIANGULATION SYSTEM. Sometimes it unavoidably happens that the field survey precedes the traverse survey, working on the triangulation system. The traverse-surveyor must in this case take up the interior surveyor's stations. This is a *sine quâ non.* The system referred to has sometimes to be resorted to, although this rarely happens, when the traverse camp is unable, owing to any cause, to provide the cadastral camp with a sufficiency of plots to keep all the field-surveyors in work. Two conditions may present themselves in this regard: (i) the traverse-surveyors may have already gone over the ground, laid down their survey-marks, and completed their angular and linear measurements, but after this a sufficient time has perhaps not elapsed to complete the necessary calculations and prepare the plots; or (ii) the traverse-surveyors may not yet have entered upon the ground. In the former case it will be essential for the field-surveyors to pick up *all* the stations met with in the village under survey either by sight-vane measurements and the re-chaining (with double measurements) of the traverse lines, or by adopting the stations as the apexes of the triangles into which the land is broken up for the purposes of the detail survey, or partly by each of these methods. In the latter case the field-surveyor will ordinarily carry his lines along the boundary of the village, just as would have been done by the traverse-surveyor himself, and then lay out in the interior of the village a series of large and symmetrical triangles to suit the requirements of the work in hand, upon which to base his detail survey. The stations thus laid down in the course of his work both on the boundary as well as in the interior of the village should be marked in some conspicuous and fairly lasting manner, as by the burying of a stout post, 3 to 5 feet in length, in the ground, covering the base of it with earthwork a foot or so high, and digging a narrow trench all round it. These would be capable of being readily found and picked up for his own work by the traverse-surveyor when he entered upon the ground to do the traverse of the village. As soon as the survey of a village is completed in this manner by the field-surveyor, a trace of the sheet or sheets belonging to it, as the case may be, should be prepared on tracing paper and forwarded without delay by the officer in charge of the cadastral camp to the officer in charge of the traverse camp. These traces are supposed to show the boundaries of the villages, exteriors of cultivation, and principal topographical features, together with the names of

villages, creeks, etc. All the station-marks fixed right round the boundaries of villages and which are to be taken up by the traverse-surveyor should be shown by the mark ◬ and connected with each other by firm blue lines, the remaining station marks in the interior of the villages of the entire triangulation should be shown by the mark ⊙ and connected with each other according to the field-surveyor's chain lines by dotted lines. The traces will be most useful to the traverse-surveyors and will enable them to pick up the boundary stations, or as many of them as possible, and thus connect on the field-work properly. It is important, however, that some of the stations of the net-work of triangles are picked up, as far as can be, on opposite sides of the village; this would ensure almost absolute accuracy in fitting the work off the original sheets on to the plots to be subsequently supplied by the traverse camp. This is said because in some instances it might be found that the triangulation-work is confined to the centre of a village and the boundary stations are entirely, or almost entirely, laid down by sight-vane merely, and some of these connected on with the net-work of triangles by sight-vane also. Work done on the triangulation system, whether the stations have been previously laid down by the traverse section or afterwards, is necessarily slow; and the surveyors who are entrusted with such work should be well instructed in their duties, as much judgment is required to do the survey in an efficient manner. Where patches of detached work done on the triangulation system are situated at long distances from main circuits, and which it is desirable to connect, in an inexpensive manner, with the main work, these may be so connected by a series of what are known as unclosed traverses, emanating from a station of a main circuit and joined on to a station or stations of the detached work. In this way the detached work is capable of being accurately mapped; but as there can be no absolute check on the work of an unclosed traverse, the duty of executing such a survey should only be entrusted to a trustworthy surveyor.

94.* OBSERVATION OF AZIMUTHS. (Fig. 448– 489*a*.) It is desirable to choose stars of north declination only for azimuth purposes, but not exceeding the co-latitude of the station; and also to observe the star at an altitude of between 10° and 25°. Never less than two stars should be observed at any one azimuth station. For places, owing to their distance from an encampment, it is found inconvenient to visit at night, the sun may be observed for azimuth; in this case afternoon observations are preferable, owing to there being lesser refraction then, to morning ones. There are several large stars which may be observed early of an evening, or in the morning when there is just sufficient daylight to enable an illuminated referring mark to be seen, and also to enable the wires of the theodolite to be seen without illumination. In such cases the angles may be measured with no more trouble than if a terrestrial object was observed instead of a star. When selecting stars for observation, a reference should be made to the star charts given in Figs. 488 –489*a*† or those given in maps LXXII and LXXIII of the "Calculating Tables" (5th edition); these contain all the stars of the first, second, and third magnitudes, while the stars that are encircled in the maps referred to, and which also show stars of the fourth magnitude, are Nautical Almanac stars.

95. CONDITIONS OF A TRAVERSE.‡ The conditions of a traverse are—

(i) that the sum of all the inward angles must be equal to twice as many right angles as the figure has sides, less four right angles;

(ii) that the sum of the northings must be equal to the sum of the southings; and

(iii) that the sum of the eastings must be equal to the sum of the westings.

96. LIMIT OF ERROR. The limit of error, in regard to the distances on the meridian and perpendicular, allowed in practice is one link in every 10 chains for village circuits and ½ a link for main circuits; in angular work it is one minute in every five angles for main and minor circuits, and one minute in every three

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OBSERVATIONS.

* The astronomical terms which occur in Arts. 94, 120 and 126 are explained in Chapter XVI.

† Figs. 488 and 489 are for use in the northern and Figs. 488*a* and 489*a* in the southern terrestrial hemispheres, and respectively refer to the projections about the northern and southern celestial poles. In the former the north and in the latter the south polar regions are wholly exhibited.

‡ *See* also Chapter XIV, Art 9.

angles for village traverses and sub-traverse lines or "purdahs." If greater corrections than these be required in the traverse computations, a re-survey becomes necessary, although practically errors of 50 per cent. over and above these limits would not affect the quality of the work to any appreciable extent—and under certain circumstances there would be no objection to adopting the higher percentages as the limits of the work to be passed.

97. SIGNALS. The best description of signal for observing to with the theodolite is a thin straight bamboo about 10 or 12 feet long with a red and white flag on the top, and an iron spike fixed at the bottom. It is a good plan to intersect signals as near the ground as possible. Where, owing to the nature of the undergrowth or other cause, this cannot be done, a fine string of the length of the bamboo, with a stone or other heavy object as a plummet suspended at one end of it, should be let drop from the top of the bamboo, and care should be taken that the staff and the string correspond all the way down; this would ensure the perpendicularity of the signal.

98. SIZE OF VILLAGE. The village area most convenient for survey purposes is about 1 square mile, and should not ordinarily exceed 2 square miles; but where the village contains nothing but forest or other jungle, there would be no objection to having the area even larger than this, up to 4 square miles or so.

99. VILLAGE BOUNDARIES. Conspicuous natural features of country should, where such exist, be adopted as boundaries of villages, unless the adoption of them would make the areas of the villages much in excess of 2 square miles.

100. NUMBERING OF PIPES. All pipes buried in the ground to represent theodolite station marks should be numbered, the requisite figures being written on the outer portion of the pipes which projects above the surface of the ground with a little coal-tar applied by means of a small stick. The same number should be written on a slip of paper and placed inside the pipe a couple of inches or so below the top of the earth which is to be rammed inside each pipe.

101. SYSTEM OF NUMBERING STATIONS. Every main-circuit surveyor and every village-circuit surveyor should be given a distinctive letter, as A, B, C, etc., so that his stations might be capable of recognition on the ground. Each surveyor should assume this letter as his distinguishing mark along with the serial order of numbering he has to adopt for all the pipes imbedded by him in the traverses during the entire season. Thus surveyor B would number all his pipes whatsoever, as laid down by him, B1, B2, B3, and so on up to the end of the field-season. The references in his field-book to these stations should be shown with the corresponding numbers therein. Where the stations of another surveyor are picked up, his numbers should be invariably adopted, no fresh numbers being given in such cases. The only distinction to be observed in the use of the letters, and this is material, is that all main circuit stations should be denoted by *capital* letters, as A, and village-circuit stations by *small* letters, as *b*. In the traverse computation forms and the 16″ and 2″ plots the same numbering as adopted by the surveyors should be preserved, otherwise confusion would most probably result.

102. LINE-CUTTING. If a creek is fringed with forest or jungle, not more than 10 chains wide, the lines should run outside the jungle. If the jungle is more than this in width, the lines should follow the creek, while another set of lines must also run outside the jungle. For main-circuit lines, or sides of the greater polygons, about 20 chains are best lengths. A main-circuit should run along boundaries as much as possible. A line-cutting chainman must be thoroughly up to his work. A good line-cutter will take advantage of all high ground for his stations, and will avoid low spots, which are apt to be covered with water; along the banks of rivers and streams stations should always be placed above high water mark. He would at the same time select his lines in such a way as to necessitate the least amount of jungle-clearing. He will fix his flags so that they can be intersected near their bases. About 10 chains are the best lengths for village lines, or sides of the lesser polygons. In all traverses, the longer the lines are, the better will the work close; but the requirements of the detailed survey have also to be kept in view. When a cultivated village is more than 40 chains wide, it should be divided, with one or more sub-traverse lines as required, running through

it in convenient and suitable directions. Sub-traverse lines should be about an equal distance apart from each other. Where the line-cutting through jungle in a village is found to be heavy, it will often be found possible to get the cultivators of the neighbourhood to assist in this work. Opportunities such as these should always be taken advantage of, as the progress of the work is thereby much expedited.

103. FIXING STATIONS. In fixing stations traverse-surveyors should select them in such a manner that field-surveyors will derive the maximum amount of advantage from the stations so fixed, also so that pagodas and other conspicuous objects can be intersected from them. The speed of a field-surveyor greatly depends on how the theodolite stations are situated for his work. If a village is surrounded by jungle, the cultivation should be bounded by theodolite stations. When a village boundary is not a natural boundary, but runs through jungle, the survey will follow it of course, but another set of lines should run outside the jungle. In the midst of cultivation survey stations should be fixed on field boundaries and not inside fields, and preferably at junctions of field divisions. Along village boundaries theodolite stations should be established on the salient and re-entering angles of village limits.

104. ROUGH CIRCLE-MAPS. The line-cutter's guide to the boundaries of villages are the rough circuit maps previously supplied to him and the posts which have been put up in advance at every bend of the boundary. These posts should have a rim of paint round them, and should not be put up along natural boundaries, such as creeks and roads. Stations should be placed at posts where possible.

105. TRAVERSE LINES. These should run along the boundaries of villages exactly, where this is possible. Where a river forms the boundary, the lines should be run on both sides for the two villages. The lines should be taken as near as practicable to the banks of rivers within 1 or 2 chains of them and certainly not more than 5 chains distant unless there was cultivation on the outer side to within a distance of 10 chains, when it would be advisable to hug the cultivation and thus save excessive line-clearing. Where a range of hills forms the boundary between two villages, the line. should be run along the ridge which separates the villages: another line should be run along the foot of the hills bordering on cultivation. Blocks of cultivation should have lines running along the borders or through the centre, depending entirely upon the size of the blocks and the general run of the ground. Sub-traverse lines running more or less parallel to each other should, in cultivated tracts, not be more than 30, or at the outside 40, chains apart. The distance betw en two adjoining stations should average about 10 chains length. Circumstances may necessitate lengths varying from 5 to 20 chains, but the latter distance should not be exceeded. Through or by cultivation, village-sites, and the like, the distances should not ordinarily be much over 10 chains, as starting and closing points will be chiefly required here for the detailed survey.

106. PERMANENT MARKING OF STATIONS. Every theodolite station without exception should be permanently marked by the use of some lasting material, such as a pottery cylinder (which may be 18″ long × 5″ diameter at either end × 1″ thickness of clay). Such a pipe should be buried in the ground to the extent of 14″, four inches being allowed to project above the surface of the ground. A wooden peg should be driven inside this and the pipe well rammed in with earth. A mound of earth 3 feet high should then be raised over the mark with a trench dug all round; this would ensure the better preservation of the station. Each of the pipes or other similar mark thus placed should have marked on the outer side of it on the part projecting above the surface of the ground, with coal-tar, the serial number and letter by which it is to be denoted by the surveyor in his field-book. In some parts of the country it may be a matter of great difficulty, or one of excessive expenditure, to get clay pipes manufactured for the permanent marking of survey stations. In such cases, stone, cut in the shape of right prisms, say 18″ in length, with equilateral triangles of 4·33″ sides at the ends or any other suitable material, may be employed instead. In a rocky country it may be possible to utilize prominent rocks and stones for survey stations, when the necessity for imbedding pipes would no longer exist. Rocks and stones so selected should have cut on them with a

sharp piece of iron *in situ*, just as is done in cases where masonry wells, walls, etc., are similarly made use of, a small circle having two diameter lines drawn at right angles to each other with a whole punched in the centre, to mark the position of the station. But in studying economy of money in this direction, that of time is also to be considered, and only such rocks should be so made use of for stations as are conveniently situated for the survey lines. It would hardly be worth while, for instance, to take lines out of the way of the ordinary survey requirements to get to a rock to convert into a survey station, or to cut extra lines through jungle in order to take in a particular rock for a station. The only exception to be made in regard to the permanent marking of all survey stations should be when the work is being carried through hilly country, or through heavy forest jungle, where the survey lines are necessarily small, often as short as 1 and 2 chains, that it would manifestly be a simple waste of money to carry out the rule in question. For such cases it would be sufficient to permanently mark stations at about every 10 chains or so apart only, care being taken to so mark all the trijunctions and "jawabs," or adjoining stations, whatsoever. For stations not permanently marked, good stout pegs should be imbedded in the ground instead of the pipes. In the field-books and plots permanently marked, stations should be shown by the symbols ◬⊕, according as a pipe has been employed or a mark on a rock or other material *in situ*, and by the plain ⊙ for the temporarily marked stations. In this manner a considerable saving of both time and money might be effected. If pipes are imbedded at stations before observation, well and good; but where circumstances do not allow of this being done, great care should be taken to see that the person entrusted with the subsequent imbedding of pipes does so at the *correct* points only.

107. USE OF CHARCOAL. A handful of charcoal or broken red-brick might with advantage be put inside each cylinder which is imbedded in the ground to mark the stations. Such a system has been found to work well for the purpose of the identification of the exact positions of marks on the ground, where the pipes have been purposely shifted by interested parties, or have been otherwise lost or destroyed. Surveyors should not fail to attend to this matter; charcoal is obtainable in every village, and in the heart of the jungles can be easily made on the spot by burning sufficient wood, which is found in abundance everywhere, for the purpose.

108. PLOTTING OF STATIONS. The plotting of the stations on the 16″ sheets of a village should be so done as to allow ample room for the detailed work to be introduced afterwards. Paper, however, should be economized to the utmost, as far as can be judged from the rough 2″ index plots of the traverse-surveyors and on which it would be advisable for the traverse-surveyors to introduce the approximate boundaries of the villages surveyed by them. The following symbols should be employed for denoting the positions of survey stations on the sheets, namely, for stations permanently marked by pipes (including stone or other material which it may be found more convenient to use in places, instead of pipes) by a small equilateral triangle, thus ◬; for those cut *in situ* on rock, masonry wells, walls, house-tops, bases of pagodas, and the like, by a small circle with two cross lines running through its centre at right angles to each other, thus ⊕; and for those temporarily marked by wooden pegs merely by a small plain circle, thus ⊙. All such symbols, together with station letters and numbers and traverse lines, should be entered in the sheets in blue paint, while the headings of the sheets, village names, and references should be in black pencil alone, before the sheets are forwarded by the Traverse to the Cadastral camp.

109. IMBEDDING OF PIPES BEFORE OBSERVATION. Pipes should invariably be buried at survey stations *before* observations are made with the theodolite. This rule is important. If, however, owing to any unforeseen cause, it is not found possible to bury the pipe before observation, a short wooden peg should be inserted in the proper place in the ground and observed over; while six smaller pegs should be driven into the ground about 2 feet away from the centre peg, equidistant around it, and so that a line drawn from one peg to the corresponding opposite one may fall exactly over the station mark at the point of observation. If these smaller pegs are correctly placed, the three cross lines formed by these

pairs of pegs respectively should intersect at the centre point. Afterwards, when pipes were procurable, a responsible chainman or traverse-surveyor should be sent round with a sketch map, to replace the temporary marks referred to by permanent ones; this work should be carefully performed and the final position of the imbedded pipe well tested by means of the six smaller pegs previously laid down. If thought necessary, by way of a further precaution, the man to whom the duty of replacing marks is delegated might be provided with a wooden framework, in the shape of an equilateral triangle or a square, of 2 feet sides, with which to fix the pipe in position. This would be done by placing the framework in position round the mark before extracting the peg, and by leaving it there undisturbed till a hole has been dug and the pipe placed in its proper place and well secured and tested before removal of the framework.

110. CHAINING DISTANCES. All traverse lines should be measured twice, with two chains, one a 66 feet, and the other a 100 feet one. The measurements should be made by four chainmen, two for each chain, and both their figures recorded in the field-book, as shown in the form given in Art. 86. The calculations in the office refer to the Gunter's chain measurements; the surveyor should similarly convert the feet measurements into links by Table II of the "Calculating Tables" (5th edition), and if it differs by more than two links in 10 chains from the other, the distance should be re-measured by both chains, and their value also entered in the field-book over the first entries. In the chaining of distances one chain should always be made to keep ahead of and the other to follow the observer. At trijunction stations, lines belonging to an adjoining finished circuit need only be measured once, with the Gunter's chain, for record of the distance in the field-book.

111. ENTRIES TO BE IN INK. All entries in the angle-books and field-books ought to be in ink only. Erasures should not be allowed: where corrections become necessary, these should be made in ink; for, although the books may not present a perfectly clean appearance in this way, the entries in them are more reliable.

112. SURVEYED AREA TO BE KEPT TO LEFT. In executing the traverse survey of a main circuit or of a village, the area surveyed should always be kept to the *left* of the observer when going round the circuit.

113. STARTING POINTS. In starting the survey of a traverse, the surveyor should always start operations, whenever possible, from a trijunction point, that is, from a station that is or may become common to three village traverses; in the case of two adjoining villages bordering on the sea, the start could not perhaps be made from any other than a bijunction point.

114. SPECIMEN TRAVERSE. (Fig. 363.) A specimen sketch of a traverse survey is given in (Fig. 363) to illustrate the methods of survey. *A1* is the starting point of the main-circuit *A1*, *A2*, *A3*, etc., back to *A1*. The five interior blocks are village-circuits; while three sub-traverse lines have been run through villages Nos. 1, 3, and 4. The main-circuit is surveyed by surveyor A; village No. 1 has been surveyed by B, Nos. 3 and 5 by C, No. 2 by D, and No. 4 by E. The figure explains itself: and shows the system of numbering stations to be followed, the way in which the traverses have to be run, and so forth. It is not necessary for a village-circuit surveyor to re-measure the lines or re-observe the angles of a common boundary already measured by another village-circuit surveyor or by the main-circuit surveyor, except at the starting and closing points of his village traverse where it should invariably be done (with *single* angular and linear measurements only), otherwise there might be difficulty in the office in connecting the work of the various traverses. The *interior* angles of a polygon under survey would of course have to be rigidly observed at starting and closing points; it is only in regard to the outer or adjoining angles that a single reading (of vernier A alone) need be recorded. Great care should be exercised by the Camp Officer in apportioning work to the village-circuit surveyors so that no confusion may result amongst the men, and the same ground may not be gone over unnecessarily by different parties. The practice in this respect should be that where a village has not had any portion of its boundary laid out by the surveyor of an adjoining village, it should be entirely surveyed by the first arrival, the surveyors

of the adjoining villages having merely to connect on their work with the common boundaries; where a surveyor is engaged upon or has already completed the work appertaining to a common boundary, the later arrival should connect his own work on to such boundary, in every instance commencing at and closing upon a trijunction point. Where a portion of the boundary of a village is to be or has already been surveyed for a main-circuit, the village traverses need only be taken up to the connecting points, the requisite "jawab" or connecting lines being of course thrown out and the interior angles at the connecting stations being duly observed. In country where hill and jungle villages predominate, if these contain little or no cultivation and are not likely to be cultivated for a long period, it would be well not to survey or plot them at all, even when falling inside of a main or minor circuit. The plotting should be confined as much as possible to cultivated tracts, as the detailed survey is a very expensive one and much time is usually lost in doing hilly country in which the contour lines have to be necessarily shown. It will not matter if the numerous gaps form "eye-sores" to the work so long as the entire survey is well connected, either within main or minor circuits, or bordering on them on their outer sides.

115. FACE LEFT OBSERVATIONS. For main-circuits and village traverse work all the interior angles of the polygons need only be observed on "face left" of the instrument.

116. CROSSING RIVERS. (Fig. 387.) When lines are taken across rivers and creeks, it is not possible to chain the distances; so that in such cases it would be necessary to take a base line on one side of the water at least one-third of the width of the river or creek, and observe all the three interior angles of the triangle (the sum of which should be equal, if not quite, very nearly so, to 180°). (*See* Fig. 387.) The base line should be measured *twice*, with both chains, to ensure accuracy of results. The angular and linear measurements should be recorded (separately if necessary) in the field-book. (*See* the field-book given below, also Art. 131.) The calculations for the required distance would be worked out in the office, so that the surveyor need not trouble about these. To ensure accuracy, in each case the single and double angles should be observed, with zero set in the first instance to the magnetic north; and if it can be managed, on both faces of the instrument; but this is not essential. It is well to remember that the nearer each of the angles of the triangle is to 60°, the better it is for the work, as well-formed triangles give more correct results than ill-formed ones. If possible, angles of less than 20° should be avoided.

		11·69 16·84		=16·81 chains.
° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″
28 51 00 78 24 40	52 00 08 00	245 51 00 353 07 00	52 00 07 30	245 51 40 △ B14 353 07 20
28 50 50 78 24 40	27 00 43 00 18 00	167 27 30 274 42 00 196 18 00 △ B16	27 00 43 00 18 00	167 27 10 △ B14 274 42 40 196 18 00
Cross River.		(37·72)		250 50′ 55″ Side deduced. 55° 30′ 30″ △B14
264 21 50	45 30	279 45 00	46 00 24 00	279 45 30
264 22 30	24 00 01 00	15 23 00 111 10 00 △ B15	01 30	15 23 40 111 01 10 95° 37′ 50″ 22·06
		14·58 22·06		=22·09 chains.
55 30 50 207 49 20	50 00 20 00	193 51 00 138 19 30	51 00 20 00	193 50 40 △B16 138 19 50
55 30 10 207 49 10	01 00 30 30 41 00	346 00 00 290 30 30 82 41 00 △ B14	01 00 30 30 24 00	346 00 40 △ B16 290 30 30 82 41 21

117. JUNCTION LINES AND POINTS. At all junction lines and stations, angular and linear measurements should invariably be made to the adjoining stations and recorded in the field-book, with corresponding diagrams; all lines, angles, distances, and names relating to the same being duly written in, as a guide to the computers and plotters in the office. In this way there can be no doubt afterwards as to the closing and starting points of 'the various traverses.

118. PERPENDICULARITY OF FLAG. Care should be taken to see that flagmen hold their flags perpendicularly, and that the point of a flag is made to rest on the centre of the mark, otherwise the angular work will be rendered erroneous.

119. METHOD OF CHAINING. The chaining of distances should always be carefully attended to; for any serious mistake in this respect may necessitate the whole of the measurements of a traverse, the linear as well perhaps as the angular work, being re-done. The alignments should be carefully made from flag to flag by the leader and follower being careful to see that the respective forward and back flags are seen by either over the head of the other throughout the measurement. The chain must be drawn tight, but not pulled away from the pins by overdrawing it, and the return of the 10 pins from the follower to the leader should be correctly made and noted after every distance of 10 chains measured over of the same line.

120. CHECK AZIMUTHS. The angular work of a main-circuit should be tested at intervals of about every 30 stations or so, by observations for azimuth to stars or the sun. The convergency, in latitude 20°, for successive intervals of 10 miles east or west of origin, would be about three minutes. Convergency has to be allowed for in all cases where check bearings are observed.

121. FIELD-BOOK. The field-book entries are made from the bottom upwards, as in the form given in Art. 86. At the commencement of a survey the name of the village under survey, the date of survey, and the name of the surveyor should be written at the bottom of the first page. Above this a diagram is to be made and the name of the village under survey and those adjoining are to be entered; the angles made by the forward line with the two village traverse lines on either side are to be likewise recorded, as also the linear measurements of all three lines emanating from the starting-point. With the angles and distances thus known, it will be an easy matter to find out in the office the station from which a particular traverse starts. When a triple junction of the village under survey is reached, a line is to be drawn across the side of the page in the field-book to the right and the name of the village next in succession entered therein. So also when the traverse is closed either on the starting-point, or a station of another traverse, a diagram is to be drawn in the field-book and the observed angles at the connecting stations together with the measured distances of the lines emanating from there are to be duly recorded therein. The proper symbol denoting whether a station has been permanently marked on the ground or not should invariably be entered in the field-book alongside of the number appertaining to the station and always *preceding* it.

122. BEARINGS. As the zero at a station for the first set of readings is set to the magnetic north, it follows that the first reading to a back station + or — 180° should coincide to within a degree or so with the station of observation's first reading from such back station.

123. SUBTENSE POLE OR BAR. (Figs. 387, 435*a*.) In taking survey lines across rivers, it would be as well if vertical observations were taken to a subtense pole to form a check on the horizontal measurements. To do this set up a 10-link rod on a peg at the station on the opposite side of the river; the *perpendicularity* of the rod must be ensured. With the theodolite on the side of the river and at the point from which it is desired to take the line across to the other side observe the vertical angle of elevation or depression, as the case may be, carefully to both the top as well as to the bottom of the rod precisely, on faces left and right of the instrument. Record these angles in a spare corner of your field-book as given below, the necessary calculations being left to be done in the office (*See* Fig. 387; also Art. 131.) In chaining over very undulating ground, or

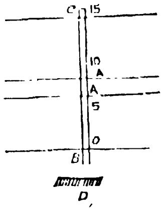
along slopes or the sides of precipitous hills, it would be advisable also to take subtense pole measurements by way of a check, in addition to the customary chaining.

SUBTENSE POLE.

		° ′ ″	′ ″	° ′ ″	° ′ ″
Bottom	L.	D. 0 48 30	48 00	D. 0 48 15	D. 0 48 20
,,	R.	D. 0 48 10	48 40	D. 0 48 25	
Top	R.	D. 0 39 30	39 10	D. 0 39 20	D. 0 39 13
	L.	D. 0 39 00	39 12	D. 0 36 06	
		△B15△B16			

Another method which has virtually superseded the vertical subtense pole is that which is known as the horizontal subtense bar method. This should be employed in preference to the other if the requisite bars are available, as it is more accurate, although not quite so convenient in regard to portability. It is also preferable to use the horizontal bars instead of the vertical poles, firstly, because the horizontal angular measurements in all instruments are more to be relied upon than the vertical ones; secondly, because surveyors are more accustomed to the horizontal than to the vertical reading; and, thirdly, because a greater number of horizontal readings of an angle can be taken in a like space of time as of vertical ones. For ordinary purposes a 10 feet, or even a 10 links, bar would be all that would be required, as these suffice respectively for rays of 1½ miles and 1 mile in length. For greater distances the subtended angle would require to be repeated 20 or more times instead of 10 or 15 times, the number ordinarily required. The measurement of the angle is effected by means of the tangent screws only. The screws of both the upper and lower plates require to be unscrewed to their full extent and the lower plate then clamped. The board of the left end of the bar is then intersected by moving the upper plate, then clamping and finishing the intersection with the tangent screw. The limb here is read off and recorded, all the verniers being entered in the record. Then by means of the upper plate tangent screw only the board at the right end of the bar requires to be intersected and one vernier read off to give an approximate value to the subtended angle, which would be found useful as a check against gross errors. Next, with the lower plate tangent screw, the telescope is to be brought back to the left end of the bar and the process repeated as often as necessary (ordinarily 10 or 15 times), finishing off on the right end board of the bar. Finally, all the verniers are to be read and recorded and the difference of the means of the first and last readings divided by the number of repetitions. This gives the value of the subtended angle, and a comparison with the approximate value obtained as above described would guard against errors, or against a wrong reckoning of the number of repetitions made. If so many measures were made that the tangent screws had not sufficient play to include them all, the angle would have to be measured again, care being taken that the *lower* tangent screw was screwed while the *upper* plate remained clamped, and the *upper* tangent screw was unscrewed while the *lower* plate remained clamped. It should be noted that, if the bar is held horizontally instead of vertically, a difference of level between the observer and the bar does not prejudice the deduced distance, since the fact of the theodolite measuring angles on the plane of the horizon makes automatically the necessary correction from the hypotenuse of the slope to its base. Another reason, already given, for using the bar horizontally instead of vertically, is that the subtended angle is much more easily measured in the former case by the employment of the two tangent screws. The bar (*see* Fig. 435*a*) is generally made 10 feet long, having at each end a diamond-shaped board, about 1½ feet along arc, diagonal, and 1 foot along the other, facing the observer. The longer diagonals are vertical and so adjusted as to be exactly 10 feet apart. The boards are painted white with a streak of white in the centre along the longer diagonal, for convenience of intersection. A short iron bar about a foot long and about $\frac{1}{10}$ inch thick, carrying a V-sight at one end and a head at the other, is firmly screwed to the middle of the subtense bar, and so adjusted once for all that its sight-line is at right angles to the bar. A small level is also inserted in the upper side of the wooden bar to ensure its horizontality when in use, and the bar itself so constructed as to be capable of adjustment on an ordinary plane-table stand. It will of course be necessary to be careful that the bar is held at right angles to the ray under measurement. The attendant, who is told off for the work, should hold it in the proper position until he receives a signal of some preconcerted kind from the observer to signify that the observation is completed. It is desirable for the attendant in charge of the bar to be well drilled in his work, as there is no check on him unless the ray be short enough to enable the observer to see the sights himself when looking through the theodolite telescope; this is generally possible up to 300 or 400 yards. On the other hand, it is worth noting that the error arising from a slight misdirection of the bar is much less than would at first sight be supposed so long as the error in pointing—this is, the excess or defect from 90° of the bar's direction with the ray under measurement, does not exceed 35′ (an almost impossible quantity, unless the bar be purposely misdirected)—the error in measurement will not amount to $\frac{1}{2}$ foot per mile. Tables of distances, in chains and links, in connection with the subtense bar measurements corresponding to the angles subtended by 10 feet and 10 links bars, are to be found in the "Calculating Tables" (Tables XVIII and XIX, 5th edition).

In route-surveying of a rapid nature,* the traverse may be made with a theodolite, having a micrometer † fitted to the eye-piece, enabling the observer to measure the number of revolutions of the screw-head that a known height subtends, the angular value of one revolution of the screw-head having been previously determined Thus, CD is a staff marked 0, 5, 10, 15 feet. The micrometer has one fixed wire, A, and two moveable wires, B and C, attached to the screw-heads. In observing, the fixed wire is made to cross the staff at any point, A; and if a 15-feet subtense bar is being measured, the moveable wires B and C are made to coincide with the marks on the staff at 0 and 15; and number of revolutions 15 feet subtends = rev. AC + rev. AB. In all cases, to prevent mistakes, a second reading should be taken, the fixed wire being made to cross the staff, say at A^1: then 15 feet subtends rev. A^1C + rev. A^1B; = rev. AC + rev. AB, if the observation has been made correctly. The computation is made very simply by means of a table showing the distance in miles from the instrument to the staff corresponding to the number of revolutions that 15 feet subtend; the 10 and 5 feet marks are necessary for short distances, and the same table is used, by multiplying the revolutions by $\frac{3}{4}$ for a 10 feet staff, and multiplying by 3 if a 5 feet staff is observed to. In all cases, the angle of elevation or depression to the staff is to be taken, and the distances reduced to the horizontal plane. The whole of the traverse may be computed out by tables of latitude and departure and then plotted. This is a better method of plotting a long route, for a small scale map, than plotting the traverse on a large scale, and reducing it by squares or any other method.



124. AREA OF MAIN CIRCUITS. The area of main circuits should be limited, as far as the conditions of the country allow, to from 50 to 100 square miles, and in exceptional cases up to 200 square miles.

125. CONNECTING OLD WORK. In taking circuits along boundaries over which former traverses have been run, it is imperative upon the surveyor to pick up for his own work as many of the theodolite stations of the old survey as possible.

126. THE ORIGIN. The origin to be assumed for the survey in a district should be situated as much as possible in the centre of the tract for survey, and if a G. T. station is thus conveniently situated, it should be adopted for the origin.‡

OBSERVATIONS.

* Such as was so successfully performed by Major Carter, R.E., in the Survey Operations in Abyssinia on the return journey of the force from Magdala to the coast.

† This instrument was introduced by General Sir James Walker, R.E., Superintendent, G. T. Survey of India, for mountain route surveying, or traversing over raviney and marshy ground; in the former case, it is particularly useful in enabling the surveyor to avoid all the small bends at which the instrument would have to be put up where measurement has to be made with chain or perambulators. The accuracy of the instrument when tested by running a perambulator over the same ground, has been found to agree to the third place of decimals of miles in the comparative results.

‡ In the absence of a suitable G. T. S., the practice is to assume the intersection of convenient geographical lines of latitude and longitude, *e.g.*, 17° 00′ 00″ N. and L. 96° 00′ 00″ E. for the survey of Pegu district, as the origin of the survey.

If the survey to be done is to be connected with old work, the origin assumed for the former survey might be accepted, if convenient, for the new work. For the village plots furnished to the cadastral section, each village is plotted on its own origin, called the "first of its series." The meridian of the station, or of the assumed point of origin, is to be regarded as the initial meridian of the survey, and all azimuth observations made at other stations are to be reduced to it, by allowing for convergency, before being used as elements of correction or check.

127. REDUCED AZIMUTHS. If the deduced and reduced azimuths differ by more than two minutes, corrections must be applied in the computations and the angular work made to agree with the reduced value.

128. MAIN CIRCUITS AND VILLAGE AREAS. The areas of village traverses enclosed in a main circuit should agree within ½ per cent. with the computed area of the main circuit. The area by summation of villages, however, is what is adopted for the record of district areas. The village traverse areas should always be checked by the planimeter, and the area thus obtained noted alongside of the other computation; the areas found by the universal theorem and by the planimeter should agree within 1 per cent.

129. DEFECTIVE DEMARCATION. When the demarcation of village boundaries is found to be defective, the surveyor should report the circumstance to the Camp Officer. He should at the same time do all in his power to remedy any defects, and get the villagers to help in this respect, if necessary, so that the survey work may not be seriously impeded.

130. PARTALLING. When village traverses are found not to close, it may become necessary to partal, *i.e.*, check, portions or even the whole of either or both the angular and linear measurements of the traverse-surveyor. This work is ordinarily performed by the Inspector or, if practicable, by the Camp Officer himself, but where this is not possible a good traverse-surveyor on whom dependence can be placed, being other than the person who executed the original survey, is sometimes deputed to this duty.

131. CALCULATION OF DISTANCE FROM ANGULAR MEASUREMENTS ALONE. (Fig. 387.) The calculation of the length of a chain line, for which the angular measurements alone have been obtained, by triangulation and the subtense pole, as described in Arts. 116 and 123, according to the readings recorded in Fig. 387 and the field-book which appears in Art 123 are given below. Such calculations, it may be observed, are not intended to be performed by the surveyor, but would be done in the office.

△ Calculation.

Station.		Observed angles.		Correction. +		Corrected angles.
B14	...	55° 30′ 30″	...	00′ 30″	...	55° 31′ 00″
B15	...	95 37 50	...	...	...	95 37 50
B16	...	28 50 55	...	00 15	...	28 51 10
Total	...	179 59 15	...	00 45	...	180 00 00

L. cosec. ... 28° 51′ 10″ = 10·316148

L. sin. ... 55 31 00 = 9·916081

Log. 22·08 $\left(=\frac{22·06 + 22·00''}{2}\right)$ = 1·343999

B15 to B16 = chains 37·72 = 1·576528

S. P. Calculation.

Stations.	Object.		Face.	Verniers. A	Verniers. B	Means.	Angles.
From B15	Bot. Rod	...	L.	D. 0° 48′ 30″	D. 48′ 00″	D. 0° 48′ 15″	D. 0° 48′ 20′
	Do	...	R.	D. 0 48 10	D. 48 40	D. 0 48 25	
To B16	Top Rod	...	R.	D. 0 39 30	D. 39 10	D. 0 39 20	D. 0 39 13
	Do.	...	L.	D. 0 39 00	D. 39 12	D. 0 39 06	

Bot. ... L. Cos. 0° 48′ 20″ = 1·999958

Top. ... „ 0 39 13 = 1·999972

Diff. ... L. Cosec. 0 09 07 = 2·576438

Rod. Log. 10 = 1·000000

B15 to B16 = chains 37·70 = 3·576368

By the horizontal subtense bar method, for an observed angle of 0° 13′ 48″ with a 10 feet bar, the distance B 15 to B 16 = 37·74 chains (according to Table

XVIII of the "Calculating Tables," 5th edition); and for an observed angle of 0″ 09′ 06″ with a 10 links bar, the distance B 15 to B 16 = 37·78 chains (according to Table XIX of the "Calculating Tables," 5th edition).

(ii) THE RECESS SEASON.

132. DUTIES. The principal work that has to be done during the short recess-season consists chiefly of all the traverse computations remaining over from the field-season, azimuth computations, computation of rectangular co-ordinates, latitudes and longitudes, completion of traverse tables, plotting of balance of 16″ sheets, preparing a 2″ plot of the traverses, writing up tables of co-ordinates and plotting the graticule* and station points on the 2″ sheets.†

CADASTRAL SECTION.

(i) THE FIELD SEASON.

133. FIELD-SURVEYOR'S DUTIES. The field surveyor's duty is to survey all fields and open ground, and all topographical features including gardens, village-sites, roads, creeks, houses, tanks, jungle, and the like. If there is jungle in the village he should examine it for cultivation and find out from enquiry where such exists in order to survey it. Before commencing the survey of a village a field-surveyor should invariably go over the ground to be surveyed and make himself acquainted with the principal features of the land to be surveyed, taking notes at the same time as to the situation of the survey stations, cultivation, topographical features, and the like. This will help him considerably in laying out suitable lines for his survey and work will be thus minimized.

134. SYSTEM OF SURVEY. Filed-surveyors should invariably survey on one uniform system. Fixed points are provided for them at convenient intervals with the traverse lines run at about 40 chains apart. They should chain between those fixed points, cutting up the block for survey into small rectangular pieces of 8 or 10 chain widths before they commence the field survey at all. Even these chain lines for cutting up the theodolite block should be run where they are required most for taking up field boundaries. The principle is to economize chaining and to have short offsets. Under this system even a careless surveyor can hardly do bad work, and if he makes a mistake in chaining, he at once detects it. The plane-table sight ought only to be used in a close country, and the system of survey by chained triangles only when cultivation runs up into jungle, or in a survey which is in advance of the theodolite survey, when there is no help for this. In a survey which has to be done in advance of the traverse (and this may at any time become necessary), what is known as the "triangulation system" has to be adopted. This consists of the whole of the village for survey being broken up by a series of large and well-formed triangles, the sides of which are carefully measured (with double chainings) and plotted, upon which the whole of the detailed work is surveyed and mapped. For this class of work the following procedure should be adopted as much as possible. The surveyor should walk over the whole of the ground and note how the cultivation and natural features lie. He should so start his work on his sheets that the whole may approximately fit in as compactly and upon as few sheets as possible. He should start where cultivation is most extensive and generally about the centre of the village. Base lines, as long as can be conveniently got, should be carefully measured, and a complete fabric of triangles built upon these to start with. The whole of the cultivation and village sites, wherever in the village, must be connected by a complete system of accurately measured triangles. With such a network of triangles the detailed

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OBSERVATIONS.

* A graticule is the representation by a net-work of lines of meridians and perpendiculars of a small-scale map on paper.

† The Indian Survey Committee appointed by the Government of India in 1904-05 recommended that standard maps should be on the scale of 1″ = 1 mile to be represented in colours, and that topographical surveys should as a rule be made on the scale of 2″ = 1 mile for reduction and publication on the 1-inch scale. Sixteen 1-inch standard sheets will make up exactly one degree sheet of the 1 millionth series [*vide* footnote to Art. 31, Chap. VII]. Each map will cover an area of either 15′ 1° or 4° lat. or longitude according to the series to which it belongs. Thus the degree sheets atlas series would consist of (1·M) one to a millionth sheet followed by its 16 component degree sheets, while the 1-inch atlas series would consist of a degree sheet followed by its 16 component standard sheets.

work can be readily and accurately fitted in in the usual way. It will require care and judgment in laying out the triangles so that they may be as few and as large as possible; the larger the triangles and the more they approach the equilateral, the more acourate will the work be. All sides of triangles must invariably be measured *twice* and the mean of the two measurements taken for the correct one. Roads, creeks, boundaries of the village, hills and the like, may be put in by sight-vane measurements only; but in such cases the surveyor should start from some well determined point of his triangulation and close upon a similar point. Where theodolite stations are on the ground, as many of them as possible should be connected with the triangulation by means of triangles. Stations on all sides of the village should be thus shown on the sectional sheets. Where theodolite stations have not yet been laid down by the traverse camp, the work of connecting the stations laid down by the field-surveyors would be left to sub-surveyors to do when the latter arrive at the ground later on in the season. On receipt of the 16″ plots of the villages thus surveyed from the traverse camp, the officer in charge of the Cadastral camp would cause to be traced and fitted in upon them, either in the field or during the recess or both, the work of the sheets done on the triangulation system, which should be inked in beforehand. The tracing glass should be used for this. As much of this work as can be done in the field, when leisure and opportunities permit, should be so done, that as little as possible be left for the recess. Unless the triangulation work is connected with the traverse work on all sides of the village, it will not be of much value. Partal lines should be run over the triangulation work like any other detailed work. Theodolite stations, or those intended to be taken up hereafter as such, should be utilized for partalling as much as possible. The triangulation work must be very accurately done, and the larger triangles upon which it is all based must be absolutely correct or it will be worthless. If there is plenty of open ground, it would be well to have one long straight line, as a main one, running through it and triangles built upon this when required. There would be less chance of error creeping into the work in this way. In no case should the detail survey be entered on the sheets (although there is no objection to putting in of field boundary crossings, etc., in field-books at the time of chaining the main lines for plotting on to the sheets afterwards) till all the requisite triangles and survey stations have been first plotted on the map. See also, in reference to the procedure to be adopted for the survey on the triangulation system, Art. 93.

135. SURVEY OF GARDENS. For the survey of gardens a plane-table must be used. If there are, say, 30 small gardens in a village, commencing from a theodolite station, a traverse should be run round the lot first; then this traverse should be broken down into smaller blocks till all the gardens are surveyed.

136. SURVEY OF BOUNDARY. The first duty of a field-surveyor on entering a village is to survey the boundary. If he finds that the demarcation posts are not a sufficient guide to the exact run of the boundary, he should by enquiry from the villagers find out the boundary line, just as he might have to do for the theodolite stations themselves, and follow it.

137. SURVEY OF DETAILS. When the field-surveyor has cut up his village, he should survey every item, omitting nothing except patches of waste, etc., under 15 links in breadth, which cannot very well be shown on the 16″ scale. As a field-surveyor's work is not always inked in afterwards by himself, he should take great care that none of his survey lines are mistaken by the draughtsman in office for the sides of fields. He ought to do these lines.

138. VILLAGE-SITES. In village-sites all roads dividing houses should be shown, all compounds or gardens and all houses, wells, pagodas, etc. All trees forming prominent landmarks should be accurately laid down by offset; these form useful points of departure or connection for any subsequent surveys.

139. POLE AND BRUSH. When a field-surveyor precedes the traverse-surveyor, he should lay down his stations judiciously, and mark them firmly with a pole and brush, and a mound of earth raised over the station. The traverse-surveyor is bound to make these his own stations.

140. COMMON BOUNDARIES. Creeks, roads, and the like, forming the boundaries of adjoining villages, should be surveyed independently by each of the

field-surveyors concerned on his own village sheets. The common boundaries should be examined in the first instance by the Inspector before the sheets are sent in to the Camp Officer; if they differ considerably from each other, the portions affected should be re-surveyed by the surveyors who did the original surveys. The final passing and inking in of boundaries is to be done in the office of the Camp Officer.

141. SURVEY OF CREEKS. The main bends of creeks should be accurately shown and unnecessary offsets avoided. All creeks should be followed up, and laid down by sight-vane and chain mesurements, by slight clearings, if necessary, in case the surveyor cannot progress along its bed at low water.

142. SURVEY OF HILLS. All prominent peaks of high land falling within the area for survey should be first fixed, either by triangulation or sight-vane, and the requisite connections made by short traverses run, wherever necessary, from one known point to another, on the plane-table. Measurements should be made up the slopes of hills, wherever required, and the contour lines drawn in for 10 feet vertical intervals, the minor details being sketched in, based on the principal measurements. If it is found inconvenient to determine the positions of the contour lines by short horizontal chain measurements, aided by a 7½ links or 5 feet rod, a clinometer for measuring the angle of inclination may be employed and the sloping distance measured, whence, with the aid of Table XIII of the "Calculating Tables" (5th edition), the horizontal distances for vertical intervals of 10 feet may be obtained. The crests and feet of the hill ranges and their principal spurs should invariably be laid down in the first instance before the contour lines are attempted.

143. CONVENTIONAL SIGNS. When surveying jungle or gardens, the surveyor should denote these on his map by the usual conventional signs; the names of rivers, creeks, villages, adjoining villages, etc., should always be ascertained and written in in pencil.

144. PARTAL LINES. One or more partal, or check, lines should be run in every sheet of a village without exception, by the Inspector; at least one-tenth of the interior fields should be thus tested. A "pucka," or long partal, should be run across every village, while several "kacha," or short partals, are to be taken over different parts of the village. "Pucka" (or "primary") partals are those which are run from one survey station or traverse line to another survey station or traverse line; and "kacha" (or "secondary") partals are such as have one or both ends of the line terminating on any point—the field-surveyor's temporary stations (*e.g.*, auxiliary, quadrilateral, triangulation), intersections of field-boundaries, field-cuttings, and the like—other than a survey station or traverse line. Ten per cent. of the villages should also be checked by the Camp Officer, including portion of the work done by the Inspectors. It is a good practice for the Camp Officer to confine a large proportion, say, one-half, of his partals to the Inspector's identical lines, for it is clear that if the Inspectors do their duty, the survey work cannot but be good. Corrections of errors discovered should be made on the spot, except when the work is found to be so inferior in quality as to require to be re-done, when the pencilling should be rubbed out with a piece of India-rubber and the sheet returned to the surveyor for the necessary re-survey or a fresh sheet plotted and handed over to him for the purpose instead, whichever course is considered the more expedient. A freshly surveyed sheet would have to have fresh partals taken over it. The lines intended for check should be carefully selected; they need not always go from station to station, nor be confined to cultivation alone; they might sometimes be run from a survey station to a known point on a traverse line, or they may cut across a belt of jungle, thus testing the topography as well as cultivation. As much as possible, however, partal lines should start from and close on theodolite stations. As a rule the line of partal should include all features of country within two chains, right and left, and all the required measurements invariably entered in a field-book. Partal lines taken over villages still under survey should not be plotted from field-books days after the partals have been made, as this would give the field-surveyors, to whom the sheets are generally returned, opportunities of rectifying any errors of work during the interval. It is a good rule, and a practice that should not be lightly depaited from, to plot the partals of the day, whether of

board-plan or field-book, on the same day that the partals have been run, or, if this cannot be managed owing to any good cause, on the following day at the latest.

145. CLASSES OF WORK. Work is divided into three classes, namely,
1st class, which signifies agreement like a trace;
2nd class, which is work that is out generally 10 or 15 links; and
3rd class, which is work that is rejected.

146. RESPONSIBILITY OF INSPECTOR. If an Inspector passes bad work, or does not see to his surveyors working rapidly and accurately, he would be held responsible for inaccuracies found in maps. Among other things, the Inspector should see that the boundary of a village is correctly surveyed, and that fields belonging to another village have not been taken in on a sheet or any left out.

147. SIGNATURES. The signatures of the persons who have surveyed or tested the work, with dates, should always be entered on the reverse of the sheets; so also of those who have inked up the work, or numbered the fields, or passed the boundaries, etc. Such entries should be made neatly and legibly by the parties concerned.

148. BENCH-MARKS. Where lines of levels have been previously carried through a country under survey, care should be taken to find out the localities of all existing bench-marks and to accurately survey in their positions on the maps.

149. FIELD KATANS OR BOUNDARIES. When during the process of chaining, the chain crosses or cuts intervening field-boundaries, the practice is to make a long slight cutting on the ground with an axe or other such instrument in the direction the chain has taken. These cuttings are technically termed "katans," and are useful points for taking other requisite measurements from, and also for future reference by the surveyor and the partaller. Similar cuttings are made in field-boundaries to which offsets on a chain line are taken.

150. OFFSETS. Offsets should be short, and should not ordinarily be over 1 or 1½ chains in length.

151. TESTING OF CHAIN. The chain should be tested frequently, every day before use, if possible. This could be done on first arrival at a place by driving two pegs firmly into the ground, at the correct distance apart, so that the handles of the chain may, when it is stretched out to its utmost, fall into the pegs exactly. The ground selected should be fairly level. Once a month the chain should be taken into camp and tested with the standard steel bars kept there for the purpose. Each Inspector also should be provided with a chain well tested with the standard bars, which should be kept by him only for the purpose of testing every now and again, as opportunities occur, the chains in use with his field-surveyors.

152. DEFECTIVE DEMARCATION. When the demarcation of gardens, village-sites, and the like is found not to have been done, the field-surveyor should report the matter, through his Inspector, to the Camp Officer. He should at the same time do his best to get the villagers concerned to help in completing the requisite demarcation, so that his work may not be seriously delayed for want of an effort in this respect on his part.

153. CLEAN MAPS A REQUISITE. Maps should be kept clean and free from stains, the work of those not so found, although in other respects good, are usually treated as of the second class and paid for at the rates for such class. A map sheet mounted on the plane-table when not in use should be covered with a sheet of Bally paper over which a piece of oil-cloth of the required size should be tied. Soiled sheets may be cleaned with stale bread-crumbs.

154. AREA STATEMENTS. The area of a village as found by the summation of fields should agree within 1 per cent. with that obtained by traverse (+ or — the offsets); with careful work the difference should rarely exceed ½ per cent. The difference of area between the two methods is to be recorded at the end of a village area statement. For all revenue purposes the village area, as obtained by the summation of fields, is always accepted. Area statements are prepared by two computers working independently of each other, the areas of the items being at first entered in pencil; both copies are then examined by the Inspector, and, when the

corresponding entries agree within 4 per cent. of each other, their mean is taken and entered at once in ink; but when the difference is greater than the limit mentioned, the area should be carefully worked by a third party and his results accepted by the Inspector as final. For the smaller items the acre-comb is employed for calculating the areas; for the larger ones, the planimeter.

Although for the ordinary average-sized fields the acre-comb workings of two independent workers are usually passed (their means being accepted) when not differing by over 4 per cent. of each other, it is essential that, when the fields are very small or very large, a different system be adopted for the passing of the areas. When therefore fields are found to vary a good deal from the ordinary size, the following table might be used in determining the areas that are to be re-worked, namely :—

Where the area, in acres, of the lesser working is—

					Difference allowed.
From	0·01	to	0·19	=	0·02
,,	0·20	,,	0·49	=	0·03
,,	0·50	,,	0·99	=	0·04
,,	1·00	,,	1·49	=	0·05
,,	1·50	,,	1·99	=	0·06
,,	2·00	,,	2·99	=	0·07
,,	3·00	,,	4·99	=	0·08
,,	5·00	,,	6·99	=	0·09
,,	7·00	,,	9·99	=	0·10
,,	10·00	,,	14·99	=	0·11
,,	15·00	,,	19·99	=	0·12

For the larger items, where the areas are calculated (by two workers) with the planimeter, the results should ordinarily agree within ¼ per cent., but in no case should the areas be passed when differing by over ½ per cent.

(ii) THE RECESS SEASON.

155. DUTIES. The principal work done by all the hands in the recess consists of the passing of common village boundaries, the inking up and typing in of all the 16″ original sheets, numbering of the fields and other items of the map, printing of names on the maps, preparing the area statements, drawing of the litho sheets for reproduction on stone, and the preparation of the final 2″ reductions for publication. The margins of the original 2″ sheets should be carefully examined and compared with those of the adjoining sheets and all necessary rectifications made before the sheets are finally passed. These 2″ sheets occupy 7½′ of longtitude × 7½′ of latitude, eight going to form a section comprising 30′ of longitude × 15′ of latitude. The 16″ sheets are 26″ long by 18″ broad in respect to the space worked upon, and enclose an area of 1·83 square miles.

CHAPTER XIV.

Gale's Traverse System.

1. DEFINITION OF A TRAVERSE. (Fig. 444.) A *Traverse* is the course taken in proceeding from one point on the earth's surface to another by a circuitous route, along lines of various lengths and situated in different directions. If a traverse is made from any point, as station 1, to any *other* point, as station 12, the traverse 1 2 3 4 5 6 7 8 9 10 11 12 is termed an *unclosed traverse*; but if the traverse is continued by some other route, 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 1 back to station 1, the starting-point, thus forming a complete irregular polygon, it becomes a *closed traverse*.

2. BEARING, DISTANCE, LATITUDE, AND DEPARTURE. (Fig. 335.) The angles which the successive lines of a traverse form with the meridian are called *bearings*, and the lines themselves *distances*. The quantity which one end of a line is due north or south of the other end, is called the difference of *latitude* of the two ends of the line, or its "northing" or "southing"; and the quantity which one end of a line is due east or west of the other is called the *departure*, or its "easting" or "westing." Thus in the Figure, with respect to AB the first

line of a traverse, the re-entrant angle NAB is the bearing of B at A, the length of AB is the distance; the latitude of B with reference to A is represented by the quantity *b*′B=A*b* north, and its departure by the quantity *b*B=A*b*′ west.

3. RECTANGULAR CO-ORDINATES. (Fig. 335.) *Rectangular co-ordinates* are the latitudes and departures of stations reckoned from the first station of a Traverse. Thus, in the Figure, the co-ordinates of station D as referred to the station of origin A, are *d*′D=A*d* latitude south, and *d*D=A*d*′ departure west. A traverse plotted in the ordinary way by the protraction of successive bearings and distances is subject to much accumulative error, hence the methods of co-ordinates, known as Gale's system, where each station is laid off from a common meridian independently of the preceding station, is resorted to where any pretence to accuracy is made. This is invariably the case when a theodolite is employed on the traverse. If a prismatic compass or sextant were used, in a small traverse of course, the application of the rigorous method of Gale's system would hardly be warranted, as the angular work itself would be approximate.

4. CALCULATION OF CO-ORDINATES. (Fig. 335.) Let it be required to find the co-ordinates of C with reference to the origin A, the latitude and departure of B from A, and of C from B being known:—

Latitude of C (south) or Ac = $bc - b$A;
Departure „ „ (west) „ cC = A$b' - c'b$.

In the same way to obtain the co-ordinates of D, we have:

Latitude of D (south) or Ad = A$c + cd$;
Departure „ „ (west) „ dD = cC + $c'd'$.

Thus the co-ordinate values of the other stations E, F, G, and H can also be found, being always calculated to the east or west of the meridian NS or to the north or south of the perpendicular WE, by summing up the successive latitudes and departures up to each point, which may be comprised in the following rule:— "When the distances run north of the first station, add them one to another, until they change to south, then deduct them one by one until the southing exceeds the northing, when deduct the latter from the former, changing the denomination to south; all distances then going south are added, and those going north deducted, and so on, until arrived back at the original starting-point. Likewise, in the distances on the perpendicular, when the distances run east of the meridian of the first station, add them one by one until they change to west, then deduct them until the westing exceeds the easting, when deduct the latter from the former, changing the denomination to west; all distances then going west are added and those going east deducted, and so on, until arrived back at the original starting-point."

5. MERIDIANS. *Meridians* are north and south lines, which are supposed to pass through every station of a survey, running parallel to each other. In reality they are not parallel, but converge towards the poles of the earth, but so insensibly as to be unworthy of notice. Hence all traverses are made on the supposition that the earth is a plane.

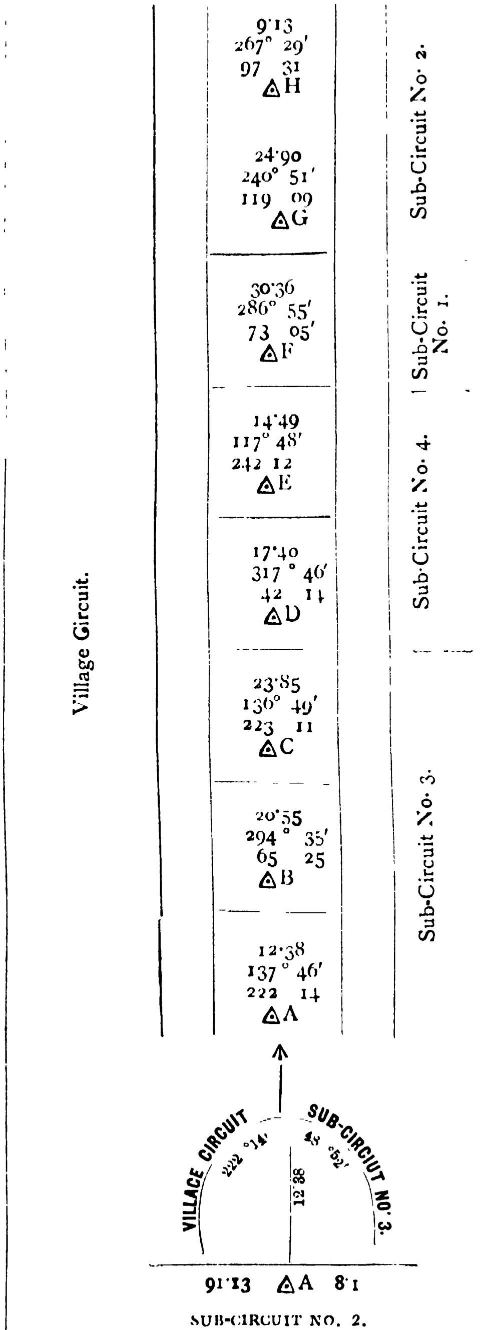
6. MODE OF TRAVERSING. (Fig. 335.) To perform a traverse such as is shown in the Figure, set up the theodolite at the station of origin A, and send flags to be put up at all the other stations, measure the *inward* angle HAB (222° 14′), by setting zero to the back station H, turning the telescope to the *forward* station B, and noting down in the field-book the reading of the vernier A; also measure the *outward* angle BAH (137° 46′), by setting zero to the *forward* station B, turning the telescope to the *back* station H, and noting down in the field-book the reading of the vernier A. The sum of the two angles should of course make 360°, and the reason for observing the outward angle at each station is to afford a check against inaccuracy of reading of the inward angle. Next chain the distance AB (12·38 chains), which also record in the field-book. Again, set up the theodolite at B, measure the inward angle ABC (65° 25′), and the outward angle CBA (294° 35′) as a check on the former, and chain the distance BC (20·55 chains). Proceed in this manner setting up the theodolite in succession at C, D, E, F, G, H, and measuring the inward and outward angles and distances, until the station of origin A is arrived at. Do not omit to take intersections to all conspicuous objects within the polygon, such as temples, large trees, buildings, etc.; also in

chaining the distances between the stations, take offsets right and left to streams, roads, fences, boundaries, tanks, etc., as in a chain survey, making a careful record of the same in the field-book. Be careful always to keep the boundary of the polygon under survey on the *left* hand while carrying on operations from station to station.*

7. Field-Book. (Fig. 335.) The field-book for a theodolite traverse is precisely the same as that used for a chain and prismatic compass survey, except that angles are recorded in place of bearings at the beginning of each line. The following field-book contains a record of the angular and linear measurements made of the polygon shown in the Figure, and entered in the central column. The narrow columns on either side of the central column are intended for the record of verniers B and C of the theodolite, when such are read. In the right and left margins are to be entered the names of adjoining lands and estates, and the points where their boundaries commence and terminate to be noted ; also for the record of offsets, which however has been omitted in this field-book. No other remarks seem necessary, as the entries in the field-book will, with the aid of the Figure, sufficiently explain themselves.

8. Magnetic and True Meridian. In every traverse the meridian of the station of origin has to be obtained ; this may be either the magnetic or the true meridian. In the case of a small traverse, the magnetic or true meridian as obtained in (Chap. X, Art. 24) will suffice, but when a polygon of several hundred square miles has to be traversed, it becomes necessary to observe the true meridian astronomically, as explained in (Chap. XVI). In either case, the magnetic or true meridians, as the case may be, of all the other stations of a traverse are deduced from the given one of the station of origin.

9. Conditions of a Traverse. (Fig. 335.) When a traverse makes a complete circuit, *i.e.*, returns to the starting-point, there are three different conditions that must be fulfilled :—



Survey of (Twante) VILLAGE CIRCUIT menced at the junction of Sub-Circuits 2 and 3 and Village Circuit.

FIELD-BOOK.

1*st.*—That the sum of all the inward angles must be equal to twice as many right angles as the figure has sides, less four right angles.

OBSERVATION.

* Traverse lines for 16″ scale surveys are ordinarily taken at from 10 to 20 chains in length, as the distance between two adjoining stations, and in cultivated tracts the sub-traverse lines are usually laid out in parallel columns, more or less, about 30 chains apart. This, however, depends entirely upon the size of the blocks for survey and the general run of the ground.

2nd.—That the sum of the northings, or distance travelled north, must be equal to the sum of the southings, or distance travelled south.

3rd.—That the sum of the eastings, or distance travelled east, must be equal to the sum of the westings, or distance travelled west.

For example, in the given traverse, which is an 8 sided figure,

(1) the sum of the inward angles must equal $8 \times 180^\circ - 360^\circ = 1{,}080^\circ$;

(2) the northings or $Ab + de + fg + gh$ must equal the southings or $bc + cd + ef + Ah$; and

(3) the eastings or $b'c' + d'e' + e'f' + f'g'$ must equal the westings or $Ab' + c'd' + g'h' + h'A$.

10. ANGULAR AND LINEAR CORRECTIONS. In practice it will be found that the three conditions mentioned in (Art. 9) are not absolutely true, owing to unavoidable errors of measurement in the field. The sum of the inward angles may be found to be in excess or defect by a few minutes of the required number of degrees, and the latitudes and departures may not agree to within several links, the limit of error allowed in the Revenue Survey of India being 1 link in every 10 chains; hence it becomes necessary to apportion these errors. The total error being known, it serves all practical purposes to apply a minute here and there against the largest angles with the sign + or — prefixed as the case may require; so also with the latitudes and departures, apply a link here and there against the longest lines, with the sign + or — prefixed, as the case may require, in their respective columns of northings, southings, eastings and westings. Strictly the error should be apportioned to each of the distances by the following proportion:—"As the sum of all the distances is to the whole error, so is each distance to its correction;" but the rule is seldom, if ever, observed in practice.*

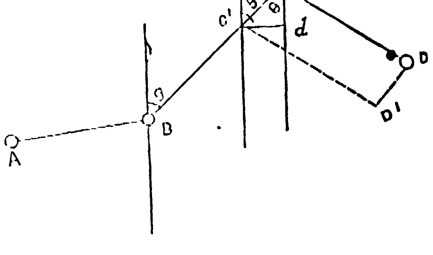
11. DEDUCED BEARINGS. (Fig. 335.) The *deduced bearings* of the various lines of a traverse are found by the following rule:—"To the bearing of the line preceding that of which the bearing is sought, add the inward angle formed by these two lines, and the sum increased or diminished by 180°, according as it may be less than, or in excess of, 180°, will be the bearing of the next line. If the sum be 540° or more, deduct 540° from it for the next bearing; and if 180° exactly, subtract 180° from it." For example, in the Figure, the true observed bearing of the line AB at A was 281°40′, and the inward angle ABC, 65° 25′, then the true deduced bearing of the line BC at $B = (281^\circ 40' + 65^\circ 25') - 180^\circ = 167^\circ 05'$.

12. CARDINAL DIRECTION. (Fig. 335.) The magnitude of the deduced bearing shows at once the cardinal direction of the line. If the bearing is

---

OBSERVATION.

* A frequent source of error in the angular work is to record the forward angle for the back and *vice versâ*; if this is in *only one station* it is readily detected, for if the difference between the forward and back angles of a station be equal to the total angular error of the traverse, the error is probably in that station; if the traverse closes when worked out with the transposition of the back and forward angles, it indicates that the error has been properly located. If the error is in the linear work in *only one line* in a circuit the exact line can be found. Suppose in a line BC, instead of the correct measurement, a measurement equal to BC^1 was recorded, so that the co-ordinates of the point C will by the error be those of C^1; then all the angles and all the other distances of the circuit being correct, it is evident that the total difference of northings and southings instead of being 0 will be Cd, owing to the error in the position of the point C, and that of the easrings and westings C^1d. So that if θ be the reduced bearing of the line where the error was made, the total difference of eastings and westings, *viz.*, C^1d, divided by the total difference Cd between the northings and southings, will be the tangent of θ or reduced bearing; and from tan. θ the angle θ can be obtained from any book of Logarithms; and θ being known, the line of which θ is the reduced bearing is obtained by reference to the traverse sheet. [Adopted from D'Cruz's Surveying and Levelling].



between 0° and 90°, *i.e.*, between north and east, the line must lie in the N.E. quadrant; if between 90° and 180°, *i.e.*, between south and east, in the S.E. quadrant; if between 180° and 270°, *i.e.*, between south and west, in the S. W. quadrant; and if between 270° and 360°, *i.e.*, between north and west, in the N.W. quadrant. Thus the line AB, whose bearing is 281° 40′, is in the N.W. quadrant.*

13. REDUCED BEARINGS. (Fig. 335.) Since the logarithmic tables only extend from 0° to 90° of arc, equivalents to all those bearings which are over 90° must be obtained. These equivalents are termed *reduced bearings* and are what are used in the traverse computations to take out the differences of latitude and departure corresponding to each line. To obtain, therefore, the reduced bearings, or angles formed with the meridian either from north or south, proceed thus :—

When the bearing (observed or deduced) is—

between 0° and 90°, or N.E., no reduction is necessary,
„ 90 „ 180 „ S.E., subtract it from 180°,
„ 180 „ 270 „ S.W. „ 180° from it,
„ 270 „ 360 „ N.W. „ it from 360°.

For example, in the Figure, having given the bearing of the line, AB 281° 40′, its reduced bearing would be 360°— 281° 40′ = 78° 20′.†

14. CALCULATION OF LATITUDES AND DEPARTURES. (Fig. 335.) The latitude and departure of each station from the preceding station is obtained by either the Traverse Table (given in Table XXXIII, " Calculating Tables," 5th edition), or by logarithms (Tables XXXI and XXXII, " Calculating Tables," 5th edition), in both of which cases the reduced bearing is employed. In the calculation by logarithms,

the Latitude = Distance × cos. Bearing (reduced),
and Departure = „ × sin. „ „ .

For example, to find the latitude and departure of station B from A, having given the reduced bearing at A = 78° 20′, and the distance AB = 12·38 chains.

By the Traverse Table—

$$\begin{aligned}\text{Latitude} &= 12{\cdot}38 \times 0{\cdot}2022 = 2{\cdot}50 \text{ chs.},\\ \text{Departure} &= 12{\cdot}38 \times 0{\cdot}9793 = 12{\cdot}12 \text{ chs.}\end{aligned}$$

By logarithms—

$$\begin{aligned}\text{Latitude} &= \text{Distance} \times \text{cos. Bearing.}\\ \therefore \text{log. Lat.} &= \text{log. Dist.} + \text{L. cos. Bearing} - 10\\ &= \text{log. } 12{\cdot}38 + \text{L. cos. } 78^\circ\ 20' - 10\\ &= 1{\cdot}092721 + 9{\cdot}305819 - 10\\ &= 0{\cdot}398540.\\ \therefore \text{Lat.} &= 2{\cdot}50346 \text{ chs.};\\ \text{Departure} &= \text{Distance} \times \text{sin. Bearing.}\\ \therefore \text{log. Dep.} &= \text{log. Dist.} + \text{L. sin. Bearing} - 10\\ &= \text{log. } 12{\cdot}38 + \text{L. sin. } 78^\circ 20' - 10\\ &= 1{\cdot}092721 + 9{\cdot}990934 - 10\\ &= 1{\cdot}083655\\ \therefore \text{Dep.} &= 12{\cdot}1243 \text{ chs.}\end{aligned}$$

15. TRAVERSE COMPUTATION FORM. (Fig 335.) Subjoined is the traverse Computation Form of an actual survey showing the system of entering the angular work and measurements made in the field with the whole process of calculation thereof :—

OBSERVATIONS.

* When the given bearing is 0° or 180° exactly, the reduced bearing will be entered at 0° ; and when 90° or 270° exactly, the reduced bearing will be entered at 90°.

† If the bearing of the line is 0°, 90°, 180° or 270° exactly, the cardinal direction will of course be the N., E., S. or W., respectively.

TRAVERSE COMPUTATION FORM.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	OBSERVED INWARD ANGLE.		DEDUCED BEARING.		REDUCED BEARING.	DISTANCE MEASURED.	DIFFERENCE OF LATITUDE AND DEPARTURE FOR EACH DISTANCE.								DIFFERENCE OF LATITUDE AND DEPARTURE OF EACH STATION REFERRED TO MERIDIAN OF FIRST STATION.		SUMS OF EACH SUCCESSIVE PAIR OF CO-ORDINATES IN COLUMN 17.	AREA.			
							LATITUDE.				DEPARTURE.				LATITUDE.	DEPARTURE.		PRODUCT OF COLUMN 18 WITH 8 AND 9 OR 10 AND 11.		OFFSETS.	
							North.		South.		East.		West.		N. or S.	E. or W.		North. N. × E. or S. × W.	South. N. × W. or S. × E.	+	—
Station.	° ′	Cor.	° ′	Cardinal direction.	° ′	Chains.	Chains.	Cor.	Chains.	Cor.	Chains.	Cor.	Chains.	Cor.	Chains.	Chains.	Chains.	Acres.	Acres.	Acs.	Acs.
A		—										—		+							
B	65 25	...	281 40*	N.W.	78 20	12·38	2·50	...	...	...	...	...	12·12	...	N. 2·50	W. 12·12	W. 12·12	...	3·03	...	1·47
C	223 11	...	167 05	S.E.	12 55	20·55	...	...	20·03	...	4·59	...	...	...	S. 17·53	W. 7·53	W. 19·65	39·36	..	3·14	...
D	42 14	...	210 16	S.W.	30 16	23·85	...	...	20·60	...	...	...	12·02	...	S. 38·13	W. 19·55	W. 27·08	55·78	...	...	2·21
E	242 12	1	72 30	N.E.	72 30	17·40	5·23	...	...	...	16·59	1	...	...	S. 32·90	W. 2·97	W. 22·52	...	11·78	0·52	0·44
F	73 05	...	134 41	S.E.	45 19	14·49	...	...	10·19	...	10·30	...	...	...	S. 43·09	E. 7·33	E. 4·36	...	4·44	...	1·64
G	119 09	...	27 46	N.E.	27 46	30·36	26·87	...	...	...	14·14	1	...	...	S. 16·22	E. 21·46	E. 28·79	77·36	...	...	3·61
H	92 31	...	326 55	N.W.	33 05	24·90	20·86	...	...	...	...	...	13·59	1	N. 4·64	E. 7·85	E. 29·32	61·16	...	1·63	1·46
A	222 14	...	239 26	S.W.	59 26	9·13	...	...	4·64	..	...	...	7·86	...	0·00	0·00	E. 7·86	...	3·65	...	0·73
	1,080 01	1	Proof of Angles = 8 × 180° − 360° = 1,080°.				55·46	...	55·46	...	45·62	2	45·59	2				233·66 22·90	22·90 ...	5·29	11·56 5·29
			* The observed true bearing = °281 40′.															210·76 = Double area of polygon.			6·27
																		105·38 = Area of polygon. —6·27 = Area of offsets.			
							SUB-TRAVERSE.											99·11 = Area of required Figure, in acres.			
B		—										+									
C	118 00	...	167 00	...	...	...	...	...	...	...	...	...	...	...	S. 17·53	W. 7·53					
x	126 50	...	105 05	S.E.	74 55	9·40	...	...	2·45	...	9·08	1	...	...	S. 19·98	E. 1·56					
y	230 50	...	51 55	N.E.	51 55	10·40	6·41	...	...	...	8·18	1	...	...	S. 13·57	E. 9·75					
G	44 12	2	102 45	S.E.	77 15	12·00	...	..	2·65	...	11·70	1	...	...	S. 16·22	E. 21·46					
H	...	...	326 55	...	...	...	...	...	...	...	...	...	...	...	...	...					
	519 52	2					6·41	...	...	...	28·96	3	...	...							

Column 1.—Contains the letters representing the *stations* of the survey. (*See* field-book, Art. 7.)

Column 2.—Contains the *inward angles* as observed in the field. (*See* field-book, Art. 7.)

Column 3.—Contains the *corrections* made in those angles so that the 1st condition (of Art. 9) may be fulfilled. (*See* Art. 10.)

Column 4.—Contains the *deduced bearings* of the several lines, as per rule in (Art. 11).

Column 5.—Contains the *cardinal direction* of each line.

Column 6.—Contains the *reduced bearings* of the several lines, for the purpose of calculation. (*See* Art. 13.)

Column 7.—Contains the *distances* as measured in the field. (*See* field-book, Art. 7.)

Columns 8, 10.—Contain the differences of *latitude* reckoned on the meridian, *between every two stations*. (*See* Art. 14.)

Columns 12, 14.—Contain the *departure* of each station *from the meridian of the preceding one*. (*See* Art. 14.)

Columns 9, 11, 13, 15.—Contain the *corrections* applied to the latitudes and departures mentioned in columns 8, 10, 12, 14, so that the second and third conditions of (Art. 9) may be fulfilled. (*See* Art. 10.)

Column 16.—Contains the *latitude* of each station *from the first in the series*. The letters N. and S. show whether the several stations are situated north or south of the first station. (*See* Art. 4.)

Column 17.—Contains the *departure* of each station *from the first in the series*. The letters E. and W. show whether the several stations are situated east or west of the first station. (*See* Art. 4.)

Column 18.—Contains the sums of each *successive pair of co-ordinates* in column 17, for area.

Columns 19, 20.—Contain the north and south *products* of column 18 with columns 8 and 9, or 10 and 11, for area.

Columns 21, 22,—Contain the area of plus or minus *offsets* calculated by means of the talc-square, on either side of the survey lines, to be added or subtracted from the total area of the polygon.

16. CALCULATION OF ITS SEVERAL COLUMNS. (Fig. 335.) If each quantity is compared with the diagram, the mode of obtaining its corresponding result will be easily understood. The following calculations refer to the Traverse Computation Form of (Art. 15).

For col. 1.—Enter the station letters, which are taken from the field-book in (Art. 7), beginning and ending with the starting-point A.

For col. 2.—Enter the observed inward angles, which are extracted from the field-book in (Art. 7), leaving the space opposite the first letter A blank, and beginning with the angle (65° 25′) at B between A and C against B, the angle (223° 11′) at C between B and D against C, and so on, ending with the angle (222° 14′) at A between H and B against the last letter A. Next, total up all the angles (1,080° 01′) which in the given 8-sided polygon, should, by the rule in (Art. 9), be 8 × 180° — 360° = 1,080°.

For col. 3.—The sum of the angles in column 2 is in excess by 1′, hence this is rectified by subtracting 1′ from the largest angle at station E, by the rule of (Art. 10).

For col. 4.—The bearing of the first line AB was observed at A to be 281° 40′. This is placed opposite station B. The bearings of the other lines are found by the rule in (Art. 11). Thus

(Bear. AB) 281° 40′ + (inward ∠ B) 65° 25′ — 180° = 167° 05′ (Bear. BC), placed opposite C;
(„ BC) 167 05 + („ „ C) 223 11 — 180 = 210 16 („ CD), „ „ D;
(„ CD) 210 16 + („ „ D) 42 14 — 180 = 72 30 („ DE), „ „ E;
(„ DE) 72 30 + („ „ E) 242 11 — 180 = 134 41 („ EF), „ „ F;
(„ EF) 134 41 + („ „ F) 73 05 — 180 = 27 46 („ FG), „ „ G;
(„ FG) 27 46 + („ „ G) 119 09 + 180 = 326 55 („ GH), „ „ H;
(„ GH) 326 55 + („ „ H) 92 31 — 180 = 239 26 („ HA), „ „ A;
(„ HA) 239 26 + („ „ A) 222 14 — 180 = 281 40 („ AB), „ „ B.

For col. 5.—Opposite each of the deduced bearings of column 4, enter the cardinal directions of the respective lines they represent, according to the rule in (Art. 12).

For col. 6.—Convert the deduced bearings into reduced bearings for the purpose of calculation, by the rule in (Art. 13), and enter the same opposite their corresponding deduced bearings of column 4. Thus: —

— 281° 40′ (Bear.) + 360° = 78° 20′ (Red. Bear.);
— 167 05 („) + 180 = 12 55 („);
+ 210 16 („) — 180 = 30 16 („);
+ 72 30 („) = 72 30 („);
— 134 41 („) + 180 = 45 19 („);
+ 27 46 („) = 27 46 („);
— 326 55 („) + 360 = 33 05 („);
+ 239 26 („) — 180 = 59 26 („).

For col. 7.—Enter the measured distances of each line which are taken from the field-book in (Art. 7), beginning with the distance AB (12·38) opposite the letter B, the distance BC (20·55) opposite the letter C, and so on to the end.

For cols. 8, 10, 12, 14.—Work out by either of the rules in (Art. 13), the latitude and departure of each station with reference to the preceding station, entering the results of station B opposite the letter B, of C opposite the letter C, and so on, in their respective columns marked north, south, east and west, as indicated by the corresponding cardinal direction of col. 5. Then add up each of the columns, when the north should balance with the south, and the east with the west, in conformity with the rule of (Art. 9). Thus, by the Traverse Table:

Lat. B (Ab) = AB × cos. 78° 20′ = 12·38 × 0·2022 = 2·50 N.,
Dep. „ (Ab′) = „ × sin. 78 20 = 12·38 × 0·9793 = 12·12 W.;
Lat. C (bc) = BC × cos. 12° 55′ = 20·55 × 0·9747 = 20·03 S.,
Dep. „ (b′c′) = „ × sin. 12 55 = 20·55 × 0·2235 = 4·59 E.;
Lat. D (cd) = CD × cos. 30° 16′ = 23·85 × 0·8637 = 20·60 S.,
Dep. „ (c′d′) = „ × sin. 30 16 = 23·85 × 0·5040 = 12·02 W.;
Lat. E (de) = DE × cos. 72° 30′ = 17·40 × 0·3007 = 5·23 N.,
Dep. „ (d′e′) = „ × sin. 72 30 = 17·40 × 0·9537 = 16·59 E.;
Lat. F (ef) = EF × cos. 45° 19′ = 14·49 × 0·7032 = 10·19 S.,
Dep. „ (e′f′) = „ × sin. 45 19 = 14·49 × 0·7110 = 10·30 E.;
Lat. G (fg) = FG × cos. 27° 46′ = 30·36 × 0·8849 = 26·87 N.,
Dep. „ (f′g′) = „ × sin. 27 46 = 30·36 × 0·4659 = 14·14 E.;
Lat. H (gh) = GH × cos. 33° 05′ = 24·90 × 0·8379 = 20·86 N.,
Dep. „ (g′h′) = „ × sin. 33 05 = 24·90 × 0·3459 = 13·59 W.;
Lat. A (hA) = HA × cos. 59° 26′ = 9·13 × 0·5085 = 4·64 S.,
Dep. „ (h′A) = „ × sin. 59 26 = 9·13 × 0·8610 = 7·86 W.

For cols. 9, 11, 13, 15.—The difference between the total north and south latitudes is 0, hence no corrections are necessary for cols. 9 and 11; and the difference between the total east and west departures is 3 links, two of which are subtracted from the greatest lengths (16·59 and 14·14) in column 13, and 1 added to the greatest length (13·59) in column 15, to equalize them. The corrections are arbitrary and applied according to the rules of (Art. 10).

For cols. 16, 17.—By the rule in (Art. 5) calculate the latitude north or south and the departure east or west, of each station, placing the result opposite it with its indicative letter N or S and E or W, as the case may require, prefixed. The latitude (N. 2·50) and departure (W. 12·12) of B from A will be the same as in columns 8 and 14; that of C from A will be S. 20·03 — N. 2·50 = S. 17·53 latitude, and W. 12·12 — E. 4·59 = W. 7·53 departure, and so on for the rest. A shorter rule for finding the quantities in columns 16 and 17 may be framed as follows:—

"Take the sum or difference of the preceding $\frac{\text{latitude}}{\text{departure}}$ in col. $\frac{16}{17}$ and the following $\frac{\text{latitude}}{\text{departure}}$ in col. $\frac{8}{12}$ or $\frac{10}{14}$, for the following $\frac{\text{latitude}}{\text{departure}}$ in col. $\frac{16}{17}$; in the case of like symbols taking their sum and prefixing their symbol, and in the case of unlike symbols taking their difference and prefixing the symbol of the greater of them."

For example, the latitude of F = sum of S. 32·90 and S. 10·19 = S. 43·09, and the departure of F = diff. of W. 2·97 and E. 10·30 = E. 4·36; and so on for the remainder.

Thus:—

(Col. 16).
Lat B = sum of 0·00 and N. 2·50 = N. 2·50,
C = diff. „ N. 2·50 „ S. 20·03 = S. 17·53,
D = sum „ S. 17·53 „ S. 20·60 = S. 38·13,
E = diff. „ S. 38·13 „ N. 5·23 = S. 32·90,
F = sum „ S. 32·90 „ S. 10·19 = S. 43·09,
G = diff. „ S. 43·09 „ N. 26·87 = S. 16·22,
H = „ „ S. 16·22 „ N. 20·86 = N. 4·64,
A = „ „ N. 4·64 „ S. 4·64 = 0·00.

(Col. 17.)
Dep. B = sum of 0·00 and W. 12·12 = W. 12·12,
C = diff. „ W. 12·12 „ E. 4·59 = W. 7·53,
D = sum „ W. 7·53 „ W. 12·02 = W. 19·55,
E = diff. „ W. 19·55 „ E. 16·58 = W. 2·97,
F = „ „ W. 2·97 „ E. 10·30 = E. 7·33,
G = sum „ E. 7·33 „ E. 14·13 = E. 21·46,
H = diff. „ E. 21·46 „ W. 13·60 = E. 7·86,
A = „ „ E. 7·86 „ W. 7·86 = 0·00.

For col. 18.—Transfer the first quantity (W. 12·12) of col. 17 opposite it; then take the sum or difference of the 1st and 2nd quantities of col. 17, for the 2nd quantity in col. 18; the sum or difference of the 2nd and 3rd quantities of col. 17, for the 3rd quantity in col. 18; and so on to the end—in the case of like symbols taking the sum and prefixing their symbol, and in the case of unlike symbols taking their difference and prefixing the symbol of the greater of them. Thus, for the 2nd quantity in col. 18 opposite the letter C, take the sum of W. 12·12 and W. 7·53=W. 19·65; and so on for the others. Thus:—

Quantity opposite B = sum of 0·00 and W. 12·12 = W. 12·12,
„ „ C = „ „ W. 12·12 „ W. 7·53 = W. 19·65,
„ „ D = „ „ W. 7·53 „ W. 19·55 = W. 27·08,
„ „ E = „ „ W. 19·55 „ W. 2·97 = W. 22·52,
„ „ F = diff. „ W. 2·97 „ E. 7·33 = E. 4·36,
„ „ G = sum „ E. 7·33 „ E. 21·46 = E. 28·79,
„ „ H = „ „ E. 21·46 „ E. 7·86 = E. 29·32,
„ „ A = „ „ E. 7·86 „ 0·00 = E. 7·86.

For cols. 19, 20.—Multiply each quantity of col. 18 by the corresponding quantity of col. 8 or 10, and place the products opposite them in col. 19 or 20, as indicated in the heading, namely, *north*, where the muitipliers are north by east or south by west; and *south* where they are north by west or south by east. Thus, opposite to the letter B, the product will be N. 2·50 × W. 12·12 = 3·13 in col. 20, and so on. It must be recollected that the multipliers are chains, hence the product will be in square chains, which must have the decimal point removed to one place on the left to reduce it to acres; for instance, 2·50 × 12·12 chs. = 30·3000 sq. chs. = 3·03 acres, kept to two places of decimals. Next total up the two columns separately, and subtract the lesser sum from the greater, dividing the remainder by 2, which will be the area of the polygon without offsets.—
To obtain the products, proceed thus:—

Product opposite B = N. 2·50 × W. 12·12 = 3·03 (col. 20, south),
„ „ C = S. 20·03 × W. 19·65 = 39·36 („ 39, north),
„ „ D = S. 20·60 × W. 27·08 = 55·78 („ „ „),
= N. 5·23 × W. 22·52 = 11·78 („ 20, south),
F = S. 10·19 × E. 4·36 = 4·44 („ „ „),
G = N. 26·87 × E. 28·79 = 77·36 („ 19, north),
H = N. 20·86 × E. 29·32 = 61·16 („ „ „),
A = S. 4·64 × E. 7·86 = 3·65 („ 20, south).

For cols. 21, 22.—Having calculated the areas of offsets to the right and left of all the lines of the survey by the tale-square, enter those outside of the polygon in the additive column (col. 21) and those inside in the subtractive column (col. 22), opposite the respective lines to which they appertain. Then total up each of the columns separately, and subtract the lesser sum from the greater, the difference being the total area of the offsets, additive or subtractive, as the case may be, to the area of the polygon already found, for the area of the crooked figure surveyed, in acres. The area as found in the Traverse Computation Form of the Figure = 99·06 acres.

17. PLOTTING. (Fig. 335.) To plot the traverse from the Computation Form in (Art. 15), fix upon some convenient point A on the paper for the station of origin of the survey, through which draw a line NS to represent its meridian, and another line WE perpendicular to the former. Now in a corner of the paper lay down the scale on which the plot is to be drawn. Then to plot station B take from col. 16 its latitude N. 2·50, and lay it off, by means of a pair of compasses and the given scale, along the meridian line from A north, namely, Ab; again take from col. 17 its departure W. 12·12, and lay this off along the perpendicular line from A west, namely, Ab^1; through b draw a line bB west parallel to WE, and through b′ draw a line b′B north parallel to NS, thus the position of B is fixed; join AB, and take off from the given scale its distance 12·38, obtained from col. 7, which should correspond with the length of the line AB on the plot. Similarly plot—

	(Lat. col. 16.)	(Dep. col. 17.)
C by the co-ordinates	Ac = S. 17·53,	Ac′ = W. 7·53;
D „ „ „	Ad = S. 38·13,	Ad′ = W. 19·55;
E „ „ „	Ae = S. 32·90,	Ae′ = W. 2·97;
F „ „ „	Af = S. 43·09,	Af′ = E. 7·33;
G „ „ „	Ag = S. 16·22,	Ag′ = E. 21·46;
H „ „ „	Ah = N. 4·64,	Ah′ = E. 7·86;

and join the points BC, CD, DE, EF, FG, GH, and HA, testing the distances taken from col. 7, as before.

18. SECTION PAPER. (Figs. 335, 474.) If the survey be too large to admit of the co-ordinates being conveniently laid down from the initial meridian and perpendicular, section paper is employed for the purpose, which is paper ruled in squares of 1 inch sides. (Fig. 474) represents section paper so ruled, and if the circuit is to be plotted on a scale of 10 chains = 1 inch, each square would therefore contain 10 × 10 = 100 square chains or 10 acres. Supposing it is required to plot the traverse shown in (Fig. 335), on a scale of 10 chains = 1 inch but with all the distances in chains doubled, as follows:—

Ab = N. 5·00,	Ab′ = W. 24·24,	AB = 24·76;
Ac = S. 35·06,	Ac′ = W. 15·06,	BC = 41·10;
Ad = S. 76·26,	Ad′ = W. 39·10,	CD = 47·70;
Ae = S. 65·80,	Ae′ = W. 5·94,	DE = 34·80;
Af = S. 86·18,	Af′ = E. 14·66,	EF = 28·98;
Ag = S. 32·44,	Ag′ = E. 42·92,	FG = 60·72;
Ah = N. 9·28,	Ah′ = E. 15·72,	GH = 49·80;
		HA = 18·26.

Then proceed to plot the work thus, having first provided yourself with section paper with lines ruled 1 inch apart, both meridionally and perpendicularly:—On a convenient part of the paper fix upon a point A (*see* Fig. 474) at the intersection of two lines, to represent the first station; the north and south line through it is therefore the initial meridian, and the east and west line the initial perpendicular. Write 0 at each extremity of these lines; at the extremities of each line parallel to the initial meridian and away from it to the east and west sides, write successively 10, 20, 30, etc.; and at the extremities of each line parallel to the initial perpendicular, and away from it to the north and south sides, write successively 10, 20, 30, etc.; draw the scale of 10 chains = 1 inch at the bottom of the paper, which is now ready for plotting. To plot B, lay off from the given scale $bB = 5{\cdot}00$ north of the initial perpendicular, and $b'B = 4{\cdot}24 (= 20 + 4{\cdot}24)$ west of the meridian marked 20; for C lay off $cC = 5{\cdot}06 (= 30 + 5{\cdot}06)$ south of the perpendicular marked 30, and $c'C = 5{\cdot}06 (= 10 + 5{\cdot}06)$ west of the meridian marked 10; and so on for the rest, always protracting from the meridian and perpendicular nearest the station on the side of the initial meridian and perpendicular.

19. REDUCTION TO TRIANGLES. Fig. 454.) Another accurate method of finding the area of a theodolite traverse is to reduce it to triangles, calculate their areas separately by means of two sides and the included angle (Chap. XIII, Art. 76), and total them up for the area of the entire figure, when any triangle falls without the figure it being excluded; for instance, the area of the polygon ABCDEFA = ABC + CDE + EAC — EFA. If the polygon is very small, or in the case of a traverse made with either a prismatic compass or sextant, its area may be calculated by means of the acre-square or by casting (*see* Chap. VIII).

20. INITIAL MERIDIAN. In small surveys for engineering and other purposes, the magnetic meridian or the true meridian as found by the rules in (Chap. X), answers very well for the initial meridian to which all the co-ordinates are referred, but for surveys carried over a great extent of country, like the Revenue Surveys, it becomes necessary to observe the initial meridian astronomically. In small surveys above referred to, however, it is better to obtain the true meridian, as it is invariable, whereas the magnetic meridian is constantly, though slightly, varying.

21. REDUCTION OF CO-ORDINATES OF STATIONS AS REFERRED TO ORIGIN. (Fig. 433, 435.) In extensive surveys it is frequently necessary, for the plotting of small-scale maps, to reduce the co-ordinates of traverse stations at tri-junctions and others with reference to the origin of the series; so that having given a certain number of the points so obtained, the filling in of the details off the larger scale map (such as 16″) to a smaller scale one (such as 1″ or 2″) may be readily and accurately done by means of a pantagraph or any other method. To facilitate this reduction the following form has been prepared, which is self-explanatory, and which will tend to reduce the labour of the ordinary calculations. The two examples (Nos. I and II) given have reference to Figures 433 and 435. In both cases the main circuits and the village traverses are denoted, respectively, by thick and thin broken lines, and only the stations referred to in the form shown, while the intermediate ones (A_2, A_3, etc., B_2, B_3, etc., of Fig. 435) have been omitted. In Fig. 435 the main circuit commences with ⊙ A 1 (the origin) and proceeds in serial order till it gets back to the starting-point; it encloses 6 village traverses each having its own distinct series also serially numbered for all new sides. In Fig. 433 the main circuit begins from its origin O (not shown in the Figure) and similarly encloses a certain number of village traverses, only 2 of which appear in the figure. Now, in order to obtain the co-ordinates of any required station (as A16 of series No. 4 of Fig. 435), called "deduced," as referred to the origin (A1), all that it is necessary to know (and these data are given in the Traverse Computation Form) are the co-ordinate values (i) of the "duduced station" (A16) from the first (A11) of its series (No. 4), (ii) of the "station of connection" (also A11) from the first (A11) of the series (No. 4), and (iii) of the "station of connection" (A11) from the origin (A1), and to take the algebraic sum of their corresponding entries (under "Meridian" and "Perpendicular"), the directions of the entries for (ii) however being previously reversed, that is, S. read for N., W. for E., and *vice versâ*.

In this connection, also, for the purpose of the preparation of small-scale maps, it is often required to obtain the rectangular co-ordinates, as referred to origin, of trigonometrical and other stations of which the latitude and longitude are known, including the corners of the squares of such projections, and likewise, conversely, to find the latitudes and longitudes of points of which the co-ordinate values are given. The examples (Nos. III to V) which appear below illustrate the methods of work for arriving at the desired results; the calculations are self-evident.

If, again, it is desired for any purpose (i) to find the direct distance between any two stations of a traverse with the bearing of such line, or (ii) to compare the direct distance by traverse with that by triangulation, the computation for the obtaining of these results will be as follows. To take the traverse given in (Art. 15)—

(i) Required the direct distance between stations A and F and the bearing of the line:

Meridian co-ordinate (Y) S. 43·09 chains ...	... Log.	1·634376
	A. C. of Log.	8·365624
Perpendicular co-ordinate (X) E. 7·33 chains ...	... Log.	0·865104
Bearing or *a* = 9° 39′; sum = Log. tan. *a* ...	...	9·230728
	Log. sec *a*	10·006189
	Log. Y.	1·634377
Direct distance by traverse 28·85 chains ...	... Log.	1·610566

(ii) Compare the direct distance by traverse with that by triangulation between stations A and F:

Meridian co-ordinate (Y) S. 43·09 chains ...	... Log.	1·634376
	A. C. of Log.	8·365624
Perpendicular co-ordinate (X) E. 7·33 chains ...	... Log.	0·865104
Bearing or *a* = 9° 39′; sum = Log. tan. *a* ...	...	9·230728
	Log. sec. *a*	10·006189
	Log. Y.	1·634377
	Log. 66	1·819544
Direct distance by traverse 2,885 feet ...	... Log.	3·460110
Direct distance by triangulation 2,888 feet ...	... Log.	3·460597
	Difference of Logs.	0·000487
Error of traverse per mile in feet * ...	...	6

OBSERVATION.

* The error per mile in feet is obtained by dividing the difference of Logs. of direct distances by ·000082.

EXAMPLES I AND II.—*Reduction of Co-ordinates of Traverse Stations (at tri-junctions or other), as referred to Origin.*

Stations.				A. Co-ordinates of Deduced Station (I) from First of Series (III).				B. Co-ordinates of Station of Connection (II) from First of Series (III).				C. Co-ordinates of Station of Connection (II) from Origin (IV).				D.* Co-ordinates of Deduced Station (I) from Origin (IV).								Remarks.
				Meridian.		Perpendicular.		Meridian.		Perpendicular.		Meridian.		Perpendicular.		Meridian.				Perpendicular.				
Deduced I.	Of Connection II.	First of Series III.	Origin IV.	N.	S.	E.	W.	N. (S.)	S. (N.)	E. (W.)	W. (E.)	N.	S.	E.	W.	N.		S.		E.		W.		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17		18		19		20		21
				Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Chs.	Cor.	Chs.	Cor.	Chs.	Cor.	Chs.	Cor.	
Example No. I.																								
A16	A11	A11	A1	43·71	...	...	65·31	0·00	0·00	0·00	0·00	100·61	...	49·62	...	144·32	...	...	...	...	...	15·69	...	See Fig. 435.
A18	A16	A16	A1	4·49	...	...	42·70	0·00	0·00	0·00	0·00	144·32	...	...	15·69	148·81	...	...	...	...	...	58·39	...	
A22	A18	A16	A1	...	54·51	...	86·39	4·49	..	...	42·70	148·81	...	...	58·39	89·81	...	...	...	...	...	102·08	...	
A25	A22	A22	A1	...	27·02	...	22·16	0·00	0·00	0·00	0·00	89·81	...	...	102·08	62·79	...	...	...	...	...	124·24	...	
A29	A25	A22	A1	...	87·87	36·03	...	...	27·02	...	22·16	62·79	...	...	124·24	1·94	...	...	...	...	...	66·05	...	
A33	A29	A29	A1	...	47·80	0·69	...	0·00	0·00	0·00	0·00	1·94	...	...	66·05	...	...	45·86	...	...	...	65·36	...	
A1	A33	A29	A1	...	1·83	66·08	...	...	47·80	0·99	...	...	45·86	...	65·36	0·11	...	...	...	0·03	...	...	...	
D2	A11	A1	A1	70·17	...	20·16	...	100·61	...	49·62	...	100·61	...	49·62	...	70·17	...	...	...	20·16	...	...	...	
E5	D2	A11	A1	...	5·99	...	85·86	...	30·42	...	29·46	70·17	...	20·16	...	94·60	...	...	...	...	...	36·24	...	
C1	D2	E5	A1	...	31·68	...	18·89	...	24·44	56·42	...	70·17	...	20·16	...	62·93	...	...	...	...	...	55·15	...	
B4	C1	E5	A1	...	51·02	...	24·59	...	31·08	...	18·89	62·93	...	...	55·15	42·99	...	...	...	...	...	60·85	...	
B1	B4	E5	A1	...	72·00	28·81	...	...	51·02	...	24·59	42·99	...	...	60·85	22·01	...	...	...	...	...	7·45	...	
A33	B4	A29	A1	...	47·80	0·99	...	41·71	...	5·21	...	42·99	...	...	60·85	...	...	46·52	...	...	...	65·07	...	
A18	C1	A16	A1	4·49	...	...	42·70	...	81·32	...	39·44	62·93	...	...	55·15	148·74	...	...	...	...	...	58·41	...	
Example No. II.																								
K20	M20	B1	O	65·62	...	...	67·07	117·53	...	...	12·84	4,958·03	...	...	1,623·06	4,906·12	...	...	...	...	...	1,677·29	...	See Fig. 433.
B1	K20	B1	O	0·00	0·00	0·00	0·00	65·62	...	...	67·07	4,906·12	...	...	1,677·29	4,840·50	...	...	...	...	...	1,610·22	...	

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* D. Alegebraic sum of corresponding entries in columns A, B and C; the directions of the entries in column B being previously reversed, that is, S read for N, W for E, and *vice versâ*.

EXAMPLE III.—*Computation of Rectangular Co-ordinates of Trigonometrical and other stations of which the Latitude and Longitude are known.*

From Origin λ. 17° 00′ N., L. 96° 00′ E. (Pegu District).

Number of Sheet ...	279 E.	279 W.	279 W.	280 E.
Name of Station (S) ...	⊙ *a* 52	⊙ *a* 368	1 {λ. 17° 30′ N. L. 96 15 E.	2 {λ. 17° 15′ N. L. 96 15 E.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Long. of Origin (O) ...	96 00 00	96 00 00	96 00 00	96 00 00
„ (S)	96 17 00	96 14 00	96 15 00	96 15 00
Difference	0 17 00	0 14 00	0 15 00	0 15 00
„ in seconds ...	1,020	840	900	900
Log. „ „ ...	3·008600	2·924279	2·954242	2·954242
Log. from Tab. LIV* for Lat. of (S).	1·986375	1·986219	1·985747	1·986337
A. C. of Log. 66 ...	8·180456	8·180456	8·180456	8·180456
Sum = log. X	3·175431	3·090954	3·120445	3·121035
X (Easting or Westing) in chains.	E. 1,497·72	E. 1,232·97	E. 1,319·61	E 1,321·40
Twice log. of Diff. of Long. (4 places).	6·0172	5·8486	5·9085	5·9085
Log. from Tab. LV* for Lat. of (S).	7·8389	$\bar{7}$·8404	$\bar{7}$·8447	$\bar{7}$·8303
Sum = log. Arc-versine ...	1·8561	$\bar{1}$·6890	$\bar{1}$·7532	$\bar{1}$·7478
Arc-versine in seconds ...	01	00	01	01
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Lat. of (O)	17 00 00	17 00 00	17 00 00	17 00 00
Lat. of (S)	17 14 00	17 18 00	17 30 00	17 15 00
Difference (*d*)	+0 14 00	+0 18 00	+ 0 30 00	+ 0 15 00
Arc-versine in seconds ...	01	00	01	01
Algebraical sum = Y in arc ...	+0 14 01	+0 18 00	+ 0 30 01	+ 0 15 01
Y in seconds	841	1,080	1,801	901
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
± ½ Y in arc	+0 07 00	+0 09 00	+ 0 15 00	+ 0 07 30
Lat. of (O)	17 00 00	17 00 00	17 00 00	17 00 00
Algebraical sum	17 07 00	17 09 00	17 15 00	17 07 30
Log. from Tab. LIII* for Algebraical sum.	2·003680	2·003681	2·003685	2·003680
Log. of Y in seconds ...	2·924796	3·033424	3·255514	2·954725
A. C. of Log. 66 ...	8·180456	8·180456	8·180456	8·180456
Sum = log. Y	3·108932	3·217561	3·439655	3·138861
Y (Northing or Southing) in chains.	N. 1,285·09	N. 1,650·09	N. 2,752·04	N. 1,376·77

* Of the "Calculating Tables," 5th edition.

NOTE.—X is East or West of Origin according as the Longitude of the Station is greater or less than that of the Origin.

(*d*) is ± according as the Latitude of the Station is greater or less than that of the Origin.
Y is North or South of the Origin according as Y in arc is ±.

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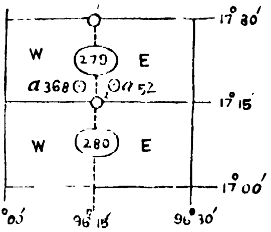
(Scale 1 inch = 1 mile, for ¼ Degree sheets)



EXAMPLE IV.—*Computation of Latitudes and Longitudes of Stations of which the rectangular co-ordinates from Origin are known.*

Origin of Survey = λ. 17° 00′ 00″ N., L. 96° 00′ 00″ E.
Co-ordinates of S^3 (Station) = 1,390·87 N. (+) and 1,392·54 E. (+).
Y = 1,390·87 chains = 91,797 feet (Table I *).
X = 1,392·54 chains = 91,908 feet (Table I *).
(i) Y in feet = 91,797 ... ... log. = 4·962828
∴ Y in arc = 918 seconds, approximately
= 15′ 18″;
∴ λ. of Origin + ½ Y = 17° 07′ 39″
(corresponding log. in Table LIII*) ... = 2·003680

∴ Y in arc = + 15′ 10·22″ = 910·22″ ... log. = 2·959148

(ii) X in feet = 91,908 ... ... log. = 4·9634
Latitude of Origin + Y in arc = 17° 15′,
from Table LIV* ... ... log. = 1·9863

Difference = 2·9771
2

5·9542
From Table LV* corresponding to λ. 17° 15′ log. = 7·8393

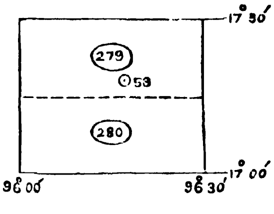
∴ Arc-versine = 0·62″ ... ... log. = $\bar{1}$·7935

(iii) Latitude of S^3 = 17° 00′ 00″ + 15′ 10·22″ — 0·62″ = 17° 15′ 09·60″ N.
(iv) X in feet = 91,908 ... ... log. = 4·963353
Table LIV* corresponding to λ. 17° 15′ 09·60″ log. = 1·986330

X in arc = + 15′ 48·47″ = 948·47″ ... log. = 2·977023
∴ Longitude of S^3 = 96° 00′ 00″ + 15′ 48·47″ = 96° 15′ 48·47″ E.

INDEX.

(Scale 1 inch = 4 miles, for ½ Degree sheets.)



OBSERVATION.

* *See* "Calculating Tables," 5th edition.

EXAMPLE V.—*Computation of Rectangular Co-ordinates of Intersected Points (Pegu District).*

Number of Sheet	279	......
Intersected Point C	Pagoda.	......
Name of Station A	⊙T³	......
Name of Station B	⊙S³	......
	° ′ ″	
Direction at A	5 29 39	......
Direction at B	349 34 42	......
log. ($X_B - X_A$)=log. + 14·31	1·155640	......
+ log. cot. A	11·016887	......
Sum	2·172527	......
± Natural Number	+ 148·87	......
($Y_A - Y_B$)	+ 0·39	......
K=Algebraical Sum	149·26	......
log. tan. B	9·264641	......
+ log. cot. A	11·016887	......
Sum	0 281528	......
L=1 ± Natural Number	1+ 1·91	......
log. K	2·173944	......
— log. L	0·463893	......
Diff.=log. ($Y_C - Y_B$)	1·710051	......
± ($Y_C - Y_B$)	+ 51·29	......
± Y_B	+ 1,390·87	......
± Y_C	+ 1,442·16	......
log. ($Y_C - Y_B$)	1·710051	......
+ log. tan. B	9·264641	......
Sum = log. ($X_C - X$)	0·974692	......
± ($X - X$)	— 9·43	......
± X_B	+ 1,392·54	......
± X_C	+ 1,383·11	......

NOTE.—Take as Station **A** that one the abscissa of which has the lesser numerical value.

Y_A, Y_B, Y_C are the rectangular ordinates (North or South) and X_A, X_B, X_C the rectangular abscissæ (East or West) of the stations of Observation **A** and **B**, and of the Intersected Point **C** as referred to Orign.

The angles **A** and **B** are the directions of **C** from **A** and **B** with respect to the meridian of the origin, *i.e.*, the centre of the Degree Sheet.

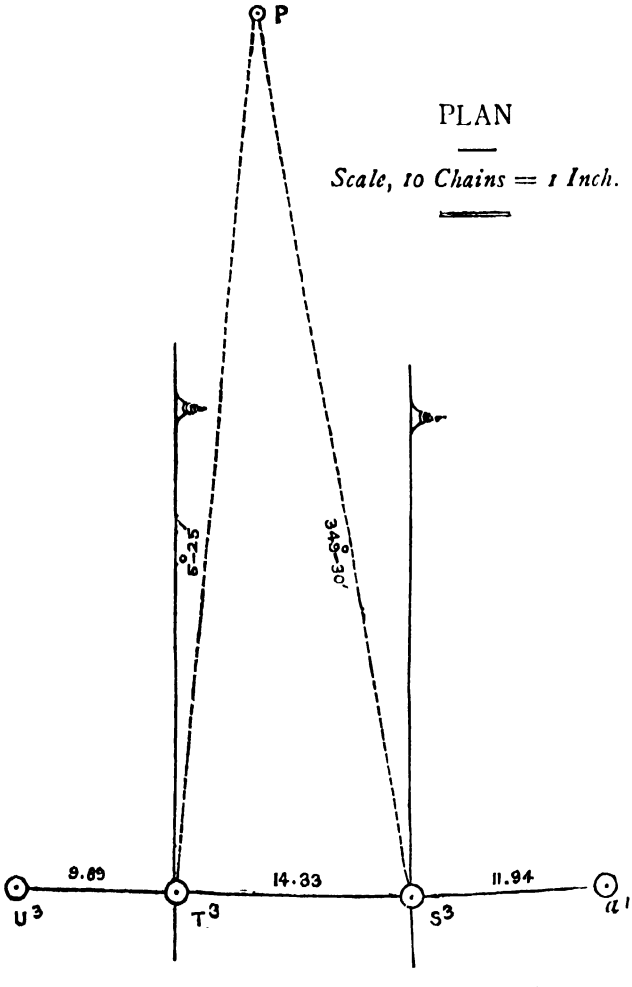
Where the double sign ± is given, the sign of the term varies according to the relative positions of **A, B,** and **C.**

Extract from Minbu Circuit No. 1, Paunglin Survey, Pegu District.

Stations.	Observed angles.	Bearings	Distances	CO-ORDINATES. Each from the preceding. Meridian.	Perpendicular.	From origin. Meridian.	Perpendicular.
	° ′	° ′	Chs.	Chs.	Chs.	Chs.	Chs.
↓							
U ...	...	...	...	...	...	N. 1,391·56	E. 1,368·34
T³ ...	179 48	91 46	9·89	S. 0·30	E. 9·89	N. 1,391·26	E. 1,378·23
S³ ...	174 58	91 34	14·33	S. 0·39	E. 14·31	N. 1,390·87	E. 1,392·54
A¹ ...	32 10	86 32	11·94	N. 0·72	E. 11·91	N. 1,391·59	E. 1,104·45
↓							

Intersected Point = P.

Station of observation.	Forward station.	Bearings.	Intersected Point.	Bearings.	Convergency angle.*	Direction with reference to Meridian of Origin.†
		° ′		° ′	′ ″	° ′ ″
T³ ...	S³	91 34	P	5 25	+ 4 39	5 29 39
S³ ‡ ...	a¹	86 32	P	349 30	+ 4 42	349 34 42



PLAN

Scale, 10 Chains = 1 Inch.

OBSERVATIONS.

* Convergency at ⊙T³ = 0·2023″ × 1,378·23 = 278·82″ = 4′ 38·82″ (*See* Table XLII of "Calculating Tables," 5th edition).

Convergency at ⊙S³ = 0·2023″ × 1,392·54 = 281·71″ = 4′ 41·71″ (*See* Table XLII of "Calculating Tables," 5th edition).

† Origin of Survey = λ. 17° 00′ 00″ N., L. 96° 00′ 00″ E.

‡ Calculated values of ⊙S³ are : λ. 1,0° 15′ 09·60″ N., L. 96° 15′ 48·47″ E.— *vide* Example IV, Art. 21 *ante.*

22. CONVERGENCY OF MERIDIAN. (Fig. 476.) If a traverse is carried over a large tract of country extending several miles in length, as is practised in the Revenue Survey, it becomes necessary to check the accuracy of the work by astronomical observations for the determination of the meridian at stations about every 10 miles apart. The meridian thus observed is the *true* meridian at the station of observation, but as the Traverse System is based on the assumption that the earth is a plane, certain corrections must be applied to the observed meridian to make it parallel to the meridian of origin. This correction is called the *convergency of the meridian*, and is to be *added* to, or *subtracted* from, the observed meridian, as the station of observation is *west* or *east* of the meridian of origin. This will be clear from the following illustration :—

Let *a* be the starting-point of a survey, where the *true* meridian *a*N, called the "meridian of origin," has been astronomically laid down, and to which it is necessary to refer all other points connected with the survey operations, in order to preserve their relative positions by the application of the traverse system of rectangular co-ordinates, drawn respectively parallel and perpendicular to the meridian of origin. Suppose the survey to proceed along the lines *ab*, *bc*, etc., either *eastward* or *westward*, to *e*, where, owing to the distance traversed, and the number of angles observed between it and the first station, it becomes necessary to check the work by re-observation of azimuth. *e*N′ is the true meridian passing through *e*, converging to the north pole and thus inclined to the initial meridian. This angle of inclination is the measure of convergency at *e*, with reference to *a*. Now, the true meridian being determined at *e*, the bearing of the forward line *ef* is the angle N′*ef*, to which if the convergency N′*en* is applied (being *subtracted* if *e* is to the *east* of *a* or *added* if to the *west*), there will be obtained the *reduced bearing nef* of the line *ef*. But the *deduced bearing* of the line *ef*, as obtained from the Computation Form (Art. 15), is also measured by the angle *nef*; hence the *deduced* and *reduced* bearings of *ef* should agree, any difference showing the error of the former between the points *a* and *e*. This error is then to be divided amongst the bearings of the preceding survey lines, *plus* or *minus* as the case may be, and the bearings corrected accordingly.

To find the convergency of the meridian at a place, the *latitude* of the place and the *departure* from the station of origin must be known; and it may then be found by the following rules :—

$$\frac{\text{For departure in Miles}}{\text{For departure in Chains}} : \text{ To the constant log. } \begin{matrix}\bar{1}\cdot938792\\ \bar{2}\cdot035702\end{matrix}$$

add log. tangent (less 10 in the index) of the latitude and log. departure in $\frac{\text{miles}}{\text{chains}}$; the sum is the log. convergency, in *minutes*.

Example: Find the convergency for $\frac{\text{12 miles}}{\text{960 chains}}$ of departure, in latitude 18°

For departure in Miles.		*For departure in Chains.*	
Constant log.	= 1·938792	Constant log.	= $\bar{2}\cdot035702$
L. tan. lat. (less 10)	= 1·511776	L. tan. lat. (less 10)	= 1·511776
log. 12 (miles)	= 1·079181	log. 960 (chains)	= 2·982271
Conv. (=3′ 21·2″) = 3·386′	= 0·529749	Conv. (=3′ 21·2″) = 3·386′	= 0·529749

Convergency for every degree of latitude from 0° to 60° has been calculated in (Table XLII, "Calculating Tables," 5th edition) for 1 mile or 1 chain of departure, and given in *seconds*; if the departure is more than 1 mile or 1 chain, multiply it by the corresponding convergency given in the Table, for that required; as the convergency is proportional to the departure. If the latitude contains part of a degree, the convergency for it can be found by simple proportion. Thus in the above example :—

Convergency = 12 (miles) × 16·93″ = 203·2″ = 3′ 23·2″;
" = 960 (chains) × 0·2117″ = 230·2″ = 3′ 23·2″.

23. THE RELAYING OF BOUNDARY MARKS.* (Fig. 502.) This subject follows that of convergency in natural sequence. Unless the principle of

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* From Kelly's Hand-book of Practical Surveying for India.

convergency be thoroughly understood, all attempts to trace up old survey lines and stations from traverse data must result in failure, and until the survey stations have been identified, it is hopeless to attempt the re-demarcation of any boundary which depends entirely on the traverse lines from which it was originally measured by offsets.

The first step, then, is to find the theodolite stations of the traverse survey of the boundary in question. For this purpose it is necessary to identify at least one of the survey stations as a point of departure. The surveyor must also be furnished with the following records of the old survey :—*1st*, A copy of the old plan on the largest scale available, showing the survey stations and lines, the run of the boundary, all boundary pillars, posts, or other marks taken up by the old survey ; also rivers, streams, roads, and paths crossing the boundary line, and any temples, pagodas, or other conspicuous buildings that may be situated near the boundary. *2nd*, The traverse data of angles, bearings, distances and co-ordinates. It is absolutely necessary that the *origin of survey* should be distinctly recorded on the traverse. *3rd*, The field-book of the boundary survey, showing the offsets from the traverse lines to the pillars and bends of the boundary. It may happen that the field-book is not forthcoming, in which case we must measure the offsets on the large scale boundary plan with scale and compass, and set off on the ground the measurements thus obtained. It is true that we here depend entirely on the accuracy of the old boundary plot, but there is no alternative, and once we have traced up and marked the main circuit or village traverse lines and stations of the old survey skirting the boundary, the boundary marks themselves can be relaid, if not exactly, yet with the closest approximation possible under the circumstances.

In work of this nature the theodolite is indispensable and the chain (Gunter's) must be carefully measured with a standard chain or bar before commencing work. Two chains are required, one for use and the other for reference as a standard.

Let us suppose that we are required to relay a certain boundary, the marks on which have all been destroyed. The theodolite stations of the boundary survey have also, with one exception, been removed. We proceed to this station (A), set up the theodolite over it, and there lay down a true north and south line AN, on which we set up a referring mark. We now look at the traverse data and find that the reduced bearing of the next line AB is 84° 50′. From the meridian already laid down by us we could easily lay off this bearing, 84° 50′, but it would not be correct to do so, because the bearings recorded in the traverse are not true bearings in the abstract, but are true only *with reference* to the *origin* of survey. This origin may be miles distant, east or west, from the station on which our theodolite stands, in which case the true bearing will differ from the traverse bearing by the amount of convergency due at that point. In the present instance we know the distance of the station observed at east of the origin, and find by computation that the convergency is 5 minutes *minus*.

In the figure, AN is the true meridian, AO is parallel to the meridian at the origin of the survey, and NAO is the convergency angle.

As the traverse (reduced) bearing is 84° 50′, it is clear that the true (observed) bearing at our station must be 84° 50′ plus convergency angle, or 84° 55′. Now direct the theodolite to the referring mark just fixed by us on the true meridian line AN, clamp the lower limb (set to 360°) in that position, and move the upper plate round until the vernier (A) reads 84° 55′, at which angle clamp the vernier. Leaving the theodolite in this position, send a flag-man in the direction indicated by the telescope, and motion to him to move right or left until the flagstaff is intersected by the cross-wires of the telescope.

It must be understood that the flag-staff cannot in the first instance be set up at the required distance shown in the traverse which has yet to be measured, but its distance from the station should never fall short of the traverse distance, and may safely exceed it. A trained flag-man can judge the required distance very closely. Having obtained the true direction of the line, chain the distance given in the traverse. The flag-staff should then be brought up to the end of the line just measured, and again intersected by the cross-wires of the telescope to make sure that there is no deviation in the direction of the line. The survey station B is thus fixed.

Set up the theodolite next on the station B, clamp the upper plate at 360°, direct the telescope to A, clamp the lower limb, and bring the telescope round until the vernier (A) reads the angle 135° 25′ at station B, as recorded in the traverse, in which position it is evident that the telescope will point to station C, which must be fixed in the same way as B. Next set up the instrument at station C, and read off the angle 197° 30′ given for that point by the traverse, the result being that the telescope now points to station D. Proceed thus from station to station, observing the angles and measuring the distances recorded in the traverse form, and while chaining from point to point carefully measure the required offsets right and left of the chain line, leaving marks on the ground to indicate the salient points, the bends and pillars or posts of the boundary thus relaid, which can be permanently marked after the completion of the survey.

Let us assume another case. We want to fix the position of station D from A, without reference to the intermediate stations B and C, which are not required. Having been supplied with the traverse co-ordinates, we know the meridian and perpendicular distances of A and D from the origin, and the difference of co-ordinates will give the distance of one from the other. Thus :—

A from origin	... ...	N. 565·78 chs.	E. 824·45 chs.
D „ „	... ...	N. 615·30 „	E. 905·65 „
Distance of D from A	...	N. 49·52 „	E. 81·20 „

We lay down the true meridian AN at station A, and from it lay off a line AO bearing 5 minutes (the amount of convergency) from AN. The line AO is parallel to the initial meridian, and on it we place a referring mark and measure the distance 49·52 chains north, up to *a*. We next plant the theodolite on *a*, read off an angle of 90° with reference to a flag at A and send another flag in the direction of D, to which the telescope points, and measure the distance 81·20 chains east, which fixes the position of D.*

24. STATIONS. All stations should be marked by a wooden peg driven into the ground or by some other means whereby the station can be found when required. The intervening ground between stations should be cleared of jungle, tall grass, or other obstacle, to prepare the way for chaining and to secure mutual visibility of points.

25. REVENUE SURVEY TRAVERSE. (Fig. 444.) Suppose it was required to perform a traverse according to the Revenue Survey method of a village and ground called Twante. Select a starting-point as station 1, at which observe the true meridian ; then proceed to traverse round the boundary of the lands of Twante village called the *main circuit,* in the direction of 2, 3, 4, etc., back to station 1, as explained in (Art. 6). Next take up the survey of *sub-circuit* No. 1, beginning at 10 round by G, F, 27 to 1 ; the remaining angles and distances being supplied by the main circuit. For *sub-circuit* No. 2, begin at 16 by 28, A, H, G to 10. For *sub-circuit* No. 3, begin at 21 by D, C, B to A. For *sub-circuit* No. 4, begin at F by E, D to 21. The *village circuit* or *sub-circuit* No. 5, enclosing the village itself, there is no necessity to survey, as all the required data can be obtained from the adjoining sub-circuits by reversing their angles. Through the village two or three *minor circuits* may be run, as from F to C, and from C to H. The details within the several *sub-circuits* and village circuits must be filled in with the aid of a plane-table.

OBSERVATION.

* By the rules of Trigonometry (Chap. III, Art. 24), having given A*a* (= 49·52 chains) and D*a* (= 81·20 chains), the direct line AD (=95·10 chains) may be found and also the angle *a*AD (=58° 38′). And, if the back station (not shown in the Figure) be traceable on the ground, the contained angle between it and D at A (through their respective reduced bearings) could be obtained, and the point D thus readily determined by means of that angle and the distance AD.

CHAPTER XV.

LEVELLING.

1. LEVELLING DEFINED. *Levelling* is the art of determining and measuring, with the aid of scientific instruments, the relative heights and depths, or elevations of places, points, and objects on and comparatively near the earth's surface.

2. TRUE AND APPARENT LEVEL. (Fig. 458.) A line of *true level* is such that all points in it are equally distant from the centre of the earth, and hence must be curved, as EHJ. A line of *apparent level* at any place is a horizontal line passing through that place, and hence must be straight, as EB. The point B, on apparent level with E, is found by means of a spirit level placed horizontally at E.

3. DIFFERENCE BETWEEN TRUE AND APPARENT LEVEL. (Fig. 458.) Let LEM represent the surface of the earth, G its centre, EB a tangent to its circumference at E. Then E and B are in the same apparent level when B is seen from E; also E and H are on the same true level; and HI is the difference between the true and apparent level in reference to E at a distance from it equal to EH. The further the distance is extended from a point on the earth's surface, the higher will the line of apparent level rise above the line of true level; in other words, at J the distance JK will be greater than the distance HI at H. The difference between the true and apparent level may be found by the following formula (Euclid III, 36):—$(LH + HI) \times HI = EI^2$. But the diameter of the earth LH is so great with respect to the quantity HI, at all distances to which a levelling operation usually extends, that LH may be taken for LH+HI without sensible error; the equation will then become—

$$LH \times HI = EI^2; \therefore HI = \frac{EI^2}{LH},$$

that is, the difference between the true and apparent level is equal to the square of the distance divided by the earth's diameter. The *mean* diameter of the earth is 7,916 miles (the polar diameter being 7,908, and the equatorial 7,924 miles), then for a distance EI of 1 mile, the excess of the apparent above the true level, or HI, will $= \frac{EI^2}{LH} = \frac{1}{7,916}$ of a mile = 8·004 inches; at 2 miles it is 4 times that quantity, or 32·016 inches; at three miles 9 times or 72·036 inches; and so on, increasing in proportion to the square of the distance. Rejecting the decimal ·004, and assuming the curvature in 1 mile to be 8 inches or two-thirds of a foot, the following simple rule is obtained for computing the correction due to the curvature of the earth, namely, the correction for curvature, *in feet*, is two-thirds of the square of the distance, *in miles*.

Examples.—(1) What is the difference, in feet, between true and apparent level, at the distance of $3\frac{1}{2}$ miles? ... ... *Ans.*: $8\frac{1}{6}$ feet (short rule).

(2) Required the difference, in inches, between true and apparent level, at the distance of 3,100 feet ... ... *Ans.*: 2·76 inches (long rule).

4. CURVATURE. (Fig. 462.) The effect of the curvature of the earth is to *depress* the apparent place of an object; hence the correction for curvature is *additive*; for instance, if the observer is stationed on a rising piece of ground on the earth's surface at O, from which he observes two objects A and B on two other rising pieces of ground, the former above and the latter below his horizon MN; and if he observes the apparent level, or AD, of A = + 863 feet, and the apparent level, or BC, of B = —8·63 feet, and that the distance from each of the points from O is 1,950 feet, then to find the true level, or AG, of A, and the true level, or BF, of B, the curvature must be added to each of their apparent levels. The curvature for a distance of 1,950 feet is 0·09 feet (*see* Table XLI, "Calculating Tables," 5th edition), ∴ AG, the true level of A = AD + AG = + 8 63 + 0·09 = + 8·72 feet, and BF, the true level of B, = BC + CF = — 8·63 + 0·09 = —8·54 feet. The correction for curvature and refraction together, *in feet*, is approximately $\frac{2}{3}$rd of the square of the distance, in *miles*.

5. REFRACTION. Besides the correction for curvature applied to the apparent level of an object to reduce it to its true level, another correction due to

terrestrial refraction is applied, and this latter has the effect of causing the position of an object to appear *higher* than it really is, hence the correction for it is *subtractive*.* The correction for refraction is often assumed at one-seventh of the correction for curvature, and is quite correct for all practical purposes.

6. CORRECTION FOR CURVATURE AND REFRACTION COMBINED. Since the correction for curvature is *additive* and that for refraction *subtractive*, and since the latter is always less than the former, being $\frac{1}{7}$th of it, it is usual to combine both and treat them as one correction. For instance, from A, a place of observation, the apparent level of a point B 2,000 feet away was found to be 5·97 feet; its true level would be arrived at as follows:—

Feet.
Apparent level of B ... ... = 5·97
Correction for curvature for 2,000 ft. (from Table XLI, "Calculating Tables," 5th edition) = + 0·096 }
,, ,, refraction for 2,000 ft. (,,) = − 0·014 } = +0·08
True level of B ... = 6·05

7. CORRECTIONS WHEN UNNECESSARY. (Fig. 458.) The corrections due to curvature and refraction are not applied in ordinary levelling operations, since under a distance of 1,000 feet they are inappreciable; also where the spirit-level is set up as nearly as possible midway *between* two stations, as at E between C and I, the influence of both curvature and refraction is counteracted by taking observations at equal distances from each side of the instrument, when they are similarly affected, and thereby nullified. It is only when the instrument is set up *at* one station and observations are taken to another, and such other be over 1,000 feet distant, that curvature and refraction are allowed for. An example in simple levelling (Art. 8) illustrates where the corrections should be applied. It will be well to recollect that when the instrument has been set up *at* a station, the height of the instrument has also to be considered, but not so when it is placed *between* them.

8. SIMPLE LEVELLING. (Fig. 438.) *Simple levelling* is applied to the process by which the difference of level between two points is ascertained *at one operation*.

Examples: (1) It was required to drain a swamp A by making a cut to a stream at B, a distance of 1,800 feet. Let a level be set up at C, and directed to a staff held upright at the edge of the water at B. The horizontal line CD represents the line of sight which would cut the staff at D, the reading being 11·94 feet; the height of the instrument above the ground is 4·6 feet, and the depth of the swamp 5 feet; therefore the difference of level between the bottom of the swamp and the surface of the stream is as follows:—

Reading of the staff ... ... = − 11·94
Height of instrument ... ... = + 4·60
Depth of swamp ... ... = + 5·00
Curvature and refraction for 1,800 ft. (Table XLI, "Calculating Tables," 5th edition) = + 0·07 + 9·67
Difference of level = − 2·27

(2) An instrument is set up midway between A and B, from which the staff at A reads 6·89 feet and the staff at B 10·50 feet; how much is B higher or lower than A? ... ... *Ans.*: 3·61 feet lower.

9. COMPOUND LEVELLING. (Fig. 465.) By *compound levelling* is meant the process by which the difference of level between two distant points is found through intermediate stations, and therefore at several operations.

OBSERVATION.

* *See* also Chapter XVI, Art. 39.

Examples: (1) Let it be required to find the difference of level between two distant points A and I. Set up a staff at A and another at C about 600 or 800 feet away, somewhere in the direction of I, and plant the instrument about midway between A and C at B. Direct the telescope to the back staff at A and note its reading, which, say, is 3·62 feet; then turn it round to the forward staff at C and note its reading, which, say, is 8·74 feet; thus the ground at C will have fallen 8·74 — 3·62 = 5·12 feet; again send the staff at A on to E in the direction of I, remove the instrument and set it up at D about midway between C and E, note the reading of the staff at C, which now becomes the back staff, and that of E, which becomes the forward staff; and so proceed till I is arrived at. The difference of the readings on any two successive staves shows the difference in level of the ground on which they stand; if the forward reading is greater, the ground has fallen; if lesser, it has risen; and the difference between the sum of the back sights and the sum of the forward sights will give the difference of level between the extreme points, thus:—

Back sights.		Instrument at	Forward sights	
A	3·62	B	C	8·74
C	1·79	D	E	10·83
E	4·55	F	G	6·18
G	11·24	H	I	2·07
	21·20			27·82
				21·20

Difference of level = 6·62 feet,

showing that the point I is 6·62 feet lower than the point A.

(2) What is the difference of level of A and E, taking the data from the last example? ... ... *Ans.:* 14·16 feet lower than A.

10. THE DUMPY LEVEL. (Fig. 424.) There are three descriptions of spirit-levels used for the purpose of levelling, namely, the *Y Level, Troughton's Level,* and *Gravatt's Level,* of which the last is more generally used and is much to be preferred to the others owing to its superior compactness, increased optical power, and greater stability of the adjustments. The Gravatt's Level is furnished with an object-glass of large aperture and short focal length; and sufficient light being thus obtained to admit of a higher magnifying power in the eye-glass, the advantages of a much larger instrument are secured, without the inconvenience of its length: it is hence called the *Dumpy Level.*

11. ITS USE. The levelling instrument is used for measuring the rise and fall of the surface of the ground, and for taking the section of a hill, or of a proposed road, canal or railway.

12. ITS DESCRIPTION, WITH THE VARIOUS PARTS NAMED. (Fig. 424.) The Dumpy Level is constructed much after the fashion of the theodolite, and by possessing a knowledge of the latter instrument, the student will find no difficulty in mastering the former. The instrument rests on three footscrews *f*, which work in a tribrach. The telescope T, which shows objects inverted, is permanently attached to two stout horizontal bars BB, and has *only* a horizontal motion. There is a space of about half an inch left between the bar and the telescope to admit a prismatic compass C, over which is a small magnifying glass *g* on a moveable arm to read the graduations of its limb; *s* is a screw stop which settles the needle in the compass-box. The diaphragm is carried by the internal tube *a*, which is nearly equal in length to the external tube. The external tube T is sprung at the aperture, and gives a steady and even motion to the internal tube, which can thrust out, or draw in, to adjust the focus for objects at different distances, by means of a large mill-headed screw M. A very sensitive spirit-level *l*, 6½ inches in length, is placed above the telescope and attached thereto by a hinged joint at one end and a capstan-headed screw at the other, for the purpose of adjusting it; it is divided to inches and tenths. A small cross-level K, 2½ inches in length, is placed upon the telescope at right angles to the principal level *l*, by means of which the instrument may be set up at once with the axis nearly vertical,

but must only be used for levelling the instrument approximately. The mirror *m*, mounted upon a hinge joint, can be placed at the object end of the level *l*, so that the observer, while reading the staff, can at the same time see that the bubble retains its proper position, a precaution by no means unnecessary in windy weather, or on bad springy ground. The cap P can be drawn out to prevent the sun, when just above the horizon, from striking the object-glass, which it would otherwise do, and L is its lid. Finally the instrument is placed on a stand similar to that of the theodolite (*see* Chap. XIII, Art. 26).

13. THE ADJUSTMENTS. The adjustments of the Dumpy Level are of two kinds:—namely, *temporary* and *permanent*; the former are necessary every time the instrument is set up for use, and the latter should be carefully examined and rectified before commencing a series of observations or when the surveyor has reason to think the instrument has been shaken by some means, and the adjustments so displaced.

14. THE TEMPORARY ADJUSTMENTS. The temporary adjustments are—

(1) Obtaining distinct vision of wires.
(2) Eliminating parallax or "finding focus."
(3) Levelling or setting the vertical axis of motion truly vertical.

15. OBTAINING DISTINCT VISION OF WIRES. Apply your eye at the eye-end of the telescope and push in or draw out the eye-piece until the wires appear perfectly clear and sharp.

16. ELIMINATING PARALLAX. Direct the telescope to a staff set up on a station about to be observed, and by means of the large mill-headed screw draw the telescope out till the divisions on the staff are distinctly visible. Now intersect any one division of the staff with the horizontal wire by raising or lowering the footscrews, and if by the observer shaking his head *vertically* the division of the staff and the horizontal wire continue identical, parallax is destroyed; but if the division appears to move, parallax exists, and the telescope must be drawn in or out, as necessary, until such movement ceases.

17. LEVELLING. For this adjustment, *see* description in (Chap XIII, Art. 48). When the principal level is in adjustment, the short cross level will also be in adjustment, that is, its bubble will come into the centre of the short tube.

18. THE PERMANENT ADJUSTMENTS. The permanent adjustments are—

(1) Collimating.
(2) Setting the spirit-level parallel to the line of collimation.
(3) Setting the telescope perpendicular to the vertical axis.

19. COLLIMATING. (Fig. 466.) To collimate for the "Y" and "Troughton's Level" (*see* Chap. XIII, Art. 54); but for the Dumpy Level, as the telescope is firmly fixed to its supports and cannot be removed, the following process must be gone through:—On a tolerably level piece of ground set up two levelling staves at A and B about 400 feet apart. Bisect the distance between them, and plant the instrument exactly over the middle point C; go through the temporary adjustments, direct the telescope to the staff at A and note its reading which, say, is 4·56 feet; do likewise with staff B, the reading of which, say, is 7·28 feet; the difference of the two readings being 7·28 — 4·56 = 2·72 feet. Now at D, 6 feet beyond B in the direction of AB produced, again plant the instrument, put it through its temporary adjustments and observe the respective readings of the staves at A and B, when the reading of B differs from the reading of A (it being greater in the assumed case) by 2·72 feet, the line of collimation will be correct, for instance, if B read 5·13 and A read 2·41; otherwise, if the reading of B was, say, 5·13 and that of A, say, 3·09, it is rectified by raising or lowering the diaphragm wires by the collimating screws,* until the reading of the further staff A gives the exact difference in level of the two staves, or 5·13 — 2·72 = 2·41 feet, the reading of the first staff B remaining the same as before.

OBSERVATION.

* The diaphragm should be moved up if the reading on the further staff exceed that on the nearer; and *vice versâ*. It is necessary that the bubble of the large level should be kept at the centre of its run throughout the whole process at both settings up, at C and D, of the instrument. It is advisable that the bubble of the cross-level be also kept at the centre of its run. Any deviation in either bubble should be corrected by the footscrews.

20. SETTING THE SPIRIT-LEVEL PARALLEL TO THE LINE OF COLLIMATION. Move the telescope parallel to two of its footscrews, and by turning them inwards or outwards, as necessary, bring the air-bubble to the centre of its run; now turn the telescope a quarter round till it is exactly over the third screw, by means of which bring the bubble to the centre of its tube; again, repeat the above operation *carefully* twice. Finally move the telescope over the first two footscrews and by their means bring the bubble in the centre of its run; now turn the telescope a *half* round, and should the bubble not return to the centre of the level, it shows that it is not parallel to the optical axis, and therefore requires correcting. The end to which the bubble retires must be noticed, and the bubble made to return one-half the distance by the two footscrews, and the other half by turning the capstan-headed screws at one end of the level tube The telescope must now again be reversed, and the operation repeated, until the bubble settles at the same point of the tube, in the centre of its run, in both positions of the instrument. The bubble will now remain stationary in the centre of its tube during a whole revolution of the telescope, and the adjustment is therefore perfect.

21. SETTING THE TELESCOPE PERPENDICULAR TO THE VERTICAL AXIS. No such adjustment is necessary in the Dumpy level, as the telescope is a fixture so constructed by the makers; but in the Y and Troughton's Level the operation is performed by first levelling the instrument, and raising or lowering one end of the telescope by a screw for that purpose.

22. LEVELLING STAVES. (Figs. 455, 459, 460.) There are various descriptions of levelling staves that have been invented, but for simplicity and usefulness, the Roorkee pattern (*see* Fig. 455) is unequalled. The levelling staff Roorkee (pattern) consists of a rod 22 feet in length, one broadside of which is graduated into feet and tenths, each tenth division being subdivided into five equal parts to read to $\frac{2}{100}$ of a foot and by estimation to $\frac{1}{100}$ of a foot. The feet divisions and the numerals denoting the same are marked in red, and the tenths and hundredths in black. By means of the powerful telescopes applied to spirit-levels, the graduations of a levelling staff can be distinctly and accurately noted at a distance exceeding 10 chains, but they are never required to be read at such distances unless in cases of *simple levelling*. The upright figures on the staff, when viewed through the telescope of a Dumpy level, the glasses of which invert everything seen through it, appear in an inverted position, so that the surveyor must be cautious to read the divisions *from* the top downwards and also be careful in distinguishing the figures 8 and 9 from 3 and 6; the eye, however, soon becomes accustomed to this. Two staves are required in levelling, one to be held at a back station and the other at a forward station. A careful man should always be selected to hold the staff, and he should never be allowed to raise the staff off the ground in turning it. A small plummet is sometimes enclosed in the staff, about the height of the eye, which enables the staffman to hold it perpendicular.

Examples.—(1) What is the reading of the levelling staff viewed upright (*see* Fig. 459), cut by the horizontal wire $a - a$, $b - b$, $c - c$? *Ans.*: $a = 2·73$, $b = 3·42$, $c = 4·07$ feet.

(2) Required the reading of the levelling staff viewed inverted (*see* Fig. 460) cut by the horizontal wire $x - x$, $y - y$, $z - z$. *Ans.*: $x = 6·94$, $y = 7·40$, $z = 8·09$ feet.

23. DIAPHRAGM. (Figs. 424, 468.) The diaphragm fixed towards the eye-end of the telescope is provided with two parallel vertical wires, *aa*, *bb*, a little either side of the centre, and one horizontal wire, *cc* which passes through the centre; the former enables the observer to judge whether the staff is being held perpendicularly (as in the figure), and by the latter he bisects it, and thus obtains the required height by reading off the figures thereon. To replace wires on the diaphragm in the event of their breaking from damp or other cause (*see* Chap. XIII, Art. 60).

24. MODE OF LEVELLING. (Fig. 467.) In running a line of levels, for instance from point A to point E, direct a staffman to hold a levelling staff perpendicularly at the starting-point A, and send another staff to be similarly held up at a point B from 3 to 10 chains distant, either in or out of the direction AE,

as the nature of the ground admits. Have the staves to face each other. Next plant the levelling instrument *about* midway between A and B, either upon the line or on one side of it, whichever is most convenient, and so that the line of sight does not intersect the ground line before reaching the staff; it being borne in mind that the instrument need not be set up in a direct line between any two staves (although it is customary to do so when possible), unless a section of the ground is wanted. Now put the instrument through its temporary adjustments (*see* Art. 14), taking care that the bubble of the level is *accurately* in the centre of its run; direct the telescope to the back staff at A and observe what division of the staff is cut by the horizontal wire and record the reading, called the *back reading* or *back-sight*, in the field-book; also by means of the moveable magnifying glass read the degrees and minutes denoted by an index pin inside the prismatic compass-box, on the graduated limb, which indicates the bearing of A at the position of the instrument, and register the same in the field-book as the *back bearing*. Again, taking care that the level still continues in adjustment, turn the telescope, directing it to the forward staff at B, and obtain the reading thereon, and note the same in the field-book; this reading is termed the *forward reading* or *fore-sight*. Read also the bearing of B from the prismatic compass, as was before done of A, and enter it in the field-book as the *forward bearing*. Measure the *back distance* from A to the instrument (to the point on the ground where a pebble would fall when dropped from the bottom of the vertical axis of the instrument) and the *forward distance* from the instrument to B, by means of a chain, and make entries of the same in the field-book. Having finished the requisite observations and measurements between the points A and B send the staff at A onward past B somewhere in the direction of E, and set it up between 3 and 10 chains beyond B at the next inequality of the ground, C, where it becomes the forward staff, while that at B is now the back staff; the faces of both these staves will of course be turned so as to face each other. Remove the instrument to *about* midway between B and C, and go through precisely the same process as was gone through at its first situation, registering the observations and measurements in the field-book, as before, and so continue onwards till the final point is arrived at. Observe that if after the instrument has been adjusted at a place, and during the observations the level bubble has shifted from the centre of its tube, the instrument must be re-levelled and *all* the observations at that place repeated.

25. CERTAIN CORRECTIONS OMITTED. As stated in (Art. 7), the corrections for curvature and refraction as well as the height of the instrument are not considered when the instrument is placed *between* staff stations, hence the desirability of invariably setting up the instrument approximately midway between staff stations.* When practicable, however, the instrument should always be placed *exactly* midway between the staves and in their *direct* line, as it facilitates the after-plotting considerably, and for other obvious reasons. In cases of simple levelling generally where the instrument is set up *at* one of two stations, and the distance between them is *over* 1,000 feet, then the correction for curvature and refraction must be allowed for; and in all such cases the height of the instrument must be taken into reckoning as in the example to (Art. 8).

26. CHAIN MEASUREMENTS. The measurements that are made with the chain of the distances between staves are *horizontal* and not hypotenusal measurements, hence great care should be taken that they are as nearly such as possible, otherwise the section will be longer than it ought to be. For measuring over irregular ground such horizontal measurements may be obtained by holding one end of the chain close to the ground at the higher end, and holding the other end *above* the ground, stretching it in a horizontal line, a stone let fall from the latter end marking upon the ground the point to which the measurement reaches. For further particulars as to the measurement of horizontal distances (*see* Chap. IX, Art. 9).

OBSERVATION.

* By this means, too, any discrepancies due to the line of collimation being out of adjustment, or to other instrumental errors, are eliminated; for the error in *reading* on the back and forward staves will then be equal and the resulting difference of level (being the difference of the readings) will consequently be correct.

27. **CHAINMEN AND STAFF-HOLDERS.** In all levelling operations two chainmen are required for the purpose of measuring distances with the chain, and two staff-holders to hold the staves at the several staff stations. They should all be men of some experience in field-work, especially the chainmen, who otherwise would be apt to measure the distances as too little or too much.

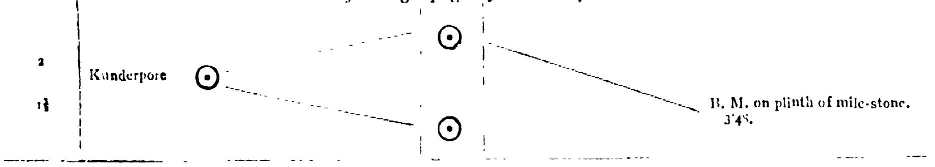
28. **INTERMEDIATE READINGS.** Besides the back and forward readings referred to in (Art. 24), *intermediate readings* are taken from where the instrument has already been placed to supplemental staves set up at great inequalities of the ground or at the level of water in a swamp, stream, or well, or a bench-mark of the Public Works or other department, or some conspicuous object the relative height of which is desired. Such intermediate readings may be taken anywhere between the staves, on or off the line; and are to be recorded together with their bearings and distances from the position of the instrument, in the field-book, precisely in the same way as for the staff stations.

29. **FIELD-BOOK.** (Fig. 467.) The field-book adopted for levelling differs altogether from that used in surveying. It is ruled with several columns, usually having printed headings, in which the observations and measurements made in the field are recorded and the reduced levels worked out and entered. Many forms of field-books are in use, of which three specimens are given below, namely, (1) the Canal Pattern, which has its adjoining page ruled for a chain or compass survey, (2) the Roorkee Pattern, and (3) the Survey Pattern; but the most useful form will be the one which follows the above, and although some surveyors may think that there are too many columns in it, yet they can easily omit filling up any which they object to.

CANAL PATTERN.

No. of back station.	BACK.		Distance from Inst to each station.	FORE.		DIFFERENCE.		Reduced level of back station.
	Read.	Bear.		Bear.	Read.	Rise.	Fall.	
		°		°				
1	4'50	7	300	187	5'24	...	0'75	451'43 R. L. of G. T. S.
2	6'18	7	300	187	5'34	0'64	...	451'28.
3								

Adjoining page for Survey.



N.B.—Station 1½ is between 1 and 2, where the instrument is set up.

ROORKEE PATTERN.

No. of station.	Back-sight.	Intermediate sight.	Fore-sight.	Distance.	Bearings.	Rise.	Fall.	Reduced level.	REMARKS.
					°				
1	4'56	...	...	...	187	...	.	500'60	R. L. of starting-point.
...	...	5'34	.	...	...	..	0'78	499'22	Bench-mark, No. 1 (G. T. S.)
2	6'71	...	4'21	500	172	0'35	...	500'35	
...	...	4'28		800	...	2'43	...	502'78	Left bank of river.
...	..	8'30	.	818	...	...	1'63	498'67	Highest flood mark, left bank.
...	...	11'73	.	.	...	.	5'21	493'14	Level of water, January 1st, 1868.
...	..	8'50		1000	...	...	1'70	498'56	Highest flood mark, right bank.
3	...	...	2'65	1054	..	4'06	...	504'41	Right bank, cross section No. 1.

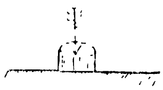
SURVEY PATTERN.

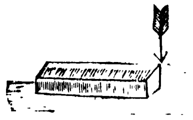
No. of station.	BACK.			Intermediate.	FORE.			Difference of Back and Forward Staves.	MEAN.		Reduced levels.	REMARKS.
	Bearings.	Distance, in chains.	Staff.		Staff.	Distance, in chains.	Bearings.		Rise +	Fall −		
	° ′						° ′					
1	266 30	5'00	6'64	...	8'99	5'00	86 30	2'35	...	2'35	455'21	R. L. of B. M. No. 37 = 457'56.
2	266 30	5'00	7'09	..	8'26	5'00	86 30	1'17	..	1'17	454'04	
3	266 30	5'00	7'43	...	8'21	5'00	86 30	0'78	...	0'78	453'26	

LEVELLING FIELD-BOOK.

No. of Staff Station.	Back.			Forward.			Intermediate.			Differences of Sights.		Reduced level.	Remarks.
	Sight.	Bearings.	Distance.	Sight.	Bearings.	Distance.	Sight.	Bearings.	Distance	Rise +	Fall −		
	Feet.	° ′	Feet.	Feet.	° ′	Feet.	Feet.	° ′	Feet.	Feet.	Feet.	Feet.	
1	...	...	...	...	..	...	..	.	...	...	..	56·27	* B. M. No. 1, 56·27 feet above mean sea level.
1—2	7·08	206 20	323	2·36	38 40	280	...	...	...	4·72	...	60·99	
2—3	4·57	207 00	250	4·72	26 50	250	...	...	...	...	0·15	60·84	At centre of road to Court-house, 45 feet broad.
"	...	...	...	...	...	...	11·47	207 00	151	...	6·90	54·09	Bottom of dry drain, 10 feet broad.
3—4	2·10	185 10	486	11·23	31 00	671	...	...	...	...	9·13	51·71	† B. M. No. 2, at foot of 3rd milestone.
4—5	11·97	213 20	577	0·64	31 10	204	...	...	...	11·33	...	63·04	
"	...	...	...	...	...	...	1·85	223 30	83	10·12	...	61·83	‡ B. M. of P. W. D., 2 feet high.
"	...	...	...	...	...	...	19·40	33 10	72	...	7·43	44·28	Bottom of stream —20 feet broad.
"	...	...	...	...	...	...	11·56	33 10	72	0·41	..	52·12	Stream-level of water.
"	...	...	...	...	...	...	23·49	142 30	133	.	11·52	40·19	Well-level of water.

* B. M. No. 1 is a circular pillar 2 feet in diameter built of masonry and is 3 feet above ground, at the southern corner of Government wharf.





‡ On top of left wall (N. E. corner) of masonry bridge near Court-house.

The last of the four forms of levelling field-books will be the one referred to hereafter in this work, the entries in which are to be made in the following order: The first column gives the number of the station, which refers to the position of the staff and not to that of the instrument; the 2nd, 3rd and 4th show the height of the sight, the bearing, and the distance of the back staff respectively; the 5th, 6th and 7th, respectively, give the height of the sight, the bearing, and the distance of the forward staff; the 8th, 9th and 10th respectively give the height of the sight, the bearing, and the distance of any intermediate object; the 11th and 12th give the difference (obtained by subtracting the lesser reading from the greater) between the back sight and the forward or intermediate sight, as the case may be, the rises being placed in the 11th and the falls in the 12th column; the 13th gives the reduced level above or below the first station or some other datum, obtained by applying the difference of sights appearing in columns 11 and 12 of a forward or intermediate station, as the case may be, to its corresponding back station; and the last or 14th column is left for remarks respecting the situation of station, crossings of rivers, roads, etc. The 11th, 12th and 13th columns can be calculated and filled up after a day's work or in the office, but the other columns must be entered up on the ground. With respect to the entries in the 11th and 12th columns, when the fore-sight is *less* than the back-sight, it is thereby shown that the forward staff must have been situated on *higher* ground than the back staff, and it hence indicates a *rise*; if, on the contrary, the fore-sight is *greater* than the back-sight, it is thereby shown that the forward staff must have been situated on *lower* ground than the back staff, and it hence indicates a *fall*; and the differences therefore are accordingly placed under the heads of Rise (+) and Fall (—). This reasoning will be clearly understood by a glance at the Figure. Field-books should be kept with clearness and accuracy as it often happens that others have to plot the work, although it is advisable that each surveyor should plot his own field-work when possible.

30. PROOF OF FIELD-BOOK. To prove that a field-book has been correctly kept, a thing which must invariably be done, work out the difference

between the totals of the back and fore sights, that between the totals of the rises and falls, and that between the reduced level of the first and last station, and compare them. If all three agree, the field-book is faultless. The following field-book, containing only the necessary entries of the field-book in (Art. 29), will sufficiently illustrate the above rule:—

STATION.	Back-sight.	Fore-sight.	Rise.	Fall.	Reduced level.
1	...	...	...	...	256·27
2	7·08	2·36	4·72	...	260·99
3	4·57	1·72	...	0·15	260·84
4	2·10	11·23	...	9·13	251·71
5	11·07	0·64	11·33	...	263·04
	25·72	18·95	16·05	9·28	263·04
	18·95	...	9·28	...	256·27
	6·77		6·77		6·77

31. STAFF STATION. At each staff station an ordinary tent peg a foot long should, where it is possible, be driven into the ground flush with the surface, upon which the staff must be held; whereas if no peg or other similar contrivance for obtaining a firm basis was used, the staff, if resting on grass, clay, or gravel, etc., would be liable to undergo a very sensible change in its elevation, when turned round from one side to the other in becoming a back staff from having been a forward staff; and a number of such changes in the course of a long section, would introduce serious errors which may be altogether avoided by the contrivance aforesaid. On metalled roads or pucka masonry work, and wherever the ground upon which the staff is to be placed is very firm, it will be evident that no peg is necessary. For bench-mark purposes and where future identification of the stations is an object, pegs about 3 feet long should be used, and a note made of their distance and bearings from any easily recognized and permanent objects, if such are available.

32. SURVEYING OF COUNTRY WHICH IS TO BE LEVELLED OVER. Before engaging in levelling operations, surveyors should provide themselves with the published maps of that part of the district or town in which their work lies, if such are to be had, as they will be found most useful guides in the selection of points and in connecting their lines of levels with known stations of the Survey or Public Works Department. If it is not practicable to get such maps, it is usual to make a survey, in ordinary levelling operations, of a chain's length on either side of the line of levels, showing the features of the country, drains, tanks, sand-hills, towns and villages, wells, buildings, roads, etc.

33. MEAN SEA LEVEL. (Fig. 469.) The tides from the springs to the neaps may be said to oscillate above and below the mean tidal line, and it is found that although the rise and fall of the tides varies in amount round the coast, the mean tidal line, called the *mean sea level*, is nearly a constant level, and it is therefore a much better line of reference for levels and contours than any other line.

34. DATUM LINE. (Fig. 470.) The words "Datum Line" in connection with a section signify a standard, a continued reference or horizontal line concentric to the earth's exterior, to which all the calculations are referred. All levels are referred to some fixed datum line, such as the level of the starting-point, an *assumed* datum of 100 or other number of feet below the level of the first station, or the mean sea level. The height or depth of the point above or below such datum so referred, is called the *Reduced Level* of that point. In the first instance, where all the subsequent levels are referred to the level of the starting-point, the inconvenience of placing the sign + against all reduced levels *above* the datum, and the sign — against all those *below* it is occasioned. In the second instance, where the first station is assumed to be 100, or any other number (which is chosen high enough to avoid the necessity of using negative quantities for the reduced heights that follow) of feet usually *greater* than 50 above an imaginary line, which is termed the "assumed datum line," all the reduced levels would

be +, or above the datum line, and therefore no need of an algebraic sign. In the third instance, where the reference is to the mean sea level, the advantages are greatest, although they are shared by the method in the second instance. The datum line being the mean sea level or lowest point of reference, the high numbers of the reduced levels will show high ground, and the lower numbers the lower ground, and in inspecting a plan covered with a network of such numbers, the comparative heights of points is seen at a glance. The accompanying example, the drawings of which are seen in the Figure, will elucidate the three instances above referred to:—

STATION.	Rise.	Fall.	REDUCED LEVEL.			REMARKS.
			1st instance.	2nd instance.	3rd instance.	
1	...	...	0·00	100·00	378·26	Height of starting-point.
2	1·76	...	+ 1·76	101·76	380·02	
3	...	2·93	− 1·17	98·83	377·09	
4	...	5·04	− 6·21	93·79	372·05	
5	10·85	...	+ 4·64	104·64	382·90	

35. BENCH-MARKS. The object of *Bench-marks* or fixed stations is to serve the purpose of future reference, namely, to check the accuracy of the work by running check levels, for correcting errors by repeating the levels between the marks where the error is detected, without its being necessary to retrace the entire work a second time, to close or start new or branch lines of levels, and to run cross sections at right angles. Bench-marks should consist of permanent objects, and be defined by a light sketch and description in the column for remarks, of the field-book, that they may be easily found again. They should be marked on gateposts, milestones, stumps of large trees, the floor of a bridge, any masonry construction, and such like. A bench-mark must be left at the close of a day's work for the continuation of the levels from that spot the following day. In levelling for a long section bench-marks ought to be chosen at a distance of about half a mile asunder. Bench-marks are usually indicated by a broad arrow scratched on the substance. The letters B.M., wherever they occur, denote Bench-mark.

36. CHECK LEVELS. In levelling there is no check on the accuracy of the work except it be gone over a second time; the second operation is termed "check levelling," and should invariably be resorted to. Check levels are run from bench-mark to bench-mark, and in them it is unnecessary to chain the distances or read the bearings; all that is required being to ascertain the relative elevations of the bench-marks and see that they correspond with the results first obtained; if they differ, the error must be found and corrected. It is immaterial by what route these levels are carried, the most convenient being chosen for the purpose, as along roads or fields, etc., but they must always commence and finish at a bench-mark. The field-book required for check levels is merely a simple entry of back and fore-sights, the difference of the sums of which will be the difference of level between the extreme points of the section line. The following is the form of a field-book for check levels:—

STATION.	Back-sight.	Fore-sight.	REMARKS.
4	5·37	7·46	B.M. on stump of tree.
5	3·51	0·60	
6	4·42	4·59	
7	8·63	6·71	
8	4·96	5·88	B.M. on 16th milestone
	26·89	25·24	
	25·24		
	1·65		

37. CROSS LEVELS. *Cross levels* are principally taken at the crossing of a line of railway or canal, of roads or rivers, to show the nature of the surface of the ground. Cross sections are generally obtained without removing the instrument, and the observations extended to about 1 chain's length on either side of the main section; when they are required at other parts, the line of cross levels should be at right angles to the base. For a cross section the levels are to be always taken from the right to the left of the forward direction of the main line, lest the section, through mistake, should be plotted in the wrong direction. The field-book for a cross section has 5 columns, the 1st for the names of objects, the 2nd for back-sights, the 3rd for fore-sights, the 4th for distance, and the 5th for remarks. The cross section may be plotted on the same scale as the main section, though some surveyors adopt a larger one.

38. LIMIT OF ERROR. The limit of error allowed in any circuit of levels is 1 foot per 100 miles traversed—which in no case ought to be exceeded if the observations and measurements have been carefully made. *Minute* errors arising from incorrect reading of the staff, it not being held vertically, high wind, and such like, are inseparable from all levelling operations, but these will never be found to accumulate, as what will be in excess in one set of observations may be deficient in another, and so on.

39. SCALES. The vertical scale to which a level section is plotted is always *larger* than that of the horizontal, in order to show the slightest variations of the ground. Engineers often take 4 inches to a mile or 5,280 feet for the horizontal scale, and 400 feet to one inch for the vertical scale. When a survey has also been made along the line of levels, the scale for the protraction of the plan should be identical with the horizontal scale of the section.

40. PLAN DRAWING. After a survey of the tract of country through which the levels have been carried has been completed, a plan of it is projected on paper, on the scale determined upon, such as 4 inches to 1 mile; its object being to show the line or situation of the whole work, and the lands in or through which the levels have been carried. Such surveys are usually only executed in a very extensive series of levels for railways and canals and such like, but for ordinary work they are omitted.

41. SECTION PLOTTING. (Fig. 470). To plot a level section, first draw the horizontal and vertical scales on a convenient part of the paper; next rule a fine straight line, horizontally, to represent the datum line, when, if it is in a level with the starting-point, make a mark upon it to denote the first station A. For convenience sake and to avoid errors of plotting in a long line, mark off on the datum line, from A, distances of 10 chains. With the aid of a pair of compasses take from the horizontal scale, already drawn, the several *horizontal* distances of the staff stations as they appear in the field-book, and set them off on the datum line from A; through these points draw sufficiently long lines perpendicular to the datum line AB on either side of it, and set off on each of them, above or below as it is a rise or fall, the reduced levels of the several stations, and connect these last points, when the section will be made, showing the delineation of the ground levelled over. After this write in *vertically* well above the ground line, the several remarks as crossing of roads, rivers, etc., that appear in the last column of the field-book. All the numbers of the stations and the figures denoting the horizontal and vertical distances should be written in on the drawing *along* the lines they represent. Sketches with small descriptions of bench-marks should be entered in a corner of the paper for easy reference, and to provide against any accidents that may happen to the field-book. When the datum line is *below* the level of the starting-point, as in the Figure, the datum line is first drawn and the same procedure for plotting the sections adopted, only that in laying off the vertical heights they will all fall *above* the datum line.

42. LEVELLING WITH THE THEODOLITE. Levelling with the theodolite is in all cases inferior in point of accuracy to levelling with the spirit level, is much more heavy and occupies much more time, hence is very seldom resorted to for this purpose. It is only in *extraordinary* cases that it is necessary, where the levelling operations are required to be conducted over very high and rapidly rising ground, or over steep and almost perpendicular rocks, where the ordinary levelling

instrument cannot be fixed. The theodolite is also well adapted for obtaining the difference of level between two distant points, which are mutually visible. To level with the theodolite set it up *at* the staff stations, put it carefully through its adjustments and observe the vertical angle of the forward staff *of the same height as the theodolite* (measured from the ground to the axis of the telescope); next measure the *hypotenusal* or sloping distance between the theodolite and the forward staff. Again, remove the theodolite to the next station, and go through the same operation there, proceeding in this manner till the final station is reached. Now to find the difference of level between each successive pair of stations, calculate it by trigonometry (*see* Chap. III), where the hypotenuse (the measured distance) and one angle (the observed vertical angle) of a right-angled triangle are given; which worked by logarithms (Chap. IV) will give the following rule:—*Add together the logarithm of the measured distance and the log.—tangent of the observed angle; the sum, rejecting ten from the index, will be the log. of the difference of level in feet, of which the corresponding number will be the difference in feet required.*

43. THE WATER-LEVEL. (Figs. 366, 384, 464.) Circumstances may occur where a surveyor has no levelling instrument, and finds it necessary to take a section. In such a case he might use an instrument called the *Water-level*, which can be made up by any village blacksmith at a trifling cost and in a very short time. It consists of a hollow tube AB (*see* Fig. 464) of brass or tin about half an inch in diameter and about 3 feet long; C and D are short pieces of brass or tin tube of larger diameter, into which the ends of the long tube are soldered, and are for the purpose of receiving the two small bottles E and F, the ends of which, after the bottoms have been cut off,* are fixed in their position by putty or white-lead; the centre of the tube is fixed on to a light three-legged bamboo stand. When the instrument is required for use, water, slightly coloured with lake or indigo to define its surface more clearly, is poured into the tube through either of the bottles, till it nearly reaches their necks, which are then corked for the purpose of carriage. To observe with this instrument set the stand tolerably level with the eye, remove the corks gently, and wait till the water is in a state of rest, when the surfaces in each bottle will be on the same level; it being clear that the instrument requires no adjustment. Now let the surveyor place his eye in the line of the two surfaces of the water, and the intersection with the back and forward staves respectively of this visual line, gives the readings required. A staff, however, with a sliding vane (*see* Fig. 366) must be used for this operation, because the observer being unassisted by a telescope, would be unable to read the divisions on the staff, except when placed very near to the instrument. (Fig. 384) shows the use of the level and staff in outdoor work. This level, when used in calm and clear weather, gives surprisingly good results.

44. EXAMPLES IN PLOTTING. (Figs. 471—473.)

Examples:—(1) Plot the horizontal and vertical section of levels from the levelling field-book (last pattern) in (Art. 29), on a horizontal scale of 500 feet = 1 inch, and a vertical scale of 20 feet = 1 inch. *Ans.*: *See* (Fig. 472).

(2) (Fig. 473) is the sectional plot of a line of levels where the instrument was always planted *exactly* midway between the staves in their direct line, consequently it has been necessary to register only the *forward* bearings and the *total* distances.

(3) Plot the plan and section from the accompanying levelling field-book, of a circuit of levels taken from B.M. No. 1 as the starting-point and closing at the same place, on a horizontal scale of 600 feet to the inch, and a vertical scale of 80 feet to an inch. A rough sketch of the plan is given for guidance, *see* (Fig. 471).

OBSERVATION.

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* This may be effected by pouring cold water into the bottles to the height required to be taken on, and placing them erect in hot-water for a few minutes, when the desired separation will of itself take place.

No. of Staff station.	BACK.			FORWARD.			INTERMEDIATE.			DIFFERENCE OF SIGHTS.		Reduced level.	Remarks.
	Sight.	Bearing	Distance.	Sight	Bearing	Distance.	Sight.	Bearing.	Distance.	Rise. +	Fall. −		
	Feet.	° ′	Feet.	Feet.	° ′	Feet.	Feet.	° ′	Feet.	Feet.	Feet.	Feet.	
1	...	...	...		...	..	...	...	...	.....	.....	150·07	B. M. No. 1, 150·07 feet above mean sea level—on bridge over Kani river.
1—2	6·50	276 40	491	6·00	96 40	491							
..	...	.	..	...	...	..	28·47	270 30	375	..	...	...	Level of water in river, 100 feet wide.
2—3	5·41	254 30	480	7·14	74 40	481							
...	...	...	...	...	...	...	..	345 10	...	...	..	...	Pala village.
3—4	5·88	277 0	375	5·76	97 10	374							
...	...	...	..		..	..	..	27 40	..	...	...	...	Maungan village.
4—5	4·68	220 20	372	5·73	40 30	372							
..	...	...	..	..	...	...	...	305 10	...	...	.	..	Maungan village.
...	...	...	.	...	..	.	...	20 00	...	...	...	...	Mutu pagoda.
5—6	4·91	239 40	474	7·33	59 30	474	...	...	..	..		...	B.M. No. 2, on pucka well.
...	...	...	...	..	...	...	4·85	59 30	160	...	...	...	Drain 50 feet wide—level of bank.
...	...	...	...	.	...	...	15·26	51 30	..	...		...	Drain 50 feet wide—level of bottom.
6—7	5·99	176 10	203	6·23	356 20	204							
...	...	...	...	..	...	...	4·27	356 20	210	...	...	...	Kani river—nearer bank.
...	...	...	..	...	...	..	10·80	356 20	...	.	...	...	Kani river—level of water.
...	..	...	...	...	...	...	5·83	356 20	204	..	...	.	Kani river—further bank.
7—8	6·82	114 40	517	5·77	294 40	517							
...	...	...	...	...	...	...	6·00	294 40	188	...	...	...	Road to pagoda—centre.
8—9	4·73	88 30	484	3·95	268 30	484							
...	...	...	...	...	...	..	...	125 00	...	...	...	...	Mutu pagoda.
9—10	7·06	107 20	405	5·84	287 30	405	..	...	...	...	...	...	B. M. No. 3, on ruins of old pagoda.
...	...	...	...	..	...	...	6·23	...	...	...	...	...	Road to bungalow.
10—11	6·80	91 40	574	7·17	271 50	574							
...	...	...	...	...	..	...	...	156 50	...	...	...	...	Bungalow.
11—12	5·11	43 00	495	6·65	222 30	495							
...	...	...	...	...	...	...	...	119 10	...	...	...	...	Pala village.
12—13	7·27	341 30	407	5·02	161 20	407							
...	...	...	...	...	...	...	...	72 10	...	.	...	...	Bungalow.
...	...	...	...	...	...	...	...	89 20	...	...	...	...	Pala village.
13—14	8·36	341 40	415	6·64	161 20	415	...	...	..	...	...	...	Starting Point.
...	..	...	...	...	...	...	7·01	341 10	142	...	...	...	Road to Pala—centre.

CHAPTER XVI.

Practical Astronomy.

1. ASTRONOMY AND ITS USE. *Astronomy* treats of the motions of the heavenly bodies. Its chief use to the surveyor is that he is enabled to determine, by its means, the meridian, time, and latitude* of any place on the earth's surface, at which observations therefor are made.

DEFINITIONS.

GENERAL.

2. SPHERE. (Fig. 483.) A *sphere* is a surface, every part of which is equally distant from a point within called the *centre*, as C.

3. DIAMETER. (Fig. 483.) The *diameter* of a sphere, as NS, is any straight line passing through the centre, and limited on each side by the surface of the sphere.

OBSERVATION.

* Longitude may likewise be obtained, but this work requires more than the ordinary skill and knowledge. Some methods of finding the longitude which may be of use to the surveyor, however, are given further on (Art. 78).

4. GREAT CIRCLE. (Fig. 485.) A *great circle*, as ABCD, is that whose plane passes through the centre of the sphere, and divides it into two equal parts or *hemispheres*.

5. SMALL CIRCLE. (Fig. 486.) A *small circle*, as ABCD, is that whose plane does not pass through the centre of the sphere, and which divides it into two unequal parts, the one greater and the other less than a hemisphere.

6. ARC AND ANGULAR DISTANCE. (Fig. 484.) Since the arcs of all great circles are proportional to the angles they subtend at the centre of the sphere, the *arc*, as AB, is often used to designate the angle; and the angle is often called the *angular distance*, as BAC, between the extremities of the arc.

THE EARTH.

7. AXIS. (Fig. 430.) The *axis* of the earth, also called the ***axis of rotation***, as NS, is the imaginary straight line about which the earth rotates.

8. POLES. (Fig. 430.) The *poles* of the earth are the extremities of its axis; one of them being called the *north pole*, as N, the other the ***south pole***, as S.

9. TERRESTRIAL EQUATOR. (Fig. 430.) The *Terrestrial equator*, as WDE, is a great circle of the earth, equidistant from the poles; and which divides the sphere into the *northern* and *southern hemispheres*.

10. TERRESTRIAL MERIDIANS. (Fig. 430) *Terrestrial meridians*, as NWS, NAS, etc., are great circles passing through the poles of the earth, at right angles to the equator.

11. LATITUDE. (Fig. 430.) The *latitude* (λ.) of a place on the earth's surface, as AP, is its angular distance from the equator, north or south, measured on the meridian of the given place, and it cannot exceed 90°. If the place is north of the equator, it is called *north latitude*; if south, *south latitude*. It is the complement of the distance of the place from the pole, which is called the ***co-latitude***.

12. LONGITUDE. (Fig. 430.) The *longitude* (L.) of the place on the earth's surface, as OP, is the angular distance of its meridian from some fixed meridian (such as that of Greenwich)* to which all longitudes are referred, east or west, measured on the equator; and it cannot exceed 180°. A place to the east of Greenwich is said to have *east longitude*, and a place to the west, ***west longitude***.

13. SENSIBLE HORIZON. (Fig. 487.) The *sensible horizon*, as SOH, is a place that is tangent to the earth at the place of the observer.

14. RATIONAL HORIZON. (Fig. 487.) The ***rational horizon***, as RCH′, is a plane that passes through the centre of the earth, parallel to the sensible horizon.

THE HEAVENS.

15. CELESTIAL SPHERE. (Fig. 481.) The ***celestial sphere***, as PEP′Q, is the apparent concave sphere, with the earth as its centre, on the surface of which the heavenly bodies appear to be situated. †

16. AXIS. (Fig. 478.) The *axis* of the celestial sphere, as PP′ is the axis of the earth produced both ways to meet the celestial sphere, about which the heavenly bodies appear to revolve.

17. CELESTIAL POLES. (Fig. 784.) The ***celestial poles***, as P, P′, are the extremities of the axis of the celestial sphere, termed the ***north*** and ***south*** poles; also called the ***elevated*** and ***depressed*** poles. The north pole is near the pole star or *Polaris*.

18. EQUINOCTIAL. (Fig. 478.) The *equinoctial* or ***celestial equator***, as ELQ, is a great circle in the heavens formed by producing the plane of the terrestrial equator to meet the celestial sphere. It is equidistant from the celestial poles, and divides the celestial sphere into the northern and southern hemispheres.

OBSERVATIONS.

* Pronounced *Grin-ij*.

† At the terrestrial poles the zenith and pole coincide, and the diurnal paths of the stars are perpendicular to the vertical line, and therefore parallel to the celestial horizon, the horizon itself coinciding with the celestial equator. No star ever rises or sets here, and the visible heavens are confined to those portions which are on the same side of the equator as the elevated pole. Such a sphere is what is sometimes called a ***parallel sphere***. At a place in the terrestrial equator the poles are in the horizon and the zenith is a point of the celestial equator. The parallels of the stars here all cut the horizon at right angles and are bisected by it, so that half their course is above, and half below—every part of the heavens becoming visible in each revolution. Such a sphere is called a *right sphere*. The ***visible portion*** of the celestial sphere comprises the vault overhead at a point on the earth's surface bounded by the rational horizon. The ancient, or terrestrial, system of Ptolemy (about 150 A.D.) supposed the earth at rest round which the heavenly bodies revolved; the modern, or solar, system of Copernicus of the earth revolving round the sun and of the earth's rotation on its axis was introduced about 1500 A.D.

19. CELESTIAL MERIDIANS. (Fig. 478.) *Celestial meridians*, as PEP′, are great circles passing through the celestial poles at right angles to the equinoctial. They are also called *hour* or *declination circles*. Such a circle passing through the place of observation is the meridian of that place. When the sun is on the meridian above the horizon it is *noon*, or *mid-day* at that place, and below the horizon it is *mid-night*. The meridian of any place is the meridian which passes through the zenith of the place.

20. CELESTIAL HORIZON. (Fig. 478). The *celestial horizon*, as HMO, is a great circle in the heavens formed by producing the plane of the rational horizon to meet the celestial sphere. It is equidistant from the zenith and nadir. The east point, *e*, in this circle is the point in the neighbourhood of which stars appear to rise above the horizon, and the west point, *w*, the point towards which they set. O and H are the north and south points, and *e* and *w* the east and west points, of the celestial horizon.

21. ECLIPTIC. (Fig. 479.) The *ecliptic*, as ♈ MB, is that great circle of the celestial sphere in which the sun appears to perform his annual motion round the earth.

22. EQUINOCTIAL POINTS. (Figs. 479, 516.) The *equinoctial points* are those two diametrically opposite points in the heavens, where the ecliptic and the equinoctial intersect; that at which the sun appears to cross the equinoctial (about 20th March*) towards the north is called the *vernal equinox*, and the other (about 23rd September*) towards the south the *autumnal equinox*. The vernal equinox is also called the *First point of Aries* (♈), to the meridian, P ♈ P′, of which all heavenly bodies are referred.†

23. SOLSTITIAL POINTS. (Figs. 479, 516.) The *solstitial points* are the middle points of the northern and southern halves of the ecliptic; the northern is called the *summer solstice*, and the southern the *winter solstice*. ‡ §

24. OBLIQUITY OF ECLIPTIC. (Fig. 479.) The *obliquity of the ecliptic* is the angle ($23^\circ\ 27'\ 08{\cdot}03''$, being the *mean* obliquity of the ecliptic at the epoch 1900) which the ecliptic makes with the equinoctial. ||

OBSERVATIONS.

* On both of these days (civil) there is equal day and night all over the globe; and the sun appears to rise *due* east and set *due* west. The more distant the sun is from the celestial equator, the more unequal will be the days and nights.

† About 4,000 years ago the position of the Vernal (or spring) Equinox coincided with the *constellation* Aries, but on account of the *Precession of the equinoxes* (which is a retrograde motion of the equinox from east to west along the ecliptic—produced by the attraction of the sun, moon, and planets upon the spheroidal figure of the earth—at the present rate of about $50''$ annually, a complete revolution occupying 25,868 years), this point is now in the *constellation* Pisces. It is still customary to call the point of the spring equinox the "first point of Aries," and no confusion arises from the term. The "first point of Libra" is the autumnal equinox. (*See* Art. 40.) Besides the line of equinoxes, the line of apsides is also in motion in the same plane, but in the opposite direction, namely, eastward. This progressive motion of the apse-line amounts to about $11''$ annually, which, added to the retrograde motion of about $50''$ annually of the line of equinoxes, gives an annual increase in the longtitude (reckoned from the vernal equinox), or a change in the position of the apse-line with respect to the first point of Aries, of about $61''$. At this rate, the line of apsides would complete a sidereal revolution in 108,000 years, or a tropical revolution in 20,900 years. At present Polaris is approaching the pole, and in 120 years will be about half a degree off it, after which it will commence to recede. Nearly 4,000 years before the present epoch α Draconis was the Pole Star, being at that time distant from the pole $3\frac{3}{4}^\circ$. To the Arabian astronomers, 2,000 years ago, β Ursæ Minoris was nearest to the pole. After the lapse of 12,000 years, Vega (α Lyræ) will be the Pole Star, approaching to within about 5° of the pole. These changes arise from the *precession of the equinoxes*, which may also be described as the slow motion of the earth's axis, owing to which the intersection of the equator with the ecliptic travels along the latter, and brings about from year to year a constant change in the right ascension and declination of the stars. The phenomena of precession was discovered by Hipparchus about 140 B.C.

‡ The average variations in a day (*see* N.A., page I) of the sun's *Apparent* Declination (or the angular distance of the sun from the equator of the date, measured on the meridian) about the periods of the equinoxes and solstices are as follows:—

(i) At the Vernal Equinox $= \pm\ 23'\ 42''$(nearly).
(ii) At the Autumnal Equinox $= \pm\ 23\ \ 24$
(iii) At the Summer Solstice $= \pm\ 0'\ 13''$ (nearly).
(iv) At the Winter Solstice $= \pm\ 0\ \ 14$

§ The summer solstice occurs about the 21st June (when the day is longest, the sun being at its maximum distance, or greatest declination, N. of the celestial equator), and the winter solstice about the 22nd December (when the day is shortest, the sun being at its maximum distance, or greatest declination, S. of the celestial equator). In southern latitudes the seasons will of course be reversed. The summer solstice is at the first point of Cancer, and the winter solstice at the first point of Capricorn. (*See* Art. 49.)

|| The obliquity of the ecliptic (or the apparent inclination of the plane of the ecliptic to that of the equator) is equal to the sun's greatest declination, and this arc is at present diminishing at the rate of about $0{\cdot}5''$ annually, or more exactly at $46{\cdot}45''$ per 100 years (the *mean* annual diminution given in Leverrier's *Tables du Soleil*, page 203, is $0{\cdot}476''$); but this diminution will not go on beyond certain very moderate limits, after which it will again increase, and thus oscillate backwards and forwards through an arc of $1^\circ\ 21'$, the time occupied in one oscillation being about 10,000 years.

25. EQUINOCTIAL AND SOLSTITIAL COLURES. (Fig. 479.) A celestial meridian passing through the equinoctial points (♈ ♎), is called the *equinoctial colure*: and that passing through the solstitial points (♎ ♑), the *solstitial colure.*[*]

26. ZENITH. (Figs. 191, 478.) The *zenith*, as Z (*see* Fig. 478), is the point in the heavens [†] directly above the head of the observer.[‡]

27. NADIR. (Fig. 478.) The *nadir*, as N, is the point in the heavens directly below the feet of the observer, hence opposite to the zenith.

28. VERTICAL CIRCLES. (Fig. 478.) *Vertical circles* of the heavens, as ZHN, ZMN, etc., are great circles passing through the zenith and nadir, and cut the celestial horizon at right angles.

29. PRIME VERTICAL. (Fig. 480.) The *prime vertical*, as Z*e*N, is a vertical circle of the celestial sphere perpendicular to the meridian of the place, and therefore passing through the east and west points of the celestial horizon. The meridian of the place intersects the celestial horizon in the north and south points.

30. ZENITH DISTANCE. (Fig. 478.) The *zenith distance* (ζ) of a celestial object, as ZS, is the arc of a vertical circle contained between it and the zenith. It is the complement of the altitude. The *meridional* zenith distance is usually denoted by the algebraical sign $\pm$ according as the celestial object is $\frac{\text{S.}}{\text{N.}}$ of the zenith.

31. POLAR DISTANCE. (Fig. 478.) The *polar distance* (p) of a celestial object, as PS, is the arc of a celestial meridian, contained between it and the pole. It is the complement of the declination; and is known as the ***north polar distance***, or *N.P.D.*, when the object is in the northern hemisphere, and as the ***south polar distance***, or *S.P.D.*, when in the southern hemisphere. DSB is the ***diurnal*** circle the object appears to describe about the pole. The N.P.D. of the sun is greatest in winter and least in summer.

32. DECLINATION.[§] (Fig. 478.) The *declination* (δ)[‖] of a celestial object, as LS, is the arc of a celestial meridian contained between it and the equinoctial. It is the complement of the polar distance; and is declination N. or S., often written + or —, according as the object is in the northern or southern hemisphere. Declination in a celestial object corresponds to latitude in a terrestrial.

33. HOUR ANGLE. (Fig. 478.) The *hour angle* (t) of a celestial object, as SPZ, is the angle at the celestial pole between the hour circle passing through it and the observer's celestial meridian. In conformity with the earth's apparent diurnal motion, it should be reckoned *positively* from the upper culmination ***westwards***, from 0° to 360°. The polar distance and hour angle of a celestial object being known, its place with respect to the observer's meridian is known.

34. ALTITUDE. (Fig. 478.) The *altitude* (h) of a celestial object, as SM, is the arc of a vertical circle passing through it, contained between the object and the celestial horizon. The altitude as seen from the surface of the earth is its *apparent* altitude, and what it would be if seen from the centre of the earth is its *true* altitude. The altitude of the pole (celestial), as PO, is equal to the latitude (terrestrial) of the place, as OP.

35. AZIMUTH. (Fig. 478.) The *azimuth* (A) of a celestial object, as O*e*M $= \angle$PZS, is an arc of the celestial horizon contained between the north or south point and a vertical circle passing through the object. It is best to reckon it always from one point, as the north.

OBSERVATIONS.

[*] The solstitial colure is at right angles both to the ecliptic and to the celestial equator, for it cuts both these circles 90° from their common intersection; that is, from the equinoctial points.

[†] *I.e.*, celestial sphere.

[‡] The zenith varies with the position of the observer on the surface of the earth; thus, suppose the earth encircled on all sides by the starry sphere A′B′C′D′ (*see* Fig. 191) and A, B, C, D, the positions of four observers on the earth's surface, then their respective zeniths would be A′, B′, C′, D′. Owing to the force of gravity, the feet of each observer are attracted towards the centre of the earth.

[§] The R.A. and δ of the principal celestial objects at particular instants are given in the *N. A.* for the meridian of Greenwich, referred to the centre of the earth.

[‖] As the earth is belted by *parallels of latitude* and *meridians of longitude*, so are the heavens belted, so to speak, with *parallels of declination* and *meridians of right ascension.* The place of a celestial object is determined by its declination and right ascension, *referred to the equinoctial*; it can also be fixed by its latitude and longitude, *referred to the ecliptic.*

36. AMPLITUDE. (Fig. 478.) The *amplitude* of a celestial object, as *e*M, is an arc of the celestial horizon contained between the east or west point and a vertical circle passing through the object.

37. RIGHT ASCENSION.[*] (Fig. 478.) The *right ascension* (*R.A.*) or (Æ)[†] of a celestial object, or R.A., as ♈ EL, is the arc of the equinoctial contained between the vernal equinox (first point of Aries) and the celestial meridian passing through the object.[‡] It is reckoned *eastward* from the vernal equinox from 0° to 360°, that is, from 0*h*. to 24*h*. in time. Right ascension in a celestial object corresponds to longitude in a terrestrial.

38. PARALLAX. (Fig. 482.) The *parallax* of a celestial object is the angular difference of its apparent from its true altitude. Parallax is greatest when the celestial object is in the horizon, and is nothing, or zero, when it is in the zenith. In the Figure, E is the centre of the earth, O the observer's position on its surface; HOR the senible horizon, H'O'R' the rational horizon, S the real place of the object; the angle SOR the apparent altitude, and SO'R' the true altitude. Then parallax$= \angle SO'R' - \angle SOR$ or the angle OSO'; hence the correction for parallax is always *additive*, and will be found in (Table XXXIX, "Calculating Tables," 5th edition). This correction[§] is only applied to the apparent altitudes of the sun, moon and planets, but not to those of the fixed stars, as the magnitude of the earth is a mere point in reference to the immeasurable distance of the fixed stars from it.

39. REFRACTION. (Figs. 445, 446, 482.) *Refraction* in a celestial object is the angular difference between its apparent from its true place in the heavens at the observer's position. Refraction is due to the following cause: a ray of light from any celestial object, upon entering the atmosphere of the earth, becomes bent or *refracted*, in a downward direction. Thus, if O (*see* Fig. 445) were any point upon the earth's surface, HR the horizon, and S a celestial object, a ray passing from S would be bent downwards towards O; and an observer at O would therefore look in the direction OS' to see S, which would consequently appear higher by the angle S'OS (*see* Fig. 446), than it really is. The true altitude of S therefore $= \angle S'OR$, the observed altitude $-$ S'OS, the refraction; hence the correction for refraction is always *subtractive*, and will be found in (Table XXXVIII, "Calculating Tables," 5th edition). This correction is applied to the altitudes of all celestial objects. Refraction[||] is greatest when the celestial object is at the horizon, decreasing in proportion to its altitude, and is nothing, or zero, when in the zenith. To denude the observed altitude of the sun, moon, or planets of both parallax and refraction (*see* Fig. 482): RES, true altitude of S $=$ SOH observed altitude $+$ OSE parallax $-$ S'OS refraction.

40. CIRCUMPOLAR STAR. (Fig. 480.) A *circumpolar star* is the name given to the star the whole of whose diurnal circle is described above the celestial horizon; as S. In other words, those stars whose polar distance is equal to or less than the latitude of the place of observation, never set; but stars whose polar distance is greater than the latitude of the place of observation both rise and set. A star whose polar distance = 90° will be above the celestial horizon the same time that it is below it.

41. MAXIMUM ELONGATION. The greatest distance attained by a circumpolar star, east and west of the north pole or the observer's celestial meridian, in the course of its diurnal revolution, is called its *maximum elongation*. When the star is at its eastern or western elongation, its motion in azimuth is almost imperceptible, during which interval it offers the most favourable opportunity for observation.

OBSERVATIONS.

[*] The R. A and δ of the principal celestial objects at particular instants are given in the *N.A.* for the meridian of Greenwich, referred to the centre of the earth.

[†] *Vide* footnote to Art. 32 *super.*

[‡] The right ascension of a celestial object, therefore, is determined by the sidereal time at which it comes to the meridian. Since, also, the sidereal time is the distance of the First Point of Aries from the meridian and the R. A. of a celestial body is its distance from the First Point of Aries, then the time which has elapsed since such body passed the meridian will be the R.A. of the body subtracted from the sidereal time, or that time + 24 hours.

[§] Called *diurnal* parallax, by which observations are referred to the centre of the earth. The horizontal parallax of the sun is 8·8″ and of the moon from 54′ to 61′. A few stars are also affected by small parallaxes, known as *annual* parallax; *e g.*, *α* Centauri = 0·75″ and 61 Cygni = 0·5″.

[||] Refraction is about 34′ at the sensible horizon, and to it is due the slightly oval shape of the sun and moon when seen in contact with the horizon, as also daylight and dawn; and by its effect, too, the duration of daylight at sunrise and sunset is each increased about 4*m*.

42. CULMINATION. When the star is on the observer's celestial meridian it is said to *culminate*; and its culmination is said to be upper or lower, according as it is then in its highest or lowest position.

43. TRANSIT. The passage of a star across the observer's celestial meridian is called its *transit*; and the star is then said to transit the meridian.

44. SIDEREAL DAY. A *sidereal day*[*] is the interval between two successive transits of the same star across the observer's celestial meridian. It is divided into 24 sidereal hours, which corresponds to 23*h.* 56*m.* 04·09*s.* of mean time, and is always of the same length. Hence a sidereal day is 03*m.* 55·91*s.* shorter than a mean solar day. A star therefore that passes the celestial meridian at a certain hour today will tomorrow pass 03*m.* 55·91*s.* earlier, and so on every succeeding day. The sidereal day begins when the vernal equinox—that is, the first point of Aries—arrives at the celestial meridian; a correct sidereal clock should then mark 0*h.* 00*m.* 00*s.*, and at any other instant the *sidereal time* (*S.T.*) will be the hour angle of ♈ reckoned westward from 0*h.* to 24*h.* A star in the equator describes about the observer's position angles at the rate of 360° in 24 sidereal hours, or 15° in 1 sidereal hour.

45. APPARENT TIME. *Apparent time* (*A.T.*), also called *solar time*, is the interval between two successive transits of the suns centre over the observer's meridian and is of variable length. This is the time shown by a sundial, the days of which are unequal. The *solar time* at any instant is the hour angle of the sun's centre reckoned westward from 0*h.* to 24*h.* The instant of the sun's passage across the meridian of a place, is called *noon* at that place.

46. MEAN TIME. *Mean time* (*M.T.*)[†] is the *mean* interval between every two successive transits of the sun's centre over the observer's meridian for a whole year, and is uniform. This is the time shown by a well-regulated clock, the days of which are all equal. The *mean time* at any instant is the hour angle of the mean sun reckoned westward from 0*h.* to 24*h.*

47. EQUATION OF TIME. The *equation of time* (*E.*)[‡] is the difference between mean and apparent time at any instant, and is given for every day in the year, for the apparent (*A.N.*) and mean noon (*M.N.*) respectively, at pages I and II of the 'Nautical Almanac' for the meridian of Greenwich. A table of this equation of time is given in (Table XX, "Calculating Tables," 5th edition) and may be considered constant for ordinary purposes, as the variation in a year is very trifling.

48. RETARDATION AND ACCELERATION. A meridian of the earth returns to the same star in a shorter interval than it does to the sun; the difference, expressed in mean time, is called the *retardation* of mean on sidereal time; and when expressed in sidereal time, it is called the *acceleration* of sidereal on mean time. Thus, the retardation for 24 hours of mean time is = 03*m.* 55·91*s.*[§], or 24

OBSERVATIONS.

[*] The distances of the stars are so great that the earth's orbit is but a point as compared with them, so that the times of transit of a star will depend only upon the rotation of the earth.

[†] For the purposes of everyday life the day is necessarily regulated by the sun, which must be on or near the meridian in the middle of the day. Owing to the irregular rate of motion of the earth in its orbit, and the inclination of the earth's axis to the plane of the orbit, the interval between successive passages of the sun over the meridian are unequal. Solar days are therefore not all of the same length, and before a correct system of time-keeping can be got some unit must be fixed. This unit is the *mean solar day*, which is the mean interval between successive meridian passages of the sun during one year. This is further divided into hours, minutes, and seconds.

[‡] It is obvious that the sun will not transit at mean noon every day, but sometimes before and sometimes after. The difference, which is called the *equation of time*, never amounts to more than 16 minutes 20 seconds. The equation of time is usually considered as the correction to be applied to the apparent time to obtain the mean time, and is therefore called positive when mean noon precedes true noon, and negative when *vice versâ*. From equinox to solstice the mean sun is in advance of the true sun (when apparent noon precedes mean noon, and the equation of time is therefore subtractive), and behind it from solstice to equinox (when apparent noon follows mean noon, and the equation of time is therefore additive). For the equation of time proper, as given in the *Nautical Almanac*, its two parts—the difference between the position of the true and mean suns (in their respective paths along the ecliptic and equator), and the change in position of the apse-line with respect to the first point of Aries—are combined into one sum, and when this is done the equation of time is found to vanish four times a year, on or about the 14th April, 14th June, 31st August and 24th December, while it is at its intermediate maximums at four other times, on or about the 14th May, 26th July, 3rd November and 10th February

[§] In other words, by this quantity the First Point of Aries anticipates the sun in a mean diurnal revolution. The effect of the earth's daily movement upon the sun is precisely similar to its effect upon the stars; that is, the sun appears to rise and set every day; but in consequence of the earth's yearly motion round it, it appears to revolve round the earth more slowly than the stars; and it is to this that is due the difference between star-time and sun-time, or, in other words, between the lengths of the sidereal and solar days.

sidereal hours = 23*h*. 56*m*. 04·09*s*. of mean time; and the acceleration for 24 hours of sidereal time = 3*m*. 53·56*s*., or 24 hours of mean solar time = 24*h*. 03*m*. 56·56*s*. of sidereal time. Tables of Acceleration and Retardation are to be found in (Tables XXXVI (*b*) and XXXVII (*b*) of the "Calculating Tables," 5th edition).

49. SIGNS OF THE ZODIAC. The ecliptic is divided into twelve equal parts or divisions of 30° each, which are called the *signs of the Zodiac*.* They are named in the order of the sun's course; Aries (♈), Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces.† The vernal equinox was once in the sign Aries, but by its retrograde motion in the ecliptic has moved into Pisces. It still, however, retains the name of the first point of Aries.‡

50. SEMI-DIAMETER OF SUN AND MOON. The *semi-diameter* of the sun or moon, is the angle at the centre of the earth subtended by the sun's or moon's semi-diameter, as the case may be. In observing the sun or moon with a theodolite when the elevation of only one of the limbs is read, a correction for semi-diameter has to be subtracted from or added to the observed altitude, according as the upper or lower limb is observed, for the true altitude of the centre. This correction is given in Tables II and III of the 'Nautical Almanac' for every day in the year.

51. FIXED STARS. The stars, though appearing to rotate in a body about the axis of the heavens, do not appear to move among themselves. This is true of the vast bulk of the heavenly bodies; so that if the position of one of them be noted on any night with respect to certain others, it will be found on all subsequent nights to occupy precisely the same position among them. The stars to which this applies are called, to distinguish them from the few exceptions§ to the rule, *fixed stars*.‖

OBSERVATIONS.

* The *Zodiac* is a zone of the heavens extending 8° each side of the ecliptic. The sun, the moon, and all the principal planets, have their motions within the limits of the zodiac.

† *See* Art. 79 *sequiter*.

‡ *See* also footnote to Art. 79.

§ *Variable stars* are those that periodically change in brightness, that is, from one magnitude to another. The stars Algol (β Persei) and Mira (ο Ceti) belong to this class. *Temporary stars* is the name that has been given to such stars as have appeared, from time to time, shining with great brilliancy, and then entirely vanished. Temporary stars, or *Novæ*, are believed to have been first discovered by Tycho Brahe in 1572. *Double stars* are those which, though appearing single to the naked eye, when viewed through a telescope are found to revolve about each other, or rather about their common centre of gravity. Similarly there are *triple, quadruple* and *multiple* stars, according to the number comprised in the system. ζ Herculis and γ Virginis are double stars. Some double stars, such as α Centauri, revolve round their common centre of gravity: these are termed *physically* double, or *binary stars*, to distinguish them from other double stars in which no periodic change of position has been discovered. Besides the binary stars, there are some triple stars, as ξ Cancri, which are proved to be physically connected, while there are believed also to be quadruple and quintuple stars, ε Lyræ and θ Orionis, similarly associated. So also in regard to various clusters of stars, such as the Pleiades, it is supposed that most of the components are in motion in what may be regarded as an *astral system*, consisting of a countless number of suns, each revolving in an orbit about the common centre of gravity. Of the binaries of the regular, short-period variables, of which there are about 15 known, the star remains at its maximum brightness for the greater part of the cycle, its change of lustre usually occupies only a few hours and is supposed to be due to the companion, a dark star, intervening between the earth and the bright star and thus eclipsing a portion of the light of the latter. Algol belongs to this type; it generally appears as a star of the 2nd magnitude, but at the expiration of every 2 days 14 hours an interval of about 7 hours occurs, during the first half of which it gradually decreases to its minimum of the 4th magnitude, when it again during the second half of the interval gradually increases until it recovers its maximum brightness. Thus, the minima (mag 3·7) of Algol in December 1906 occurred at $11^{h.}\ 17^{m.}$ P M on the 5th, at $8^{h.}\ 05^{m.}$ P.M. on the 8th, and at $4^{h.}\ 55^{m.}$ P M on the 11th, G.M.T; the maxima (mag. 2 2) would thus have been reached at $9^{h.}\ 41{\cdot}5^{m.}$ A.M on the 7th, and at $6^{h.}\ 30{\cdot}5^{m.}$ A. M on the 10th; the complete cycle of variability, or difference between successive minima or maxima periods, is $2^{d.}\ 20^{h.}\ 49^{m.}$. Suppose the maximum splendour to have been attained at noon $0^{h.}\ 00^{m.}\ 00^{s.}$ on the 1st day of the month, then the changes would for the succeeding intervals be: to $2^{d.}\ 14^{h.}\ 00^{m.}$ = 2nd magnitude; to $2^{d.}\ 17^{h.}\ 24{\cdot}5^{m.}$ = decrease to 4th magnitude; to $2^{d.}\ 20^{h.}\ 49^{m.}$ = increase to 2nd magnitude; to $5^{d.}\ 10^{h.}\ 49^{m.}$ = 2nd magnitude; etc. The periodic phenomena of Algol was discovered by Goodricke in 1782, and the appearances explained by the supposition that some opaque body revolved round the primary and intercepted a large portion of its light by interposition between the earth and the star.

Recent investigations have shown that most of the variable stars are uncondensed meteor-swarms although the swarms are not so sparse as in nebulæ. In the variables of regular period the variations are due to cometic swarms of meteorites moving around a central swarm, the maximum occurring at periastron; that is, when the revolving swarm is nearest to the centre of the uncondensed mass. In that case, the velocity is greater, and the collisions more violent, so that there is more light given out. Recent researches, also, have led to the suggestion that stars, and all other self-lumious bodies in the universe, consist of swarms of meteorites in various stages of condensaton. The luminosity is due to the heat produced by collisions. Some of the meteor-swarms are sparse, and consequently the number of collisions is small, and very little light is given out. As the swarm contracts the number of collislons increase, and more light is produced. After further contraction, the temperature of the swarm is so high that the meteorites become a mass of vapour, similar to that composing the sun. The highest stage of temperature is represented by such stars as α Lyræ (Vega). The colour of a celestial body depends upon the stage of condensation of the meteor swarm of which it is composed. In the sparse swarms the colour is almost entirely dependent upon radiation.

[For footnote to parallel (‖), see page 199.]

52. CONSTELLATIONS. The most conspicuous of the fixed stars are divided into groups, called *constellations*.*

53. MAGNITUDE. The stars are classified according to their brilliancy or *magnitude*. The stars visible with the naked eye are divided into six classes, according to their magnitude, the most brilliant being those of the first magnitude.†

54. DIRECT AND RETROGRADE MOTIONS. Motions of heavenly bodies from west to east are called *direct motions*, those from east to west being called *retrograde*. Thus, the moon's motion among the stars is direct.‡

OBSERVATIONS.

In closer swarms, absorption affects the colour; light from the meteorites is absorbed by the vapours surrounding them The sun at setting seems blood-red, in consequence of the absorption of the earth's atmosphere; if the absorption were in his own atmosphere, he would be blood-red at noonday The white stars are the hottest, the yellow ones are cooler, and the red ones are of the lowest temperature. (*Lockyer.*) The following are examples of coloured stars: white star = α Canis majoris; red star = α Tauri; blue star = α Aurigæ; green star = α Lyræ; yellow star = α Boötis [The foregoing subject, however, belongs more properly to the region of Physical Astronomy, whereas this chapter relates chiefly to Astronomy considered as a science of observation, that is, Practical Astronomy.]

The changes in the positions of the stars due to aberration and notation are merely apparent movements; and their exact amounts can be readily calculated for every star. The effects of precession can be determined with equal facility. But there are certain stars which have a really independent motion in the heavens, and to distinguish this from merely apparent displacements, it is termed the ***proper motion*** of the stars, while the solar system itself travels through space. These changes are, however, slight, though in some the proper motion is great; as for example, 61 Cygni, which is moving at the rate of over 5 seconds annually and Argelander's star (Groombridge, 1830), which travels at the rate of 7 seconds in a year.

‖ These are readily distinguished from the Planets which are constantly changing in position with respect to the stars, the latter being practically fixed in the same relative situation. The fixed stars, like the sun, shine by their own light, being hot bodies; their twinkling, or scintillation, will generally distinguish them from the planets, which, being cold bodies, shine with a steady light by reflection from the sun.

"What though no real voice nor sound
Amid their radiant orbs be found?
In Reason's ear they all rejoice,
And utter forth a glorious voice,
For ever singing as they shine,
The Hand that made us is Divine."

* These were at a very early date given the names of beasts, birds, fishes, classical heroes, etc., and although the constellations bear but a very faint resemblance to the figures they are fancifully supposed to represent, the arrangement was useful in naming the stars and in assisting to distinguish one star from another; and for this reason and to avoid confusion that may result in any change from an old and well-known system, and also for the sake of convenience, this grouping of the stars into constellations is still adhered to. For the names of the constellations (*see* Art 79 *sequitur*). L. *Constellatio*, a group of stars, from *con*, together, and *stella*, a star; Fr. *Constellation*.

† Stars are divided into classes or magnitudes, according to their degrees of apparent *brightness*, and not with reference to their real dimensions. There are about 20 which belong to the first magnitude. The next in order of brightness and of which there are about 60, belong to the second magnitude. Of the third magnitude there are about 200. Then come the fourth, fifth, and sixth magnitudes, beyond which the aid of the telescope is necessary to identify them. The system of classification extends to telescopic stars down to the 16th magnitude. The *light ratio* for stars is about 2·5 between successive magnitudes; thus, an average first magnitude star is two and a half times as bright as a second magnitude. Intermediate stages of brightness are also recognized: thus, 1·2 would represent an intermediate magnitude between the first and second magnitudes, but nearer to the former than to the latter, whereas 2·1 would indicate the reverse of this state. An average first magnitude star is about 100 times as bright as a sixth magnitude star. Some of the stars are brighter than the first magnitude, and, since the numbers expressing magnitude increase with diminishing brightness, these are usually indicated by a *minus* sign; thus, the magnitude of α Canis majoris (Sirius), the brightest star in the heavens, = —1·4. α Aquilæ (Altair) may be taken as a normal first magnitude star, and α Ursæ majoris, γ Lyræ (Vega), and γ Herculis, respectively, as average 2nd, 3rd and 4th magnitude stars. The twenty brightest stars, or those of the first magnitude, are as follows, in the order of brightness, namely, —α Canis majoris (Sirius), α Argûs (Canopus), α Centauri, α Boötis (Arcturus), β Orionis (Rigel), α Aurigæ (Capella), α Lyræ (Vega), α Canis minoris (Procyon), α Orionis (Betelguese), α Eridani (Archernar), α Tauri (Aldebaran), β Centauri, α Crucis, α Scorpii (Antares), α Aquilæ (Altair), α Virginis (Spica), α Pisces Australis (Fomalhaut), β Crucis, β Gemmorum (Pollux), and α Leonis (Regulus). These stars are nearly equally divided between the northern and southern celestial hemispheres. The number of stars of all magnitudes visible to the naked eye is about 6,000; so that the greatest number visible in the sphere of observation at any one time—as we can only see one-half of the sky at once—is 3,000. The photometric magnitudes of the Sun, Moon (full) and Venus are —26·5, —12·0 and —3·0, respectively. In star charts the magnitudes are given according to conventional shapes. The actual size represented varies slightly, since the stars are not, as a rule, of any *exact* magnitude. In recent times, though, an effort has been made to establish the *magnitude* of stars in our Stellar Universe by their *distance*, *i.e.*, according to stellar parallaxes (converted from seconds of arc to light-years) reduced to a scale of 1 inch = 1 mile (which represents the distance that light travels in one year) instead of by their *brightness* as viewed from the earth; but such a system is foredoomed to failure since the visible appearance will always appeal more strongly to the spectator on the terrestrial globe than any theoretical value, and it is not easy to overthrow a system that has the sanction of ages. With the light-year as a unit, *i.e.*, the distance over which light will travel in one year and the parallactic displacements (as viewed from opposite ends of a diameter of the earth's orbit) of certain stars known, their distances from the earth in light-years is readily ascertainable; thus, the parallaxes of Sirius and Polaris given at 0·37″ and 0·07″, their respective distances in light-years, are 8·8 and 46·5.

‡ *Direct* and (its opposite) *retrograde* motion is an expression which applies to the planets. In the direct motion, the planet, as seen from the earth, appears to move from west to east among the stars; at a stationary point, it appears motionless; in the retrograde motion, it seems to go from east to west, among the stars. These apparent vagaries result from the fact that we see the planets from the earth, which, like them, moves around the Sun. As seen from the sun, they describe ellipses round it as a focus, and ever move in the same

55. SUMMARY OF ASTRONOMICAL TERMS. (Figs. 478, 510—515, 517—525.) In the traingle PZS (*see* Fig. 478) are represented the following six astronomical magnitudes:—(i) *Sides*: PZ = co-latitude of place of observation, PS = polar distance of star, ZS = zenith distance of star; and (ii) *Angles*: ZPS = hour angle of star, PZS = azimuth of star *reckoned from the north point*, PSZ = *parallactic angle* of the star, formerly known as the "angle of position," and is the angle formed at the star by arcs passing through the pole and the zenith respectively, that is, between its vertical and declination circles. When any three of these magnitudes are given, the others may be computed. C is the place of the spectator, Z his *zenith*, and N his *nadir*. PCP′ is the axis about which the diurnal motion is apparently performed. P is the elevated pole, and P′ the depressed pole of the heavens. HMO, a great circle of the sphere whose poles are Z and N, is his *celestial horizon* (an infinite extension of the plane of the sensible horizon). PO is the *altitude of the pole*. OPZEH is his *meridian*,* and ELQ, a great circle perpendicular to PP′, is the *celestial equator* or *equinoctial* (an infinite extension of the plane of the terrestrial equator). ZHN and ZMN are *vertical circles* of the sphere, on which the altitudes of celestial objects are measured passing through Z and N and perpendicular to the horizon. Z*e*N is the *prime vertical* (not drawn in the figure), which is perpendicular to the meridian and passes through the east and west points of the horizon. If S be the position of a star, and PSP′ be a great circle passing through it, LS will be the *declination* and PS the *polar distance* of the star, and BSD will be the *diurnal circle* it will appear to describe about the pole. O and H are the north and south points, *e* and *w* the east and west points of the horizon. If the vertical circle ZSMN be drawn, OM will be the *azimuth* of the star, reckoned from the *north point*,† MS its altitude, and ZS its zenith distance. PSLP′ is an hour circle, which is a great circle passing through the poles, perpendicular to the equinoctial. SPZ is the *hour angle* of S, and is the angle at the pole between the hour circle passing through the star and the observer's celestial meridian. ♈ EL is the *right ascension* of S, and is the arc of the celestial equator, or equinoctial, intercepted between the first point of Aries (called also the vernal equinox, or equinoctial colure) and the point in the same circle to which it is referred by the hour circle (or *circle of declination*) passing through it.‡ The position, S, of a star at any instant may be determined by either of the three following systems of co-ordinates, namely—(i) SM, the altitude (or SZ, the zenith distance), and HM, the azimuth; (ii) SL, the declination (or SP, the north polar distance), and ♈ EL, the right ascension; or (iii) SP, the north polar distance, and SPZ, the hour angle.

§ NOTE.—The terms explained in this and the preceding articles are those that occur most frequently in practical Astronomy, but there are others which, though not so often used, are occasionally referred to, and an explanation of some of these therefore is here given. The *Rising* of a celestial object is when its centre appears in the *eastern* part of the horizon; and its *setting* is when its centre disappears in the *western* portion of the horizon. *Diurnal Arcs* are those parts of the parallels of declination which are apparently described between the times of the rising and setting of the object; *Nocturnal Arcs* are the parts of those parallels described from the time of setting to the time of rising. *Semi-diurnal* and *Semi-nocturnal Arcs*, or the halves of the preceding arcs, are those parts of the parallels intercepted between the meridian and the horizon. The corresponding part of the Equator answering to the Semi-diurnal Arc, gives the time between noon and the rising or setting; and the equatorial part answering to the Semi-nocturnal Arc shows the time between midnight and the time of setting or rising. The *Ascensional Difference* is an arc of the equinoctial intercepted between the horizon and the hour-circle of the object at rising and setting; with the sun, it is the time he rises before, or sets after six o'clock. The path described by a

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way round. When the R. A. of a planet is increasing from W. to E., *i.e.*, from 0*h*. to 24*h*., the motion is direct or eastward; when the R. A. is decreasing from E. to W., *i.e.*, from 24*h*. to 0*h*., the motion is retrograde or westward. (*See* Fig. 522.)

* The straight line HCO, in which the plane of the meridian meets the plane of the celestial horizon, is called the *meridian line*; and the horizontal line *eW*, which is perpendicular to the north and south line OH, is called the *east* and *west* line. The intersections of the celestial equator and the celestial horizon determine the east and west points.

† It is best to reckon the azimuth *westwards* from the point most remote from the elevated pole

‡ If the star were on the meridian at E, then its right ascension would be denoted by ♈ E. But ♈ E represents the hour angle of the first point of Aries; and therefore the hour angle of the first point of Aries is equal to the right ascension of a celestial object when on the meridian.

§ Partly from Rosser's Book of the Stars.

planet * (or comet) round the sun is called the *orbit*; and this orbit is more or less in the form of an ellipse, with the sun in one of the foci, not in the centre. That point in the planet's orbit which is furthest from the sun, is the *Aphelion*; the point nearest the sun is the *Perihelion* (*see* Fig. 523). With regard to the Earth, the terms *Apogee* and *Perigee* are used; thus, Apogee is the point in the moon's orbit furthest from the earth, and also the point in the earth's orbit furthest from the sun; the point in the moon's orbit nearest the earth, and the point in the earth's orbit nearest the sun, is the Perigee.† The imaginary

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OBSERVATIONS.

* The name *Planet* is derived from the Greek signifying a *wanderer*, and was applied by the ancients to these bodies because their apparent movements were complicated and irregular. [Greek: πλανήτης (planētēs), a wanderer, from πλανάω (planaō), to wander, and πλάνης (planēs), a planet; French: planète; Latin: planeta]. The planets Venus, Mars, Jupiter, and Saturn are given in the *Nautical Almanac* in connection with lunar distances for determining the Greenwich date and thence the terrestrial longitude. The elements of all the planets—such as R.A., declination, time of meridian passage, semi-diameter, horizontal parallax, etc.—are also given in the *Nautical Almanac*, and when an observer sees a planet, in any particular constellation, he can always determine which it is by referring to the Declination, R.A., and time of meridian passage of the several planets, and comparing them with the similar elements of the constellations in which it is moving. All planets revolve round the sun in elliptical orbits inclined at different angles to the ecliptic and also rotate on their axis, from west to east, *i.e.*, in the order of the signs of the Zodiac. The satellites revolve about their primary in the same direction, *i.e.*, direct, except of Uranus and Neptune whose motion is retrograde. Mercury and Venus are *inferior* planets; Mars, Jupiter, Saturn, Uranus and Neptune are *superior* planets (*see* Fig. 519). The former are in *inferior* conjunction when between the Earth and the Sun and in *superior* conjunction when away from the Sun (*i.e.*, with the Sun between the Earth and Planet); the latter are in *conjunction* when the Sun is between the Earth and Planet, and in *opposition* when the Earth is between the Sun and Planet (*see* Fig. 520). A superior (or outer) planet is best seen and observed about the time of opposition; it rises and sets nearly at the same time as the Sun at conjunction, and is thus invisible then. An inferior (or inner) planet is for a time invisible when at inferior conjunction, owing to the planet's enlightened hemisphere being turned away from the Earth, and is for some time invisible when at superior conjunction, owing to its proximity to the Sun's rays. A planet souths about noon when in conjunction and about midnight when in opposition. The planetary system consists of 8 major planets; of these the interior (or near) planets, which are of moderate size, are Mercury, Venus, the Earth and Mars, and the exterior (or distant) planets, which are of great magnitude, are Jupiter, Saturn, Uranus and Neptune. The minor planets, of which about 500 have been discovered, revolve in orbits between those of Mars and Jupiter. There are also 22 known satellites of which one (the Moon) accompanies the Earth, while Mars is attended by 2, Jupiter by 5 (a 6th satellite has recently been discovered at the Lick observatory and there are indications of the existence of even a 7th member). Saturn by 9. Uranus by 4 and Neptune by 1. Mercury when favourably situated shines as a brilliant star, at greatest elongations when the ecliptic makes a large angle with the horizon; he is best seen before sunrise at western elongation and after sunset at eastern elongation. Venus is the most brilliant of all the planets. Mars is red in colour and a bright planet; at quadrature he becomes gibbous. Jupiter is the biggest of all the planets, and his equatorial and polar diameters are large. Saturn is the second largest of the planets. Uranus and Neptune are the most distant planets. The Earth is nearest to the Sun, or at perihelion, early in January, farthest from the Sun, or at aphelion, early in July, and at its mean distance early in April and October. The inferior planets have greatest elongation E. or W. of the Sun, when at these points; *i.e.*, when the angular distance from the Sun as seen from the Earth is greatest, and they then set at the longest interval after or rise at the longest interval before the Sun. The greatest elongations of Jupiter vary from 16° to 29°, and of Venus from 45° to 48° The interior planets Mercury and Venus, which revolve between the Earth and the Sun, are seen only as morning or evening stars, and can never be seen at midnight. A *sidereal period* consists of the revolution of a planet round the Sun, as referred to the stars; a *synodical period* refers to the interval between two successive positions of a planet with respect to the Earth and the Sun, such as oppositions. The sidereal revolutions and the synodical periods of the greater planets are respectively: Mercury, 88 and 116 days; Venus, 225 days and 1 year 218 days; the Earth, 365¼ and *nil* days; Mars, 1 year 322 days and 2 years 50 days; Jupiter, 11 years 315 days and 1 year 34 days; Saturn, 29 years 167 days and 1 year 13 days; Uranus 84 years 7 days and 1 year 5 days; and Neptune, 164 years 280 days and 1 year 3 days. The complete cycle of a planet's phases, or its synodical revolution, may be approximately found with the sidereal period known, by a simple formula—thus, for an inferior planet as Mercury, which moves faster than the earth, the synodical period is 116 days (from $88 + \frac{88}{365} n = n$); and for a superior planet as Mars, which moves slower than the earth, it is 780 days (from $2 \times 365 \times \frac{365}{687} n - 687 + n$). *Inferior Planets* (*see* Fig. 521): at B the planet is for a time invisible, owing to its enlightened hemisphere being turned away from the Earth; at W′ it is seen as a half-moon; between B and W′ it appears as a crescent most brilliant); between W′ and A it assumes a gibbous form; at A the whole enlightened side is turned towards the Earth, but the planet is for some time invisible owing to its proximity to the Sun's rays; planets appear of largest diameter when crescents, just before or after inferior conjunction, and smallest when near superior conjunction if they are sufficiently removed to be visible; Venus is at greatest brilliancy when about 35 days *before* greatest elongation W, or *after* greatest elongation E.; Mercury should be looked for within a few days of its most favourable positions (elongations); the motions of Mercury and Venus are *direct* at superior conjunction and *retrograde* at inferior conjunction, they are *stationary* at the intermediate points—for Venus, 40 to 50 days after greatest elongation E. and before greatest elongation W. *Superior Planets* (*see* Fig. 522): at P the planet is in opposition with the Earth at E (most brilliant), and in conjunction when the Earth is at F; the motions of the planets are *retrograde* when in opposition and *direct* when in conjunction, they are *stationary* at dates intermediate between opposition and conjunction; at conjunction the planet will rise and set nearly at the same time as the Sun, and will be due S. along with the Sun about noon; at opposition it will be due S. about midnight. Superior planets are best seen and observed to about opposition, and inferior planets at elongation E. or W. Kepler's 3 planetary laws (about 1620 A.D.) consist of (i) the elliptical form of planetary orbits, (ii) the sweep of equal area, and (iii) the square of the period to the cubes of the distances. The law of universal gravitation built up by Sir Isaac Newton (about 1680 A.D.) determines that every body in the Universe attracts every other body in proportion to its mass, and the inverse ratio of the square of its distance.

† When the sun is in perigee, its distance from the earth is least, and consequently its greatest diameter greatest; and then also it attains its greatest angular velocity. The reverse of these results is the case of

According to the investigations of Sir W. Herschel (about 1800 A.D.), whose conclusions were confirmed by later observers, the sun is moving towards a point in the constellation of Hercules, in the northern sky, which point is known as the "apex" of the sun's way. It has been supposed that η Tauri (Alcyone), the brightest star in the cluster of the Pleiades, is the central sun round which all other suns revolve, but Sir J. Herschel (about 1840 A.D.) affirmed that it was scarcely probable that any ***general*** circulation can take place in this position, lying so far out of the plane of the "Galactic Circle" (Milky Way).

line joining the Aphelion and Perihelion points of the orbit is termed the *Line of Apsides*; as is also the line joining the Apogee and Perigee. The points in the orbit of a planet which intersect the ecliptic are the *Nodes*. The *Inclination of the orbit* is the angle which the path of a planet makes with the plane of the Ecliptic. When two celestial bodies have the same longitude, or right ascension, they are in *Conjunction*; when they have a difference of longitude of 180°, and are therefore in diametrically opposite parts of the heavens, they are in *Opposition*; when the difference of longitude is 90°, they are in *Quadrature*. The moon is in conjunction with the sun at the time of *new* moon; in opposition at the time of *full* moon. For illustrations of the *Phases* of the Moon and of Planets, see Figs. 524 and 525, respectively. With regard to Mercury and Venus, they are said to be in *Inferior conjunction* when passing between the sun and the earth; and in *Superior conjunction* when on the opposite of the sun to the earth. *Occultation* is the eclipsing of one celestial body by another. The occultations of the fixed stars and planets by the moon, and the occultations of Jupiter's satellites or moons, are recorded in the 'Nautical Almanac,' and the elements given for the computation of terrestrial longitude therefrom In the occultations of planets and stars by the moon, the disappearances always take place at the limb of the moon which is presented in the direction of its motion. From new to full the moon moves with its dark edge foremost, and from full to new with its illuminated edge foremost. During the former interval the object occulted disappears at the dark edge and reappears at the illuminated edge; whilst during the second part of every lunation the contrary takes place. Occultations of stars by the moon are made use of for ascertaining the longitude of places on the earth's surface. Unlike the inferior planets, Mercury and Venus, which exhibit phases (*see* Fig. 525), the superior planets, Mars, Jupiter, Saturn, Uranus, Neptune, etc., owing to their great distance have no sensible phases (*see* Fig. 511 *a*). Of the distant planets, Mars and Jupiter appear like full moons at opposition and conjunction, as at P and P′, the same hemisphere being turned to the earth and sun; in all other positions they appear slightly gibbous, but in a very small degree only (*see* Fig. 513 *a*). Seen through a telescope, Saturn's rings present the appearances, at the different positions, as represented in Fig. 523 *a*. In this planet's motion in its orbit round the sun, the plane of the rings is carried parallel to itself, so that during a revolution it undergoes, changes of position analagous to those which the earth's equator exhibits. At intervals of 15 years, that is twice in every revolution, the plane of the rings passes through the sun, and the ring then appears as a straight line; *e.g.*, 1862 and 1877, 1891 and 1907. The *aberration* of the fixed* stars is an alteration of their mean positions, caused by the earth's motion in its orbit, whereby each star appears to describe in the heavens a small ellipse, having for its centre the point which the star would occupy if the earth were at rest. *Nutation* is the oscillatory motion of the earth's axis, due chiefly to the action of the moon upon the spheroidal figure of the earth, and every object in the heavens, whether fixed or in motion, is affected by it to a certain extent. *Moon Culminating Stars* are those fixed stars nearly on the same parallel of declination with, and not differing greatly in right ascension from, the moon and are consequently selected for comparison with that satellite in the determination of terrestrial longitude In order to more readily indicate the apparent yearly motion of the sun, the celestial sphere is sometimes mapped with reference to the ecliptic as distinguished from the equinoctial, by measurements of co-ordinates of celestial latitude and longitude in the

OBSERVATIONS.

course when the sun is in apogee. The apparent diameter of the sun at different times of the year varies from 31′ 31·0″ on the 1st July to 32′ 52·0″ on the 31st December, and as the angles subtended by the sun are small, they are very nearly inversely proportional to their distances from the earth. The daily value in longitude of the sun's angular motion in his orbit varies from 0° 57′ 11·5″ to 1° 01′ 9·9″, these minimum and maximum values occurring at the same epochs of 1st July and 31st December. The moon's diameter, as seen from the earth, subtends an angle which varies from 28′ 48″ to 33′ 32″, the mean values of the sun's and moon's diameters being thus very nearly the same; so that, although so much smaller than the sun, the moon may, on account of her proximity to the earth, sometimes subtend a larger angle. In 1906, the sun was in perigee on the 3rd January at 4^h (G. M. T.) and in apogee on the 2nd July at 19h (G. M. T.). Owing to the sun being in one of the foci of the ellipse formed by the earth's orbit, the four quarters of the year divided by the sun's entry into the signs Aries, Cancer, Libra and Capricornus are unequal, the different lengths, commencing with each season, being, for 1906-07, for the vernal equinox (spring) = 92 days 20 hours (March 21 days 1 hour — June 21 days 21 hours), summer solstice (summer) = 93 days 14 hours (June 21 days 21 hours—September 23 days 11 hours), autumnal equinox (autumn) = 89 days 19 hours (September 23 days 11 hours — December 22 days 6 hours), and winter solstice (winter) = 89 days 1 hour (December 22 days 6 hours—March 21 days 7 hours); the summer half of the year was thus 7 days 50 hours longer than the winter half. Owing to her easterly motion amongst the stars, the moon rises later daily by an average of about 50^m though the retardation may vary between the limits of about ½ hour to 1¼ hours, owing to her irregular motion in her orbit which is least when she is in apogee and greatest when in perigee; at about full moon nearest 23rd September (known as the Harvest Moon) the daily retardation is less and nearest 21st March greater than at other periods of these lunations. The full moon next following the harvest moon is known as the Hunter's moon. The phenomena known as "the old moon in the new moon's arms" occurs when the moon is only a few days old and appears in the form of a thin cresent; this is due to that part of the moon which is not directly lit by the sun being faintly seen by the reflected "earth-light."

* Light travels with a velocity of 186,000 miles in 1^s and thus takes $8^m\ 17^s$ to reach the earth from the sun during which interval the earth has moved about 20^s forward in its orbit. The effect of aberration is therefore to make a celestial body, as a star, to appear somewhat in advance of its true place in the direction of the earth's motion at the instant of observation. The phenomena of aberration and mutation were discovered by Bradley (about 1750 A.D.).

former case as against co-ordinates of declination and right ascension in the latter.[*] ***Celestial latitude*** is the angular distance from the ecliptic to the *poles of the heavens*, which are 90° from it north or south; and *celestial longitude* is the angular distance, reckoned on the plane of the ecliptic, from the position occupied by the sun at the vernal equinox, reckoning from left to right up to 360°. This latitude and longitude may be either *heliocentric*[†] or *geocentric*, that is, reckoned from the centre of the sun and earth respectively. To *South* signifies to cross the meridian of a place. The *time of southing*[‡] is the time at which the heavenly bodies pass the meridian, and is so called because they are then due south; the southing will also be the time when they attain their greatest altitude above the horizon. The expression is applied to the Sun, the Moon, the planets and the stars. Thus, "souths" or "southing" refers to the time of culmination. The terms *morning* and *evening stars* are applied to planets that rise shortly before or set shortly after the Sun, respectively (*see* Fig. 521). Planets may rise on the East, S. E. or N. E., and set on the West, S. W. or N. W. They are suitable for observation only when there is a difference of over an hour between their and the Sun's rising and setting. The expression *magnitude of illumination* has reference to the illuminated portion of the discs of planets, and is given in the 'Nautical Almanac' for Venus and Mars for the 15th of every month, the number furnished being the versed sines of the illuminated portions of the discs, the apparent diameters of the planets being taken as *unity*. A planet is in the *ascending* or *descending node* according as it crosses the ecliptic from S. to N. or from N. to S. The *radiant point* is that point in the sky whence meteoric showers, or shooting stars in great number, emanate. The constellations in which the radiants lie, with dates and characteristics, of the more conspicuous showers are Serpens (Quadrantids), 2-3 Jan., swift, long paths; Lyra (Lyrids), 20-22 Apl., swift; Aquarius (Aquarids), 1-6 May, swift, streaks, 28 Jul., slow, long; Perseus (Perseids), 10-12 Aug., swift, streaks; Orion (Orionids), swift, streaks; Leo (Leonids) 14-16 Nov., swift, streaks; Andromeda (Andromedes), 17-23 Nov., very slow, trains; Gemini (Geminids), 10-12 Dec., swift, short. *Comets* are celestial bodies extraneous to the earth's surface, a discovery due to Tycho Brahé (about 1500 A. D.). They are members of the solar system and have usually elongated orbits, either parabolic or elliptical, approaching very near to the sun at perihelion and receding to a very great distance from it at aphelion. They consist of a luminous head with frequently, at or near its centre, a bright star-like point called the *nucleus* which is surrounded by the *coma* and which is accompanied by a long train of light known as the *tail* stretching into space in a direction opposite to that of the sun. One or more of these parts of a comet is often wanting. Comets and meteors are intimately connected, the latter being classed as aerolites, fireballs, shooting-stars, etc. The inferior planets have their *greatest elongation* east or west of the Sun, when they are at these points, *i.e.*, when their angular distance from the Sun as seen from the Earth is greatest, and they then set at the longest interval after the sun or rise at the longest before it. *Eclipses* occur of the Sun (*see* Fig. 518) or Moon (*see* Fig. 517); of the former when the moon, being new, intervenes between the earth and sun, and of the latter when the earth comes between the moon, at full, and Sun. An eclipse does not take place unless the moon, when in conjunction or opposition, is also near one of her nodes, the points at which her orbit intersects the ecliptic. A solar *eclipse* is *total* or *annular*, according to the length of the shadow of the moon and the distance of the moon from her node, and *partial*; a *lunar eclipse* is either *total* or *partial*. The cone of deep shadow cast on the earth is termed the *umbra* and the surrounding region of partial shadow the *penumbra* from which the Sun's light is only partially intercepted. Eclipses of the Sun always begin on the western side of the Sun and finish on the eastern side, whereas lunar eclipses always commence on the eastern side of the Moon and finish on the western side. At least one eclipse of the sun must take place at each of the moon's nodes, or 2 each year; these may, however, only be partial eclipses: there may also be a partial eclipse when the moon is new when just within the ecliptic limit west of the node, and another may take place at next new moon; two partial eclipses may similarly take place when the sun is opposite the other node. There can be but one lunar eclipse at each node, or two in the year. The greatest number of eclipses possible, and these occur only at long intervals, are 5 solar and 2 lunar in a calendar year. Ordinarily there will be 2 eclipses each of the sun and the moon annually, the lunar eclipses being distant from the solar by half a lunation. Occasionally there may be eclipses of the sun and none of the moon; or, again, there may be 3 eclipses of the moon within the calendar year. The distance from her node at which the moon must be in order that the discs of the sun and moon may just touch under average conditions is 18° on either side of the node, or 36° in all. This is known as the solar ecliptic limit, and the line joining the two points at which the moon's orbit intersects the ecliptic must point within 18° of the sun in order that an eclipse of the sun may take place. The lunar ecliptic limit, or the distance from her nodes at which the moon must be just to touch the edge of

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OBSERVATIONS.

[*] The longitudes of fixed stars vary by 50·1″, the amount of the precession, but the latitudes remain constant; the right ascensions and declinations alter with the positions of the stars in the heavens.

[†] The heliocentric conception is due to Galileo (about 1610 A.D.).

[‡] The times of southing of the principal planets are usually given in calendars containing astronomical occurrences to convey some idea of their places in the heavens on particular dates; and their meridian altitudes (*i.e.*, at transit or upper culmination) are measured from the south point of the horizon.

the earth's true shadow, is about $12\frac{1}{2}^{\circ}$ on each side of the node, under average circumstances of the relative distances of the earth, moon or sun: or 25° along the ecliptic. From the relative lengths of the solar and lunar ecliptic limits, 36° and 25° respectively, it will be seen that the chance of occurrence of a solar eclipse is greater than that of a lunar eclipse. In fact, they occur in the proportion of nearly four solar to three lunar. This refers to all types of eclipses as seen from any point of the earth's surface. Lunar eclipses are, however, seen from any point of a hemisphere of the earth's surface, or rather more owing to the earth's rotation during the progress of the eclipse, which is at the time turned away from the sun and towards the moon. Solar eclipses, on the other hand, are visible only from a comparatively narrow band traced out by the motion of the shadow in the time of eclipse, which falls within the boundary of the hemisphere of the earth which is turned towards the sun ; when annular, they are seen as such from a similar band of the earth's surface ; as partial eclipses they are seen from a more extended area, but not from a complete hemisphere. It results that lunar eclipses are more frequently seen from any particular locality than solar eclipses are. In a lunar eclipse the radius of the shadow at the point where the moon is situated may be ascertained by subtracting the Sun's radius from the sum of the equatorial horizontal parallaxes of the moon and sun as given in the 'Nautical Almanac.' As the earth's atmosphere, however, tends to increase the size of the shadow, it is usual on this account to increase the above radius by about $\frac{1}{50}$th. With this radius known and also the direction and the rate of movement of the moon through the shadow, the material is furnished for predicting any particular phase of the eclipse. The magnitude of the eclipse is measured by the ratio of the amount the innermost limb of the moon is obscured to the moon's diameter. The times of the several contacts for any meridian are obtained from the times furnished in the *Nautical Almanac* for Greenwich by adding or subtracting the East or West longitude of that meridian.

The projection of a lunar eclipse may be effected by the following method (Shackleton). Total eclipse of moon, 17th October 1902 (Civil date).

From the *Nautical Almanac* we have given—

Moon's equatorial horizontal parallax ...	=	59′	13″	=	3,553″
Sun's " " " ...	=		09″	=	09″
Sun's radius	=	16′	03″	=	963″

Adding the two former, and from the sum subtracting the latter, we get 2599,″ which represents the radius of the earth's shadow at the point of the moon's orbit.*

The earth's atmosphere, however, tends to increase the size of the shadow and on this account it is usual to increase the above radius by about $\frac{1}{50}$th ; we will therefore take—

Radius of shadow—2,650″

Take a convenient scale of equal parts (say $\frac{1}{100}$ inch = 10″), and with C as centre and radius CW equal to 2·65 in., describe the circle NESW representing the earth's shadow (*see* Fig. 510, Diagram showing path of Moon through Earth's shadow).

The declination of the centre of the shadow C will be the same as that of the sun but of contrary sign, thus since the declination of the sun is south, that of the

$$\text{Shadow} = \text{N. } 8^{\circ}\ 55'\ 21'',$$

whilst the Moon's declination $= \text{N. } 9^{\circ}\ 08'\ 53''$

make $\quad CP = 13'\ 32'' = 812'',$

which is the amount the moon is north of the centre of the shadow ; also make

$$CD = 09'\ 11'' = (10'\ 06''\text{N.} - 00'\ 55''\text{ S.}) = 551'',$$

which is the hourly motion of the moon from the shadow in declination, and since the direction of this motion is northwards, the point D is taken north of the centre C ; also make CR equal to the hourly motion of the moon from the sun in right ascension, this is 32′ 14·7″, which, to be represented on the line C W must be reduced to the arc of a small circle by multiplying by the cosine of the declination, thus :—

$$32'\ 14{\cdot}7''\ (= 34'\ 34{\cdot}7'' - 02'\ 20{\cdot}0'') \times \cos.\ 9^{\circ}\ 09'$$
$$= 1{,}934{\cdot}7 \times 0{\cdot}9875$$
$$\therefore CR = 1{,}910''.$$

OBSERVATION.

* Apparent semi-diameter of the earth's shadow at the moon ; that this is the algebraic sum of the above quantities is shown as follows (*see* Fig. 511) :—

S represents the sun.
E represents the earth.
mMm^1 represents part of the moon's orbit.

Semi-diameter of earth's shadow at moon $= mEM$.

$$mEM = EmA - ECm.$$
$$= EmA - (AES - EAC).$$
$$= EmB + EAB - AES.$$

But EmB = Moon's horizontal parallax.
EAB – Sun's horizontal parallax.
AES = Sun's apparent semi-diameter.

For further illustrations *see* Diagrams showing path of Moon through the Earth's shadow on the 11th April 1903 and the 19th February 1905 (Figs 512 and 513). For the appearances of a partial eclipse of the Sun at its greatest phase (as visible at Greenwich) and of the Moon at middle of eclipse on the 30th and the 15th August 1905, respectively (*see* Figs. 514 and 515). The elements for these are given in the ***Nautical Almanacs*** for the years referred to.

Join RD; then by the parallelogram of velocities this represents the direction and hourly rate of the moon's motion.

Through P draw MT parallel to RD; it is evident then that MT represents the path of the moon through the shadow. P is the position of the moon's centre at the time of opposition = 0h. 10m. 13·1s., and since RD represents the motion in one hour, a proportional part, PVI, can be taken equal to the motion in 10m. 13·1s., then VI will be the position of the moon at six o'clock. Make the distance from VI to V, and from VI to VII, etc., equal to RD, and we have a time scale which may be sub-divided, and from which the time of any phase may be derived.

The moon may now be drawn in at any required position, such as the first contact at M, by taking the radius of the circle to be described equal to the moon's semi-diameter. In this case,

Moon's semi-diameter = 16′ 08·4″

or the points of first and last contact, M and T, may be found by making C M or C T equal to the sum of the semi-diameters of the shadow and moon.

ELEMENTS.

		H.	M.	S.
Greenwich mean time of opposition in R. A., October 16th		18	10	13·1
☾'s Declination	N.	9°	08′	52·80″
☉'s Declination	S.	8	55	20·50
☾'s Hourly motion in R.A.			34	34·70
☉'s Hourly motion in R.A.			02	20·00
☾'s Hourly motion in declination	N.		10	06·40
☉'s Hourly motion in declination	S			55·20
☾'s Equatorial horizontal parallax			59	13·20
☉'s Equatorial horizontal parallax				08·84
☾'s True semi-diameter			16	08·37
☉'s True semi-diameter			16	03·15

Magnitude of Eclipse (Moon's diamater = 1) 1·463.

		H.	M.	
First contact with the Penumbra	Oct. 16,	15	17·9	Mean Time at Greenwich.
First contact with the Shadow	,, 16,	16	16·9	
Beginning of Total Phase	,, 16,	17	18·6	
Middle of the Eclipse	,, 16,	18	03·4	
End of Total Phase	,, 16,	18	48·2	
Last contact with the Shadow	,, 16,	19	49·9	
Last contact with the Penumbra	,, 16,	20	48·9	

In the *Nautical Almanac* are given the positions (latitude and longitude) of the places at which the moon is in the zenith at the times of contacts and beginning, middle and end of the eclipse, the angles from the north point of the moon's limb towards the east at which the first and last contacts with the shadow occur, and, where required, the times at which the moon rises or sets at Greenwich on the dates of the eclipses. In the case of a solar eclipse, the path of the moon's *shadow and penumbra* upon the surface of the earth during a *total* eclipse, and of the *penumbra* during an *annular* or *partial* eclipse, likewise the *central* and limiting lines of the eclipse from given calculated positions are usually laid down in a diagram in the *Nautical Almanac*. The hours of beginning and ending of the eclipse are expressed in Greenwich Mean Time, but by means of the dotted lines in the diagram the approximate Greenwich Mean Time of beginning or ending of the eclipse may be found for any place at which the phenomenon is visible. The elements from which the projection is made are always denoted; thus, for the total eclipse of the sun, 30th August 1905 (Civil date).

ELEMENTS.

		H.	M.	S.
Greenwich mean Time of conjunction in R. A. August 30th		0	50	13·3
☉ and ☾'s Right ascension		10	32	50·52
☾'s Declination	N.	9°	44′	25·2″
☉'s Declination	N.	9	08	48·4
☾'s Hourly motion in R. A.			35	38·4
☉'s Hourly motion in R. A.			02	16·7
☾'s Hourly motion in declination	S.		10	21·5
☉'s Hourly motion in declination	S.			53·6
☾'s Equatorial horizontal parallax			60	05·02
☉'s Equatorial horizontal parallax				08·72
☾'s True semi-diameter			16	21·25
☉'s True semi-diameter			15	50·69

Begins on the Earth generally, Aug. 29th, 22h. 37·7m., Mean Time at Greenwich, in long. 76°18′ W. of Greenwich, and Lat 37°31′ N.

Central Eclipse begins generally, Aug. 29th, 23h. 41·4m., in Long. 96° 16′ W. of Greenwich, and Lat. 50°16′ N.

Central Eclipse at Noon, Aug. 30th, 0h. 50·2m., in Long. 12°23′ W. of Greenwich, and Lat. 45° 52′ N.

Central Eclipse ends generally, August 30th, 2h. 33·6m., in Longitude 54° 52′ E. of Greenwich, and Latitude 18° 41′ N.

Ends on the Earth generally, Aug. 30, 3h. 37·3m., in Long. 36°45′ E. of Greenwich, and Lat. 5°44′ N.

The particulars necessary for indicating the times, places, etc., on the earth, at which the eclipses of the sun and moon will be visible are fully stated in the *Nautical Almanac*. The elements are given for the instant of conjunction (in cases of solar eclipses) and of opposition (in cases of lunar eclipses) of the sun and moon in right ascension. The subsequent results in the case of solar eclipses are not, however, computed from these elements unchanged, but from accurate values as interpolated for the times corresponding to each result. It should be noted that in reckoning the position-angles of the points of contact from the vertex in solar eclipses the expression "towards the east" is to be taken as indicating the positive *direction* of measurement of position-angles, *i.e.*, N.E. S.W., and does not necessarily indicate that the angles are to be measured from the vertex towards the east *point* of the sun's limb.

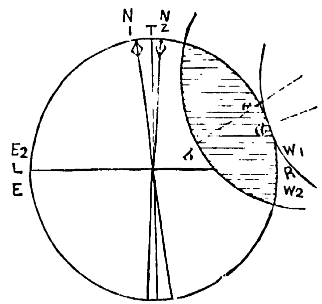
Examples for predicting the phases, times, etc., of solar and lunar eclipses are furnished below, in respect to a place in λ. 16°46′ N. and L. 96°10′ E.

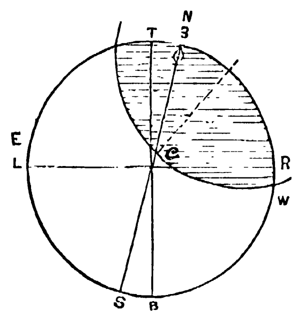
(i) Total eclipse of the sun on the 14th January 1907, visible only as a partial eclipse, at place, mag. (⊙'s diameter = 1) 0·404. Begins on the earth generally at 10-23 A.M., in L. 45° 55′ E. and λ. 27° 58′ N.; central eclipse begins generally at 11-43 A.M., in L. 42° 9′ E. and λ. 50° 29′ N.; central eclipse at noon occurs at 0-42 P.M., in L. 89° 12′ E, and λ. 38° 42′ N.; central eclipse ends generally at 1-28 P.M., in L. 131° 4′ E. and λ. 56° 45′ N.; ends on the earth generally at 2-48 P.M., in L. 131° 36′ E. and λ. 35° 12′ N. The duration of totality on the central line of the eclipse, between sunrise and sunset, in L. 42° E. and λ. 50° N. = 1m. 12s.; L. 60° E. and λ. 43° N. = 1 m. 42s.; L. 75° E. and λ. 38° N. = 2m. 12s.; L. 89° E. and λ. 39° N. = 2m. 24s.; L. 96° E and λ. 40° N. = 2m. 13s.; L. 101° E. and λ. 42° N. = 2m. 6s.; L. 117° E. and λ. 50° N. = 1m. 36s.; L. 132° E. and λ. 35° N. = 1m. 12s. The southern line of simple contact, between sunrise and sunset, extends from L. 31° E. and λ. 14° N., by L. 89° E. and λ. 2° S. to L. 147° E. and λ. 21° N. The eclipse begins, approximately, at 11-45 A.M., its greatest phase occurs at 0-55 P.M., and it ends at 2-5 P.M. Two-fifths of the sun will be obscured. The angles from N. point of first and last contacts are 296° and 47° towards the east, respectively. [The phenomenon should be viewed through the medium of a dark or smoked glass.] The eclipse begins on the western side of the sun and finishes on the eastern side. The sun rises at 6-38 A.M., and sets at 5-30 P.M.; his meridian passage occurs at 0h. 14m. 10s. P.M.; his meridian altitude (from S.) is 51° 44′ 9″; his apparent diameter is 32′34″ (in arc); the mean time of his semi-diameter passing the meridian is 1m. 10·04s; his place is in Sagittarius. Pictures (direct-image) of the various phases, including intermediate stages, at the different altitudes and bearings, are given hereunder:—

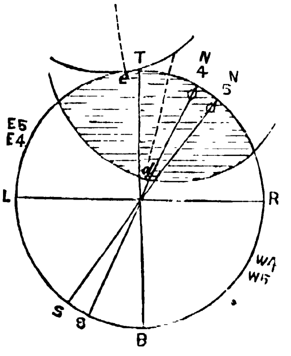
ASPECT OF SUN = SOUTH.

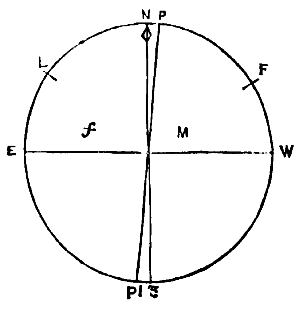
Phase.		Time.	Altitude (S).	Bearing.
(*a*) Beginning (B)	...	11h. 45m. A.M.	49°	166
(*b*) Midway (B and G)	...	0 20 P.M.	50	178
(*c*) Greatest (G)	..	0 55 ,,	50	190
(*d*) Midway (G and E)	...	1 30 ,,	47	202
(*e*) Ending (E)	...	2 05 ,,	40	218

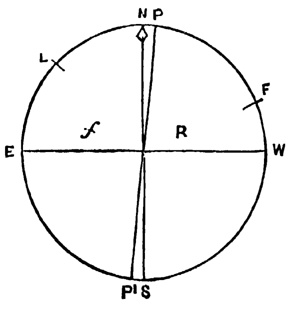
The angles, from North point, of first and last contacts, towards the East, for the places M (λ = 13° 04′ 08″ N., L = 80° 14′ 54″ E.), R (λ = 16° 46′ 00″ N., L. = 96° 10′ 00″ E.) and H (λ. = 22° 18′ 13″ N., L. = 114° 10′ 28″ E.), respectively, are exhibited in Fig. (*f*). Partial eclipses are also visible at M and H as follows:—M: mag. = 0·462, begins at 9·47 A.M., greatest phase at 11-2 A.M., ends at 0-23 P.M.; H: mag. = 0·467, begins at 1-29 P.M., greatest phase at 2-41 P.M., ends at 3-45 P.M.

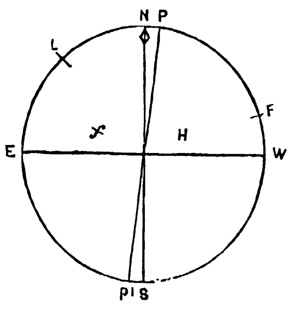








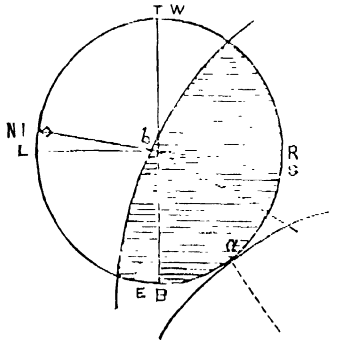


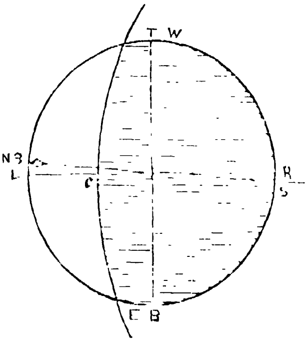


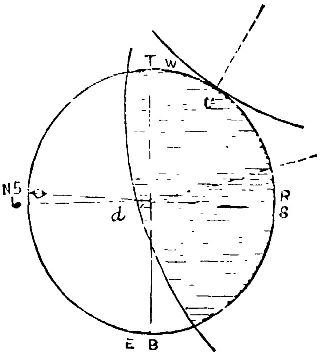
(ii) Partial eclipse of the moon on the 29th January 1907, visible at place. First contact with the penumbra at 5-16 P.M.; first contact with the shadow at 6-30 P.M.; middle of the eclipse at 8-8 P.M.; last contact with the shadow at 9-40 P.M.; last contact with the penumbra at 11-0 P.M.; magnitude of the eclipse (moon's diameter = 1) 0·715; the moon is at the zenith at these times in L. 159° W. to 117° E., and λ 19° to 18½° N.; the first contact with the shadow occurs at 137° from the N. point of the moon's limb towards the E., and the last contact at 242° towards the E. Seven-tenth of the moon will be obscured. The eclipse begins on the eastern side of the moon and finishes on the western side. The moon rises at 6-6 P.M.; her meridian passage occurs at 0h. 28m. 11s. A.M. (on 30th) ; she sets at 6-50 A.M. (on 30th); her meridian altitude (from S.) is 91° 32′ 13″; and her apparent diameter is 30′ 2″ (in arc); her place is in Cancer. Pictures (direct-image) of the various phases, including intermediate stages, at the different altitudes and bearings, are given hereunder:—

ASPECT OF MOON – – NORTH.

Phase.		Time.	Altitude (N.)	Bearing.
(*a*) First contact (F)	...	6h 30m. P.M.	10°	73°
(*b*) Midway (F and M)	..	7 22 "	20	75
(*c*) Middle (M)	...	8 08 "	31	77
(*d*) Midway (M and L)	...	8 54 "	41	79
(*e*) Last contact	...	9 40 "	52	81





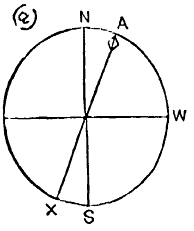


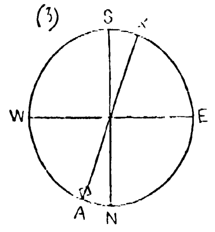
The shadow on the earth in a total solar eclipse moves from west to east at an average rate of over 1,000 miles an hour, and with an average breadth of about 90 miles, though it may be as much as 180 miles; hence the duration of totality at any point can never exceed 7½ minutes, and is usually much less The *Saros* (Chaldean) is a cycle of 18 years 11⅓ days, after which a regular recurrence of eclipses takes place in similar order, and of nearly similar type, to those which occurred in the preceding cycle. The term *transit*, as applied to a planet, is used with reference to the passage of an inferior planet across the Sun's disc. Transits of Mercury can happen only in May or November, because in these months the line drawn from the Sun's centre to the earth passes through the nodes of Mercury's orbit. They occur at intervals varying from 3½ to 13 years, *e.g.*, in May 1878 and 1891 and in November 1894 and 1907. The transits of Venus can occur only in June and December, because the earth is in these months in a line with one or other of the nodes of Venus's orbit and the Sun. They take place in pairs, between the 1st and 2nd of which there is an interval of 8 years; the interval between 1 pair and the next is 121½ years and 105½ years alternately. The last pair of transits occurred in December of 1874 and 1882; the next pair will take place in June of 2004 and 2012. As transits both of Mercury and Venus occur when these planets are at inferior conjunction, their apparent movements in the sky are then retrograde, or from east to west, consequently transits begin on the east side of the Sun's disc. The moment at which Mercury or Venus visibly enters upon the Sun's disc is called the *ingress*, and that of their departure off the disc the *egress*. When the planet first touches the limb of the sun on the outside is called the 'external contact at ingress,' and when just exactly within the disc the 'internal contact at ingress'; similarly, on the planet leaving the sun, the corresponding moments are termed the 'internal contact at egress' and the 'external contact at egress.' When the planet arrives midway between ingress and egress the 'least distance of the centres' is said to take place, because this is the moment of the nearest approach of the centres of the planet and sun respectively. The transits of Venus are made use of for ascertaining the distance of the earth from the sun. The sun rotates on its polar axis from west to east in a mean period of 25 days 9 hours, and the inclination of its equatorial plane to the ecliptic is 7° 15′. The determination of the period of rotation was first made by Galileo in 1611 A.D. Solar, or Sun, spots are intensely black patches of irregular form and various sizes which appear and disappear from time to time on the sun's surface, and appear to move from east to west (retrograde motion) owing to the rotation of the sun in the contrary direction from west to east (direct motion) as the motion of the planets round the sun, along paths parallel to the sun's equator and usually within a zone of about 35° on either side of it A large regular spot consists of the central dark *umbra*, and the surrounding grey *penumbra*. The sun-spot period is about 11 years, when the number of spots reaches its maximum; and displays of the Aurora Borealis have the same periodicity. The sun is surrounded by luminous clouds known as the *photosphere*, outside of which is a red stratum of gaseous matter called the *chromosphere*, encircling which again is the *corona* (or halo) which is seen during a total solar eclipse. The most conspicuous of the red prominences or protuberances visible above the limb of the sun when totally eclipsed are almost always associated with spots lying beneath them. The *position angle* of the sun's axis is the angle which the axis makes with the meridian passing through the sun's centre to the east (+) or west (—) of the disc; it is zero on the 5th January and 6th July, and greatest (± 26° 20′) on the 10th October and 5th April. A solar or lunar *rainbow* is an optical meteorological phenomenon of frequent occurrence consisting of a coloured arch, or arches, formed opposite the sun or moon on falling rain drops, and visible whenever the necessary conditions of a passing shower on one side and a clear and not too high sun or moon on the other occur; it is merely a reflection of a ray of light from the sun or moon to the rain shower as seen by the observer; the bow, or bows, exhibit the full spectrum of colours from red to violet, arranged in a definite order of red, orange, yellow, green, blue, indigo and violet; lunar rainbows are feebler than the solar rainbows owing to the feebleness of the moon's light owing to which there is usually an absence of colour in them.

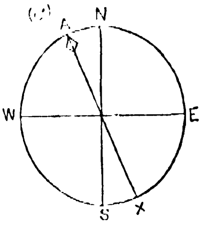
The following table gives the position-angles to the nearest degree for the first and the middle of the each month; for other days the values are readily estimated: –

Date.	Position angle.	Date.	Position angle.	Date.	Position angle.
1st January	+ 2°	1st May	— 24°	1st September	+21°
16th January	— 6	16th May	21	15th September	24
1st February	13	1st June	15	1st October	26
16th February	18	15th June	10	15th October	26
1st March	22	1st July	3	1st November	24
15th March	25	15th July	+ 4	15th November	22
1st April	26	1st August	11	1st December	16
15th April	26	16th August	17	15th December	10

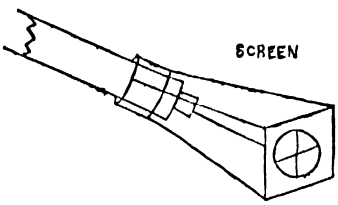
The north and south points of the sun's limb are respectively the upper and lower points of the disc cut by a meridian or great circle passing through the sun's centre and the celestial poles; the eastern limb is turned towards the east and western limb towards the west for both south and north aspects, or lie on the left and right-hand sides looking south and on the right and left hand sides looking north respectively. Viewed with the naked eye the direct image (*a*) is presented; through an inverted (astronomical) telescope the image (*b*) is inverted, the top and bottom and the left and right changing places; with a reversing eye-piece applied, or as projected on a screen towards the eye-end, the image (*c*) appears erect but the left and right change places. In the illustraticns below, AX represents the sun's axis on the west (—).







Drawings of the sun require "orientation," that is the top and bottom of the sun will have to be marked, since what may be the top at mid-day will have moved up on the left, looking south, in the forenoon and moved forward on the right in the afternoon. To do this, draw a circle the size of the disc, about 3 inches in diameter, found to be most convenient, on a sheet of paper on which rule 2 diameters at right angles to each other. Place the paper on a suitably constructed screen fitted on to the telescope a short distance from the eye-piece, at the back, and adjust the eye-piece and distance of the screen so that the image just fills the circle; this adjustment will vary for different periods of the year owing to the variation in the sun's apparent diameter. As the sun is in apparent motion relatively to the telescope, its image will also move across the screen whence the east and west line along which it moves will be obtained and the means found for orienting the drawing. Next turn the screen round until the apparent movement is in the direction of one of the diameters; then the *advancing* edge of the image will be the *western* limb, and the edge which *comes to the centre* as the telescope is slightly raised will be the *northern* one. The same result can be secured by affixing a moveable cylindrical tube of card-board or other material fitted with the sheet of paper as for the screen by adjustment at the object-end of the telescope and made to revolve in the way described for the screen till the required direction of the sun's movement is obtained.



The inclination of the course of the sun with the horizon = 90°—meridian altitude, approximately.

The *Arctic* and *Antartic Circles* are two lesser circles of the celestial sphere, parallel to the equator, 23° 27′ from the north and south poles, respectively. The *Tropics of Cancer* and *Capricorn* are two smaller circles of the celestial sphere situated on each side of the equator, at a distance of 23° 27′, and parallel to it, which the sun reaches as its greatest declination north or south. The corresponding regions on the earth enclosed by the Arctic and Antartic circles are known as the *North* and *South Frigid Zones*, those between the equator and the Tropics of Cancer and Capricorn as the *North* and *South Torrid Zones*, and those between the Arctic and Antartic Circles and the Tropics of Cancer and Capricorn as the *North* and *South Temperate Zones*, respectively (*see* Figs. 488—489*a*).

56. THE CALENDAR. A *calendar* is an arrangement of the divisions of time, as days, weeks, months, etc. The Christian calendar is reckoned from the birth of Christ, and is now calculated according to the Gregorian or New style, the change from the Julian,* or Old style having occurred in the year 1582. The Julian period is a period of 7,980 years, the year 1 of our era corresponding to the year 4,714 of the period, or the year 0 to the year 4,713 of the period. The year 1898, therefore, corresponds to the year 6,611 of the Juilan period. As a comparison between the two periods is frequently required in astronomical calculations, a table for determining the interval in days between two epochs is given in the 'Nautical Almanac.' The *Calendar month* is a period varying from 28 to 31 days, and was probably derived from the synodical revolution of the moon, the mean value of which is 29*d*. 12*h*. 44*m*. 02·87*s*., or 29·5305888 days. A *Synodical month* is the period in which the moon goes through every variety of phase, as from one conjunction to another. A *Synodic lunar month* is the interval between two consecutive full moons. A *lunation* is the interval between two successive new moons, and it is by this period that the successive phases of the moon are regulated.† An *Anomalistic month* is the period of the moon's revolution round the earth with

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OBSERVATIONS

* The Julian style was introduced by Julius Cæsar (who consulted Sosigenes, an Alexandrian astronomer, in the matter) in the year 44 B.C., and prevailed generally through the Christian world till the time of Pope Gregory XIII. The calendar of Julius Cæsar was defective in this particular, that the solar years consisted of 365 days 5 hours and 49 minutes, and not of 365 days 6 hours, as was supposed in the time of Julius Cæsar. There was a difference between the apparent year and the real year of eleven minutes. The difference at the time of Gregory XIII had amounted to ten entire days, the vernal equinox falling on the 11th instead of the 21st of March, at which period it fell correctly at the time of the Council of Nice in the year 325. To obviate this inconvenience, Gregory ordained, in 1582, that the 15th October should be counted instead of the 5th for the future, and to prevent the recurrence of this error, it was further determined that the year beginning a century should not be "Bissextile," with the exception of that beginning each fourth century The error referred to is corrected by the rule of intercalation given in the text above. The adoption of this change in the calendar was for some time resisted by States not under the authority of the See of Rome. Russians and Greeks still retain the Old style. Nearly all the nations of the Christian world now commence the year on the 1st of January, but as recently as 1752, when the correction was made by calling the day after the 3rd September the 14th September, even in England, the year did not legally and generally commence till the 25th of March. The Old style of reckoning in England thus ended on the 2nd (Wednesday) September 1752, and the New style began on the 14th (Thursday) September 1752. The computing of time by the Christian era was introduced by Dionysius, a monk. It is called the vulgar or common era, 516 after Christ. The Roman calendar, which has in great part been adopted by all nations, is stated to have been introduced by Romulus, the founder of the city, the era being the foundation of Rome. It was Dionysius who calculated that the birth of Christ took place in the year 753 R.C., but as the beginning of the Roman civil year was on the 1st January, *i.e.*, only seven days later, it was found more convenient to make the Christian year begin on the same day. It was, therefore, decided that the year 1 of the Christian era should coincide with the year 754 of the foundation of Rome. It may be remarked that there is no year 0 A.D. reckoned by chronologists—the year of the birth of Christ, that is, the year 753 R.C., being called by them the year 1 B.C.; the year preceding that 2 B.C., etc., so that to find the number of years between corresponding days of two years, the one B.C. and the other A.D., it is necessary to subtract 1 from the sum of the two.

† The time of new moon is the moment when the longitude of the moon's centre is equal to that of the sun's centre.

reference to the line of apsides, or from one apogee or perigee to another. A sidereal revolution of the moon is equal to 27·321661 days, or nearly 27⅓ days. The *Calendar* or *common year* contains 365 days. But the *Solar year*,* called also a *Tropical year*, is the period in which the sun moves from the vernal equinox to the vernal equinox again, *i.e.*, it marks the interval of time between two passages of the sun through the tropics or equinoctial points, and its average value, called a *Mean solar year*, has been found to be 365·24222 mean solar days, or 365*d.* 5*h.* 48*m.* 47·6*s.* The odd time (5*h.* 48*m.* 47·6*s.*) would soon amount to a serious error, but being nearly a quarter of a day, it is allowed to accumulate till every *fourth year*, when it amounts to a day (nearly); the year is then increased by a day, and is known as *Leap year* (or the *Bissextile year*), such a year consisting of 366 days, and the additional day being given to the month of February; that month has 29 days every fourth year. But the excess of the tropical year over 365 days is about 11¼ minutes less than a quarter of a day; hence one day every four years is too much to add, but this error (within a very small amount) is corrected by the following rule of intercalation:—For years that are not even centuries: if (i) the *year* or (ii) number of *centuries* is divisible by 4, it is bissextile (of 366 days); if not, it is an ordinary year consisting of 365 days. Thus, (i) 1898 and 1899 are ordinary years, and 1904 and 1908, bissextile years; also (ii) 1900 and 2100 are ordinary years and 2000 and 2400 bissextile years. According to this arrangement every period of 400 years consists of 97 bissextile years, or 35,502 days, and 303 ordinary years, or 110,595 days, making a total of 146,097 days, the 400th part of which is 365·2425 days, or only 0·00028 day in excess of the tropical year. The error (an excess) will therefore amount to a day in about 3,600 years. A *sidereal year* is the time which elapses between the sun's leaving a fixed star until his next return to it, and consists of 365*d.* 6*h.* 09*m.* 09·4*s.* The *Anomalistic year* denotes the time between two successive passages of the earth through its aphelion or perihelion points, and consists of 365*d.* 6*h.* 13*m.* 45·7*s.*†‡

PROBLEMS.

57. PROBLEM I.—*To convert longitude into time.*

Observation: Since in a mean solar day, or 24 mean solar hours, the earth performs a diurnal revolution of 360°, and since its motion is uniform, 1 hour of time is equal to 15° in space.

Rule: Take out separately the equivalents in time of the degrees, minutes, and seconds of the given longitude (from Table XXXIV, "Calculating Tables," 5th edition) and take their sum for the time sought.

Example: Convert 79° 15′ 26″ of longitude into time.

Equivalent of	70°			,, ,,	=	4*h.*	40*m.*	00*s.*
,, ,,	9°			,, ,,	=		36	00
,, ,,		15′		,, ,,	=		01	00
,, ,,			26″	,, ,,	=			02
				Time sought	=	5	17	02

58. PROBLEM II.—*To convert time into longitude.*

Rule: Take out separately the equivalents in longitude of the hours, minutes

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OBSERVATIONS.

* The length of the solar year is shorter than that of the sidereal one, because, owing to the precession of the equinoxes, the vernal equinox in its recession meets the sun, which therefore passes through it sooner than it would otherwise do.

† Partly from Rosser's Book of the Stars.

‡ A *Luni-solar year* is a period of time at the end of which, in the Julian calendar, the new and full moons and the eclipses of the year recur on the same days of the week and month and year as in the previous period. It consists of 532 common years, being the least common multiple of the numbers of years in the cycle (28 years) of the sun and the cycle (19 years) of the moon. *Epact* signifies the moon's age on the first day of the year or month; it is the excess of the solar year or month beyond the lunar. The *annual epact* is the excess of the solar year over the lunar year; and the *monthly epact*, or *menstrual epact*, is the excess of a calendar over a lunar month. The *lunar cycle* (Metonic) is one of 19 years which is nearly identical with 235 lunations equalling 6,939·69 days, 19 years of 365·25 days, amounting to 6,939·75 days. If the dates of the new and full moons are thus known for a period of 19 years, they can be predicted indefinitely,

and seconds of the given time (from Table XXXV, "Calculating Tables," 5th edition), and take their sum for the longitude sought.

Example: Convert 3*h.* 27*m.* 19*s.* of time into longitude.
Equivalent of 3*h.* „ „ = 45° 00′ 00″
„ „ 27*m.* „ „ = 6 45 00
„ „ 19*s.* „ „ = 04 45
Longitude sought = 51 49 45

59. PROBLEM III.—*To express civil time astronomically.*

Observations: The time adopted in the 'Nautical Almanac' is *astronomical time* reckoned from and to mean noon at Greenwich. The civil day begins at midnight and ends at the following midnight, and is divided into two periods of 12 hours each; the first 12 hours, from midnight to noon, being called A.M., and the second, from noon to midnight, being called P.M. The astronomical day is assumed to begin at noon of the corresponding civil day, and ends at the following noon, and consists of only one period of 24 hours. Astronomical time is therefore always 12 hours slower than civil time.

Rule: If the civil time be P.M., the astronomical time is the same; but if it be A.M., add 12 hours to the civil time, and deduct one day for the corresponding astronomical time.

Example: Express the following civil times astronomically:—Jan. 17, 6*h.* 21*m.* 02*s.* P.M.; and Sept. 23, 10*h.* 11*m.* 19*s.* A.M.

Jan. 17, 6*h.* 21*m.* 02*s.* P.M., civil time = Jan. 17, 6*h.* 21*m.* 02*s.*, astronomical time;
Sept. 23, 10*h.* 11*m.* 19*s.* A.M., „ = Sept. 22, 22*h.* 11*m.* 19*s.*, „ „

60. PROBLEM IV.—*To express astronomical time in civil time.*

Rule: If the hour of the astronomical time be less than 12, write P.M. after it, and it will be the required civil time; but if the astronomical time be greater than 12 hours, add 1 to the day, diminish the hours by 12, and write A.M. after it; the remainder will be the required civil time.

Example: Express the following astronomical dates in civil time:—Aug. 20, 7*h.* 09*m.* 11*s.*; and Nov. 4, 21*h.* 01*m.* 38*s.*

Aug. 20, 7*h.* 09*m.* 11*s.* astronl. time = Aug. 20, 7*h.* 09*m.* 11*s.* P.M., civil time;
Nov. 4, 21*h.* 01*m.* 38*s.* „ = Nov. 5, 9*h.* 01*m.* 38*s.* A.M. „ „

61. PROBLEM V.—*To convert a given mean solar (astronomical) time on any given day to the corresponding sidereal time.*

Observations: Problems V and VI are of constant use wherever the periods of solar observations are noted by a chronometer regulated to sidereal time, or those of stars by a chronometer showing mean time. The mean right ascension of the meridian, or the sidereal time at mean noon * (given in Table II of the 'Nautical Almanac'), is calculated for the *meridian of Greenwich*, and must, therefore, be corrected for the difference of longitudes between that place and the meridian of the observer; by the *addition* of 0·6571*s.* for each degree of longitude, if the place be to the *west* of Greenwich, but by *subtraction* if to the *east.* Thus, in 137° 31′ 30″ west longitude, the sidereal time at mean noon, January 2, must be corrected by adding 01*m.* 30·4*s.*, thus, giving 18*h.* 46*m.* 03·1*s.* + 01*m.* 30·4*s.* = 18*h.* 47*m.* 33·5*s.* for the time to be used. Conversely, the correction to be made to the mean time at sidereal noon (given as the "Transit of the first point of Aries" in Table III of the 'Nautical Almanac'), will be by the *subtraction* of 0·6551*s.* for each degree of longitude, if the place be to the *west* of Greenwich, but by *addition* if to

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OBSERVATION.

for in each subsequent 19 years the dates are reproduced in the same manner. The number which each year bears in the Cycle of Meton is called the *Golden number.* In 1900 the Golden number was 1, being the commencement of a new Cycle, and the epact 29 (*i.e.*, 0 nearly); the first year of the 19-year cycle must be a year beginning at the close of a lunation, so that the age of the moon then is 0. The *Solar Cycle* consists of a period of 28 years and is based upon the recurrence of the day of the week upon the same day of the month. In the year 1896 the Solar Cycle was 1, *i.e.*, the first year of one of these periods.

* The sidereal time at mean noon is the angular distance of the first point of Aries, or the true vernal equinox, from the meridian, at the instant of mean noon; it is therefore the right ascension of the mean sun, or the time which ought to be shown by a sidereal clock at Greenwich, when the mean-time clock indicates 0*h.* 00*m.* 00*s.*

the *east*. Thus, in 137° 31′ 30″ west longitude, the mean time at sidereal noon, January 2, must be corrected by subtracting 01*m*. 30·1*s*., thus, giving 5*h*. 13*m*. 05·5*s*.−01*m*. 30·1*s*.= 5*h*. 11*m*. 35·4*s*. for the time to be used.

Rule: To the sidereal time at the *preceding* mean noon (found in Table II of the 'Nautical Almanac'), add the sidereal interval corresponding to the given mean time (taken from Table XXXVI, "Calculating Tables," 5th edition).

Example: Convert 2*h*. 22*m*. 26·0*s*. mean (astronomical) time at Moulmein, in longitude 97° 40′ 23″ E., on the 20th January 1880, into sidereal time.

Sidereal time at the *preceding* mean noon, *namely*, Jan. 20	=	19*h*.	57*m*.	01·2*s*.
Correction for long. E. = 97·68° × 0·6571*s*. = 64·2*s*.	=	—	01	04·2
Difference	=	19	55	57·0
For 2*h*. 22*m*. 26·0*s*. solar time, the equivalent sidereal time (by Table XXXVI, "Calculating Tables," 5th edition)*	=	2	22	49·4
Sidereal time required	=	22	18	46·4

(ii) *Another Method.—Rule*: Convert the interval from the mean noon immediately preceding, from the denomination given, to that required; the result must be added to the sidereal time at the preceding mean noon for the sidereal time required. If the place of observation be to the $\frac{\text{E}}{\text{W}}$ of Greenwich, the sidereal time at mean noon must be corrected by the $\frac{\text{subtraction}}{\text{addition}}$ of 0·6571*s*. for each degree of longitude.

Example.

Mean interval from mean noon, 20th	=	2*h*.	22*m*.	26·0*s*.
Acceleration of sidereal on mean time for the interval (Table XXXVI (*b*), "Calculating Tables," 5th edition)	=	+		23·4
Sidereal interval *from* mean noon	=	2	22	49·4
Sidereal time *at* mean noon, 20th, corrected for long. E. (=97·68° × 0·6571*s*.) = 19*h*. 57*m*. 01·2*s*.—1*m*. 04·2*s*.	=	19	55	57·0
Sideral time required	=	22	18	46·4

(iii) *Another Method.—Rule*: Reduce the mean time of observation at the place to the corresponding sidereal time, by applying to the former, corrected for difference in longitude and time, the sidereal time at the preceding mean noon.

Example.

Given mean time at place, 20th	=	2*h*.	22*m*.	26·0*s*.
Diff. Long. (97° 40′ 23″) E.	=	−6	30	41·5
Corresponding Greenwich mean time, 19th ...	=	19	51	44·5
Sidereal time at Greenwich mean noon, 19th ...	=	19*h*.	53*m*.	04·6*s*.
Mean time at place	=	2	22	26·0
Acceleration (Table XXXVI (*b*), "Calculating Tables," 5th edition) for 19*h*. 51*m*. 45*s*.	=	+	03	15·8
∴ Sidereal time at place required, 20th...	=	22	18	46·4

OBSERVATION.

* Mean time.				Sidereal time.		
2*h*.	00*m*.	00*s*.	=	2*h*.	00*m*.	19·7*s*.
	22	00	=		22	03·6
		26	=			26·1
2	22	26	=	2	22	49·4

62. PROBLEM VI.—*To convert a given sidereal time on any given day to the corresponding mean solar (astronomical) time.*

Rule: To the mean time at the *preceding* sidereal noon (*i.e.*, the transit of the first point of Aries, in Table III of the 'Nautical Almanac'), add the mean interval corresponding to the given sidereal time (taken from Table XXXVII, "Calculating Tables," 5th edition).

Example: Convert 22*h*. 18*m*. 46·4*s*. sidereal time at Moulmein, in longitude 97° 40′ 23″ E., on the 20th January 1880, into mean solar (astronomical) time.

Mean time at the *preceding* sidereal noon, *namely*, Jan. 19	=	4*h*.	06*m*.	15·0*s*.
Correction for long. E. = 97·68° × 0·6551*s*. = 64·0*s*.	=	+	01	04·0
Sum	=	4	07	19·0
For 22*h*. 18*m*. 46·4*s*. sidereal time, the equivalent mean solar time (by Table XXXVII "Calculating Tables," 5th edition)*	=	22	15	07·1
Mean solar time required	=	2	22	26·1

(ii) *Another Method.—Rule*: Convert the interval from the mean noon immediately preceding, from the denomination given, to that required; the result will be the mean time required. If the place of observation be to the $\substack{\text{E}\\\text{W}}$ of Greenwich, the sidereal time at mean noon must be corrected by the $\frac{\textit{substraction}}{\textit{addition}}$ of 0·6571*s*. for each degree of longitude.

Example.

Sidereal time given	=	22*h*.	18*m*.	46·4*s*.
Sidereal time *at* mean noon, 20th, corrected for long. E. (= 97·68° × 0·6571*s*.) = 19*h*. 57*m*. 01·2*s*. — 1m. 04·2*s*.	=	19	55	57·0
Interval in sidereal time from mean noon, 20th ...	=	2	22	49·4
Retardation of mean on sidereal time for the interval (Table XXXVII (*b*), "Calculating Tables," 5th edition)	=			—23·4
Mean solar time required ...	=	2	22	26·0

(iii) *Another Method.—Rule:* Reduce the sidereal time of observation at the place to the corresponding mean time, by applying to the former, corrected for the difference of longitude and time, the sidereal time at the preceding mean noon.

Example.

Sidereal interval for place, 20th (*see* below) ...	=	2*h*.	25*m*.	41·8*s*.
Diff. long. (97° 4′ 23″) E.	=	—6	30	41·5
Corresponding sidereal interval for Greenwich, 19th ...	=	19	55	00·3
Given sidereal time at place, 20th	=	22*h*.	18*m*.	46·4*s*.
Sidereal time at Greenwich mean noon, 19th ...	=	19	53	04·6
Sidereal interval for place, 20th	=	2	25	41·8
Retardation (Table XXXVII (*b*), "Calculating Tables," 5th edition) for 19*h*. 55*m*. 00*s*.	=		—03	15·8
∴ Mean time at place required, 20th ...	=	2	22	26·0

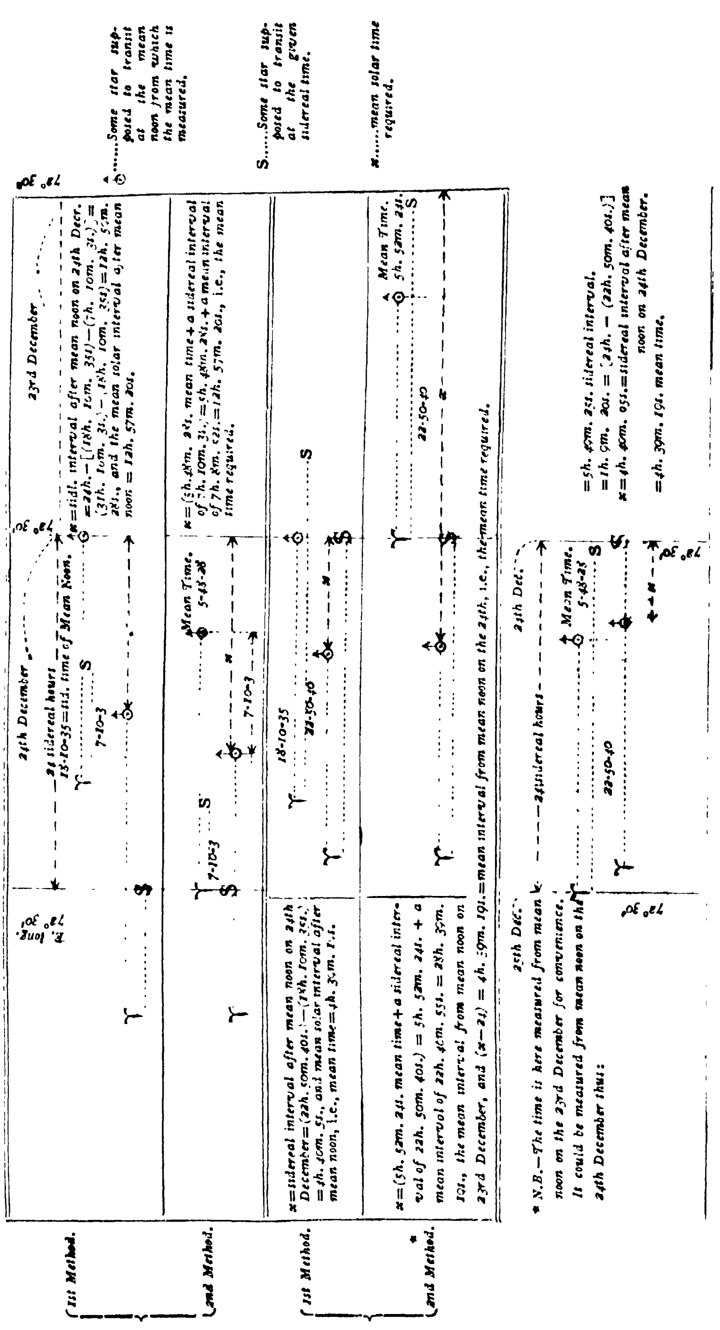
OBSERVATION.

* Sidereal time.				Mean time.		
22*h*.	00*m*.	00*s*.	=	21*h*.	56*m*.	23·7*s*.
	18	00·0	=		17	57·1
		46·0	=			45·9
		00·4	=			00·4
22	18	46·4	=	22	15	07·1

NOTE.—Illustration of the conversion of sidereal into mean solar time, by the two methods mentioned in the explanation to the 'Nautical Almanac.'

Example I.—To convert 7*h.* 10*m.* 03*s.* sidereal time, 24th December 1874, into mean time in E. longitude 72° 30′.

Example II.—To convert 22*h.* 50*m.* 40*s.* sidereal time, 24th December 1874, into mean time in E. longitude 72° 30′.



63. PROBLEM VII.—*Given the civil time at any place and the longitude of that place, to find the corresponding astronomical time at that instant at Greenwich.*

Observations: In consequence of the sun's apparent motion being from east to west, it will cross a meridian to the *east* of Greenwich *before* that of Greenwich, and a meridian *west* of Greenwich *after* that of Greenwich; therefore time in east longitude is *in advance* of Greenwich time, and time in west longitude is *behind* Greenwich time. For instance, at the instant the clocks at Greenwich are striking 12 at noon, at a place in longitude 73° 17′ 26″ E., the clocks will show 12*h.* 00*m.* 00*s.* +4*h.* 53*m.* 11*s.* (the longitude in time) = 4*h.* 53*m.* 11*s.* P.M. The positions of the heavenly bodies, as referred to the centre of the earth, are independent of meridians, and are the same for all classes at the same absolute instant; but the relative times at Greenwich and the observer's meridian would be different. The calculations of those bodies, as given in the 'Nautical Almanac,' are adapted to the meridian of Greenwich; hence, before making use of the Ephemeris, it is necessary to ascertain in every instance the distance of the sun (*in time*) of the observer's place from the meridian of Greenwich, or what is commonly called the *corresponding Greenwich* (*astronomical*) *time*.

Rule: Reduce the longitude of the place into time (by Prob. I, Art. 57); and if *east*, *subtract* it from the given time expressed astronomically (by Prob. III, Art. 59), but *add* it if *west*, and the remainder or the sum will be the required Greenwich astronomical time.

Example: Find the astronomical time at Greenwich corresponding to November 13, 9*h*. 07*m*. 53*s*. A.M., at a place in longitude 48° 57′ E.

Astronomical time at place, Nov. 12 ... = 21*h*. 07*m*. 53*s*.
Longitude of place, in time, E. ... = − 2 39 48
Corresponding astronl. time at Greenwich, Nov. 12 = 18 28 05

64. PROBLEM VIII.—*Given the astronomical time at Greenwich, and the longitude of any place, to find the corresponding civil time at that instant at the place.*

Rule: Reduce the longitude of the place into time (by Prob. I, Art. 57); and if *east*, *add* it to the given astronomical time, but *subtract* if *west*; the sum or the remainder will be the astronomical time at the place, which convert into civil time (by Prob. IV, Art. 60).

Example: Find the civil time at a place in longitude 96° 11′ 37″ E., corresponding to April 30, 7*h*. 17*m*. 19*s*. astronomical time at Greenwich.

Astronomical time at Greenwich, April 30 = 7*h*. 17*m*. 19*s*.
Longitude of place, in time, E. = + 6 24 46
Corresponding astronl. time at place, Apl. 30 = 13 42 05
and corresponding civil time at place, May 1 = 1 42 05 A.M.

65. PROBLEM IX.—*To find the time when a star culminates or passes the meridian of a given place on a given day; the longitude of the place being known.*

Rule: From the right ascension of the star (given in the 'Nautical Almanac' under the head of "Apparent places of stars"), increased, if necessary, by 24 hours, subtract the right ascension of the sun at mean noon of the day (given in Table II of the 'Nautical Almanac,' under the head of "Sidereal time at mean noon") corrected for difference of longitude (by Prob. V. Art. 61), and the remainder will be the sidereal time, which, converted into mean solar time (by Table XXXVII, "Calculating Tables," 5th edition), will show the mean astronomical time of the star's culmination at the given place.

Example: At what (astronomical) time will *Arcturus* (αBoötis) culminate at Moulmein, in longitude 97° 40′ 23″ E., on the 8th December 1880?

R. A. of star (N.A., App. places) + 24 hrs. ... = 38*h*. 10*m*. 13·8*s*.
R. A. of sun (N. A., p. II), 8th Dec., or
sidereal time at mean noon = 17*h*. 10*m*. 28·5*s*.
Correction for long. E. = 97·68° × 0·6571*s*. = −01 04·2
} = 17 09 24·3
Sidereal time of star's culmination ... ... = 21 00 49·5
∴ Mean astronomical time of star's culmination ... = 20 57 23·0

Note: Other ways of recording much the same system of work above denoted are given below.

At a station in N. lat. 27° 15′, E: long. 72° 30′, required the time of the upper transit of Fomalhaut on the 2nd February 1874.

(i) Sidereal time at mean noon, Greenwich, 2nd Feb. = 20*h*. 50*m*. 01·91*s*.
Corrn. for E. long. (by Prob. V, Art. 61) = 72·50° × 0·6571*s*. = − 47·64

Sidereal time of mean noon at station ... ... = 20*h*. 49*m*. 14·27*s*.
R. A. of Fomalhaut on 2nd Feb. ... ... = 22 50 39·61

The diffn.=Sidl. interval after mean noon of Fomalhaut = 2 01 25·34
Sidl. interval converted into mean solar interval (by Table XXXVII of "Calculating Tables," 5th edition).
Mean solar interval after mean noon=Mean solar time of transit of Fomalhaut ... ... = 2 01 05·45

(ii) A shorter method than the foregoing is—
R. A. of Fomalhaut on 2nd February = Sidereal time of his transit ... ... = 22*h*. 50*m*. 39·61*s*.
Convert into solar time, thus—
At Greenwich, mean time at *preceding* sidereal noon, namely, 1st Feb. (*vide* 'Nautical Almanac') ... = 3*h*. 13*m*. 22·88*s*.
Corrn. for E. long. (by Prob. VI, Art. 62)= 72·50°×0·6651*s*. = + 47·49

At station, mean time at *preceding* sidereal noon ... = 3 14 10·37
Equivalent mean time to 22*h*. 50*m*. 39·61*s*. Sidereal time (by Table XXXVII of "Calculating Tables," 5th edition) = 22 46 55·06

Mean solar time 26*h*. 01*m*. 05·43*s*. on 1st February ... = 2 01 05·43 on 2nd Feb.

66. PROBLEM X.—*To find the polar distance of a celestial object; the latitude of the place being known (Figs.* 436, 439*).*

Rule: When the latitude of the place and the declination (which is found in the 'Nautical Almanac') of the object are of the same name, that is, are both *north* or both *south*, the polar distance is = 90° — dec.; but when they are of opposite names, that is, the one *north* and the other *south*, the polar distance is = 90° + dec.

Example: Lat. N., dec. 13° 27′ S. gives P. D. ... = 103° 27′
" N., " 21 58 N. " " ... = 68 02
" S., " 22 02 N. " " ... = 112 02
" S., " 23 45 S. " " .. = 66 15

Note (*a*): *To find the altitude of a star, at transit, with the latitude given.*

At a station in N. lat. 27° 15′, E. long. 72° 30′, required the altitude of Fomalhaut at his upper transit on the 2nd February 1874 (*see* Note in Art. 65).

(i) (*See* Fig. 436.) The altitude of a transiting star = co-lat. ± declination.

Co-lat. ... ... = 62° 45′ 00·0″
Declination (South) ... ... = — 30 17 37·0

Altitude of Fomalhaut, measured from the south point of the horizon ... = 32 27 23·0

(ii) (*See* Fig. 439.) Or, Altitude measured from the north point of the horizon = lat. ± N.P.D.

Note (*b*) : *To find the Right Ascension and North Polar Distance of stars whose mean places only are given in the Nautical Almanac or other star catalogues.**

OBSERVATION.

* The ordinary surveyor need not trouble himself about these calculations; he would confine his observations to N.A. stars only.

(i) Find the Right Ascension and North Polar Distance of 1612 New 7 years' Catalogue on the 8th December 1873 (*vide* Explanation in N. A.)

On the 1st January 1873, mean R. A. = 13*h*. 46*m*. 04·424*s*.
and mean N.P.D. ... ... = 6° 36′ 38·704″

	R.A.			N.P.D.	
	Logarithms.	Nat. num.		Logarithms.	Nat. num.
e	9·83165		*e′*	9·84868	
E	1·46545		E	1·46545	
	1·29710	19·820*s*.		1·31413	20·612″
f	9·97272		*f′*	0·31982	
F	1·65249		F	1·65249	
	1·62521	42·190*s*.		1·97231	93·824″
g	1·35935		*g′*	1·63281	
G	0·27805		G	0·27805	
	1·63740	43·391*s*.		1·91086	81·445″
h	9·83383		*h′*	9·87652	
H	1·27369		H	1·27369	
	1·10752	12·809*s*.		1·15021	14·132″
		i = 125·055*s*.			*i′* = 69·809″
		I = 51·085*s*.			I = 51·085″
		294·350*s*.			330·907″
		— 300·000*s*.			— 300·000″
Reduction of R.A. =		— 5·650*s*.	Reduction of N. P. D. =		+ 30·907″
Mean R.A. =	13*h*. 46*m*.	4·424*s*.	Mean N. P. D. =		6° 36′ 38·704″
Apparent R.A. =	13*h*. 45*m*.	58·774*s*.	Apparent N. P. D. =		6° 37′ 09·611″

(A star like the above might be observed at transit, to see how it will answer before fixing on it for an Azimuth Star.)

(ii) On 1st January 1874, of No. 1612 New 7 years' catalogue, Mean R.A. = 13*h*. 46*m*. 02·33*s*. and Mean N.P.D. = 6° 36′ 56·74″. Required R. A. and N. P. D. of the stars on 25th February 1874.

(In the following computation Airy's Day Numbers have been used.)

	R. A.			N. P. D.	
	Logarithms.	Nat. num.		Logarithms.	Nat. num.
e	9·83165		*e′*	9·84868	
E	0·88658		E	0·88658	
	0·71823	5·227*s*.		0·73526	5·436″
f	9·97272		*f′*	0·31982	
F	1·51721		F	1·51721	
	1·48993	30·898*s*.		1·83703	68·712″
g	1·35935		*g′*	1·63281	
G	0·06480		G	0·06480	
	1·42415	26·555*s*.		1·69761	49·844″

R. A.		N. P. D.	
Logarithms.	Nat. num.	Logarithms.	Nat. num.
h 9·83383		*h′* 9·87652	
H 1·24225		H 1·24225	
1·07608 ...	... 11·915*s*.	1·11877 ...	... 13·145″
	l = 125·055*s*.		*l′* = 60·809″
	L = 111·292*s*.		L = 111·292″
	310·942*s*.		318·238″
	— 300·000*s*.		—300·000″
Reduction of R. A. =	+ 10·94*s*.	Reduction of N. P. D. =	+ 18·24″
Mean R. A. =	13*h*. 40*m*. 02·33*s*.	Mean N. P. D. =	6° 36′ 56·74″
Apparent R. A. =	13*h*. 40*m*. 13·27*s*.	Apparent N. P. D. =	6° 37′ 14·98″

67. PROBLEM XI.—*To correct the observed or apparent altitude of the sun to obtain its true altitude.*

Observations: The sun's horizontal parallax varies only ⅓ of a second, and may in practice generally be considered as invariable. The *parallaxes in altitude* for the sun at any given time, may therefore be considered the same for any other time; and thus being *constant*, they are given in (Table XXXIX, "Calculating Tables," 5th edition).

Rule: *If the sun's centre is observed*: Subtract the mean refraction (taken from Table XXXVIII, "Calculating Tables," 5th edition) corresponding to the observed altitude, to which add the sun's parallax in altitude (taken from Table XXXIX. "Calculating Tables," 5th edition).

If either limb of the sun is observed: To the above result, *add* the sun's semi-diameter (obtained from Table II of the 'Nautical Almanac'), if the *lower limb* was observed; but *subtract* it if the *upper limb* was observed.

Example: 1. On October 1st, 1880, the meridian altitude of the sun's centre was observed to be 27° 18′ 22″; required its true altitude; barometer being 30·00 inches, and thermometer, Fahrenheit, 50°.*

Observed altitude of sun's centre			=	27° 18′ 22″
Correction for refraction	=	— 1′ 52·3″	= —	1 44·5
,, ,, parallax	=	+ 07·8		
True altitude of sun			=	27 16 37·5

Example: 2. The observed altitude of the sun's upper limb on the 20th December 1880, is 41° 10′ 47″; barometer = 30·65 inches, thermometer = 80° F.; find the true altitude of the sun's centre.

Observed altitude of sun's upper limb			=	41° 10′ 47″
Correction for refraction†	=	— 1′ 04·0″	= —	57·3
,, ,, parallax	=	+ 06·7		
True altitude of sun's upper limb			=	41 09 49·7
Sun's semi-diameter			= —	16 17·9
True altitude of sun's centre			=	40 53 31·8

OBSERVATIONS.

* When the barometer and thermometer readings are not registered, the *mean* refraction, which is calculated for Bar. 30 inches and Ther. F. 50°, is only applied.

† Refn.	for 41°		=	01′ 06·9″
,,	,,	10′ 47″	= —	00·4
Bar.	,,	0·65 in.	= +	01·5
Therm.	,,	30°	= —	04·0
Total correction			=	01′ 04·0″

68. PROBLEM XII.—*To correct the observed or apparent altitude of a star to obtain its true altitude.*

Observation: The fixed stars have neither parallax nor semi-diameter on account of their immense distance from the earth, and consequently their observed altitudes require no correction for the same.

Rule: Subtract the mean refraction (taken from Table XXXVIII, "Calculating Tables," 5th edition) corresponding to the observed altitude, for the true altitude required.

Example: Required the true altitude of a star whose apparent altitude was observed to be $86^\circ\ 17'\ 20''$.

Observed altitude of star ... ... = $86^\circ\ 17'\ 20''$
Correction for mean refraction ... = — $03{\cdot}8$
True altitude of star = $86\ \ 17\ \ 16{\cdot}2$

69.* PROBLEM XIII.—*To determine the direction of the meridian at a place, by observation of a circumpolar star at its maximum elongation; the latitude and longitude of the place being known.* (Figs. 475, 499.)

Observations: To select a circumpolar star for the purpose of observation, look for one in the 'Nautical Almanac,' whose North declination therein given, is greater than the co-latitude of the place of observation.

Rule: The following elements are given:—

Latitude of place of observation;
Declination of star (from 'Nautical Almanac').

The following elements are required:—

1st. Hour angle (or time of elongation);
2nd. Star's azimuth (at time of maximum elongation);
3rd. Star's altitude.

The required elements are calculated by the following formulæ:—

N. P. D. = 90° — declination (by Prob. X, Art. 66);
Cos. hour angle = tan. N. P. D. × tan. latitude;
Sin. azimuth = sin. N. P. D. × sec. latitude;
Sin. altitude = sec. N. P. D. × sin. latitude.

The hour angle converted into time (by Prob. I, Art. 57) and *added* to the star's right ascension (as given in the 'Nautical Almanac') will give the sidereal time of *western* elongation; and *subtracted* therefrom will give the sidereal time of *eastern* elongation. This sidereal time must then be converted into mean or clock time (by Prob. VI, Art. 62).

Example: Find the maximum W. elongation of *Polaris* (α Ursæ Minoris) on the 10th December 1880, at Moulmein, in latitude $16^\circ\ 28'\ 46''$ N., and longitude $97^\circ\ 40'\ 23''$ E.

Elements given:

Latitude of Moulmein = $16^\circ\ 28'\ 46''$ N.
Declination of star (from N. A., App., places) = $88\ \ 40\ \ 51$ N.
N. P. D. = $(90^\circ - 88^\circ\ 40'\ 51'')$ = $1\ \ 19\ \ 09$

To find the hour angle:—

By formula, cos. hour angle = tan. N. P. D. × tan. latitude

$\therefore$ by logs., L. cos. h. a. = 10 — L. tan. N. P. D. — 10 + L. tan. lat. —10
= L. tan. $1^\circ\ 19'\ 09''$ + L. tan. $16^\circ\ 28'\ 46''$ — 10
= $8{\cdot}362250 + 9{\cdot}471032 - 10$
= $7{\cdot}833282$ = L. cos. $89^\circ\ 36'\ 35''$

$\therefore$ hour angle = $90^\circ\ 36'\ 35''$ = 5*h.* 58*m.* 26*s.* sidereal time,
R.A. of star, 10th Dec. (N.A., App., places) = + 1 15 43

OBSERVATION.

* The solutions given of some of the formulæ in this Article and Articles 70, 71 and 77, are arrived at by the rules of spherical trigonometry with which the ordinary surveyor need not be familiar.

Sidereal time of W. elongation = 7*h*. 14*m*. 9*s*.
∴ Mean astronomical time of W. elongation (by Prob. VI, Art. 62)*
= 13*h*. 55*m*. 35*s*. Dec. 10th
(= 1 55 35, A.M., Dec. 11th, civil time).

To find the star's azimuth:—

By formula, sin. azimuth = sin. N. P. D. × sec. latitude
∴ by logs., L. sin. azimuth = 10 + L. sin. N. P. D. — 10 + L. sec. lat. — 10
= L. sin. 1° 19′ 09″ + L. sec. 16° 28′ 46″ — 10
= 8·362134 + 10·018217 — 10
= 8·380351 = sin. 1° 22′ 33″
∴ azimuth = 1° 22′ 33″.

To find the star's altitude:—

By formula, sin. altitude = sec. N. P. D. × sin. latitude
∴ by logs., L. sin. alt. = 10 + L. sec. N. P. D. — 10 + L. sin. lat. — 10
= L. sec. 1° 19′ 09″ + L. sin. 16° 28′ 46″ — 10
= 9·999885 + 9·452816 — 10
= 9·452701 = sin. 16° 28′ 30″
∴ altitude = 16° 28′ 30″.

The next step is to take the required observation on the circumpolar star, which is done in the following manner:—About a quarter of an hour before the computed time of maximum elongation (as 1*h*. 55*m*. 35*s*. A.M.) plant the theodolite over the station at which the direction of the meridian is to be determined; put it through its temporary adjustments, set vernier A of the horizontal plate to zero, and the vertical verniers to the computed altitude of the star (as E. 16° 28′ 30″). Move the lower limb of the theodolite, and get the star into the field of the telescope,† intersecting it carefully with the cross-wires by means of the lower horizontal and the vertical tangent screws. The maximum elongation not having as yet occurred, the star will be receding from the meridian; continue therefore to follow the star, intersecting it by the *lower* horizontal and vertical tangent screws, until the star ceases to have any motion. Now release the upper horizontal and vertical clamps, and lay off the computed azimuth (as 1° 22′ 33″) towards the meridian. If the star is at its *western* elongation (as in the example), vernier A of the horizontal plate must be set to the azimuth angle *eastwards*, *plus* (as at 1° 22′ 33″); but if the star is at its *eastern* elongation, vernier A of the horizontal plate must be set to the azimuth angle *westwards, minus* (as at 358° 37′ 27″): the telescope will then be in the plane of the meridian. Next set a small bull's-eye lantern at a point, called a *referring mark*, on a survey line (where such exists) or out of it, at a distance of about 20 chains, and connect it by observation with the meridian just found. Thus, the true bearing of the referring mark having been fixed, the true meridian may at any time thereafter be set off from it, or the true bearing of any other line from the station of observation obtained.

Note (i).—*To determine the azimuth of referring mark from observations to a circumpolar star* (*see* Fig. 475).

Preliminary approximate calculation of variation in altitude about elongation:—

In the case of Polaris, on a certain day, let its N.P.D. $= \delta = 1^\circ 22' 22'' = 4{,}942''$.

In 24 hours Polaris describes the circle $= 2\pi\delta$, and in n minutes it will describe an arc″ $= \frac{2\pi\,\delta}{24 \times 60^{\text{m}}} \times n^{\text{m}} \ldots (^{a})$.

OBSERVATIONS.

* Mean time at the *preceding* sidereal noon, namely, Dec. 10 (N. A., p. III) = 6*h*. 40*m*. 33*s*.
Correction for longitude E. — 97·68° × 0·6551*s*. = 64·0*s* = + 01 04
Sum = 6 41 37
For 7*h*. 14*m*. 09*s*. sidereal time, the equivalent mean solar time (by Table XXXVII, "Calculating Tables," 5th edition) ... = 7 12 58
Mean solar time required = 13 54 35

† Small theodolites are not provided with the means of illuminating the cross-wires of the telescope at night, hence some plan as the following must be had recourse to:—Make a white paper ring about an inch broad to fit on to the object-end of the telescope, the paper rim projecting about half an inch beyond the end of the telescope. A lamp held at a short distance with the light falling obliquely on the paper will light up the cross-wires clearly.

The movement about elongation may be assumed to take place in the plane of the vertical circle OZH, and $\therefore$ the change in arc in a certain time = the change in altitude corresponding to the same time; in ($_a$) put arc$'' = a''$ and $n^m = 1$; then in an interval of 1 minute $a'' = \frac{2\pi.\ \delta.}{24 \times 60} = \frac{\pi.}{12 \times 50} = \frac{3{\cdot}14159 \times 4{,}942''}{720}$

log. 3·14159 = 0·497149
Ar. Co.—log. 720·0 = 3·142668
3·639817 = constant log;
log. 4,942″ = 3·693903 = log. N.P.D″.
1·333720 = log. a'' for 1^m

$\therefore a'' = 21{\cdot}6''$ for 1 minute,
and
$a'' = 108{\cdot}0''$ for 5 minutes.

Note (ii).—*Computation of azimuth from circumpolar star observations* (*see* Fig. 499).

For *First Factor—Hour Angle.*

Cos. P = tan. δ. tan. λ. (*see* Fig.) Let δ. = N.P.D.
λ. = Lat.
P. = Hour Angle.
A. = Azimuth.

differentiating with respect to δ. only

$-\text{Sin. P.}\ \frac{dP}{d\delta.} = \text{tan. } \lambda.\ \text{sec.}^2\ \delta.,$

or $dP = -\text{sec.}^2$ N.P.D. × tan. λ. × cosec. P × dδ. ...... (1).

For *factor of Azimuth*—

sin. δ. = cos. λ. sin. A,

or sin. A = sin. δ. sec. λ.; $\therefore$ cos. A $\frac{dA}{d\delta.}$ = cos. δ. sec. λ., and

δ. A = cos. N.P.D. × sec. λ. × sec. A × dδ. ...... (2).

(1) and (2) are the expressions used in computation.

In (1) dP = —sec.^{2} N.P.D. × tan. λ. × cosec. P × dδ.

or dP = —fp × dδ. where fp is the " factor for horary angle."

Similarly in (2) dA = cos. N.P.D. × sec. λ. × sec. A × dδ.

= fA × dδ. where fA is the " factor for azimuth."

Hence the negative sign is *always* to be prefixed to fp and the postive sign is *always* to be prefixed to fA.

70. PROBLEM XIV.—*To determine the azimuth at a place by observation of Stars of either North or South Declination, or of the Sun.* (Figs. 488-489*a*, 506.)*

(*a*) The first three examples (Nos. 1 to 3) given in this Article are specimens of a field-book and computation, for determining the meridian (by means of star observations) as adopted in the Revenue Survey of India, which will be found useful. The variation of the needle given in the forms has reference to the 'Meridian of origin' and *not* to the 'True meridian' of the station of observation.

A valuable paper on the subject was drawn up by the late General Sir J. T. Walker, R.E., from which the following passages are extracts :—

When great accuracy is required, the method of determining astronomical azimuths by observations to a circumpolar star, at or near the time of its maximum elongation, is to be preferred to any other. But when all that is required is to obtain an initial or a verificatory azimuth for theodolite and chain traverses, or for minor triangulations, a more preferable method is that given in the computation forms hereafter given and employed in the Revenue Survey Department. It is to observe simultaneously the altitude of a star situated almost due east or west of observer, and the horizontal angle between the star and a referring mark; then the azimuth of the mark may be deduced when the latitude of the place and the north polar distance of the star are known.

OBSERVATION.

*** For the logarithmic calculations and refraction and parallax values occurring in the examples (8 in number) to the subject of this Article, consult the " Calculating Tables " (5th edition), Tables XXXI (Log. numbers), XXXII (Log. Sines and Tangents), XXXVIII (Refraction) and XXXIX (Parallax). For the corrections for convergency of meridians, refer to Table XLII of the same book.**

The practical advantage of this method lies in its being readily applicable at any time between dusk and dawn when stars are visible, and also in its permitting observations being taken to stars of the largest magnitudes; whereas the circumpolar stars are mostly of comparatively small magnitudes, and they not unfrequently attain their maximum elongations at times when it is very inconvenient to have to observe them. There are several large stars which may be observed early of an evening, or in the morning, when there is just sufficient daylight to enable an unilluminated referring mark to be seen, and also to enable the wires of the theodolite to be seen without illumination. In such cases the angles may be measured with no more trouble than if a terrestrial object were observed instead of a star. And even in cases when the twilight is too short to enable the requisite observations to be taken, and arrangements must be made for illuminating the referring mark and the wires of the theodolite, it is still a great convenience to be able to take the observations early in the evening before the usual dinner hour, and thus get the work done without breaking into hours of the night which may be more profitably employed in resting than in working.

Not less than one pair of observations 'face left' and 'face right'—including both the horizontal angle between the referring mark and the star, and the vertical angle of the star—should be taken in each instance, to eliminate the several constant instrumental errors; but more than two pairs of observations are not ordinarily required to give a result within 1 minute of the truth, provided the star is favourably situated for the purpose of the observations.

Let ACB (*see* Fig. 506) be a spherical triangle in which A is the position of the observer's zenith, C that of the star, and B that of the pole;

then the side a, or CB, is = the North Polar Distance of the star,
„ „ c, or AB, is = the Co-latitude of the station,
„ „ b, or AC, is = the Zenith Distance of the star,
„ angle A is the azimuthal angle measured from the north,
„ „ C is the parallactic angle,
„ „ B is the hour angle.

Then by the fundamental formula of spherical trigonometry.

$$\cos. A = \frac{\cos. a - \cos. c \cos. b}{\sin. c. \sin. b}.$$

If we put—

$$x = \frac{\cos. a}{\cos. c. \cos. b} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\text{then, } \cos. A = (x - 1) \cot. c. \cot. b \quad \ldots \quad \ldots \quad (2)$$

and this form of computation is used in the Revenue Surveys, with the substitution of (90° + the Declination) for the North Polar Distance, of the Latitude for the Co-latitude, and of the Altitude for the Zenith Distance; but the calculation requires careful attention to the signs of the quantities involved.

Most computors would probably prefer the well-known equivalent formula

$$\cos. \tfrac{1}{2} A = \sqrt{\frac{\sin. a. \sin. (s-a)}{\sin. c. \sin. b}} \quad \ldots \quad \ldots \quad (3)$$

$$\text{in which } s = \tfrac{1}{2}(a + c + b) \quad \ldots \quad \ldots \quad (4)$$

It is not more laborious, and the signs are all positive and give no trouble.

In order to ascertain the position in which the star is most favourably situated for observation, let us *first* assume that the latitude is correctly known, but that an error db has been made in the measurement of the star's zenith distance; let dA be the corresponding error in the resulting azimuth, then

$$dA = -\frac{\cot. C}{\sin. b} \cdot db \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Thus the error of the azimuth will be a minimum when cot. $C = 0$, or $C = 90°$, *i.e.*, when the star is at its elongation; and it will always be less than the error db when cot. C is less than sin. b.

If the azimuthal angle is substituted for the parallactic angle, the formula will become

$$dA = -(\cot. c. \operatorname{cosec.} A - \cot. b. \cot. A)\, db \quad \ldots \quad (6)$$

which shows that when the azimuthal error is a minimum

$$\cos. A = \frac{\cot. c}{\cot. b}$$

as is always the case at the elongation.

Secondly, let us assume that the adopted value of the latitude is erroneous, but that the observations are exact; let dc be the error of the co-latitude c, and dA that of the azimuth, then $dA = -\frac{\cot. B}{\sin. c} \cdot dc$ ... (7)

Thus the error will be a minimum when cot. $B = 0$, or $B = 90°$, that is to say, six hours before or after the star transits the meridian; and it will always be less than the error in the latitude when cot. B is less than sin. c.

If the azimuthal angle is substituted for the hour angle, the last formula becomes

$$dA = -(\cot. b. \operatorname{cosec}. A - \cot. c. \cot. A)\, dc \;\ldots \qquad \ldots \quad (8)$$

which shows that when the azimuthal error is a minimum

$$\cos. A = \frac{\cot. b}{\cot. c}.$$

Both of the terms within the brackets of equations (6) and (8) are liable to become larger when the azimuth approaches either 0° or 180°; but up to 90° their difference only is effective on the deduced value of the azimuth, whereas beyond 90° their sum is effective, and therefore it is not desirable to observe a star when its azimuth materially exceeds 90°.

Example: An azimuth was determined at Girishk, near Kandahar, by observations to Sirius, at a time when the star was approximately in azimuth 132° 56′, and altitude 26° 22′, its N.P.D. being 106° 33′. The adopted value of the latitude was 31° 45′, which was subsequently ascertained to be 2′ too low. It may be readily found by the preceding formula that the error in the azimuth which was caused by the error of + 2′ in the co-latitude is $= - 1{\cdot}25\, dc = - 02' 30''$; it would have been less than 1 minute if a star had been observed on the prime vertical at the same altitude.

In addition to errors in the latitude and the vertical angle, there is an error in another quarter, namely, in the measurement of the horizontal angle between the referring mark and the star, which has to be provided against; when the star is at a considerable elevation, much error may be caused by defects in the construction and adjustments of the theodolite which is employed to measure that angle. Now this error will be a minimum when the star is about on a level with the referring mark; but on the other hand, the star must not be observed too near the horizon, as errors of refraction are then a maximum.

Thus we see that, taking everything into consideration, it is desirable—at least in Northern India—to avoid stars of South Declination, and to choose stars of North Declination, up to but not exceeding the co-latitude of the station; and also to observe the star at an altitude between 10° and 25°.

In observing, the following procedure should be followed:—

1st.—Level the theodolite carefully.

2nd.—Set the theodolite so that the A vernier will read 0° 0′ when the magnetic needle points to the north.

3rd.—Set the telescope on the referring mark and record the readings of the horizontal verniers.

4th.—Set the telescope on the star in such a manner that the star will be seen at the intersection of the horizontal and vertical wires; this requires a little practice, but is easy to perform; then record the readings of both the horizontal and the vertical verniers.

5th.—Change face.

6th.—Observe the star as in (4).

7th.—Observe the referring mark as in (3).

If the index and collimation errors of the instrument are large, a considerable difference between the results on 'face right' and 'face left' may arise from this cause only and not from errors of observations; consequently, two or three pairs of observations to the same star should always be taken, in order to indicate whether the discrepancies between opposite faces are instrumental or due to errors of observations.

Stars should be selected for observation in accordance with the principles laid down above.

Sufficient observation for a determination of the azimuth within 2′ of the truth may generally be taken in half an hour, with a 7-inch theodolite, so there will be no need to make a toil of what should be a pleasure.

The observations should be recorded and computed in the Form found in the Examples below.

Star charts,* as given in (Figs. 488-489*a*), showing all stars of the 1st, 2nd and 3rd magnitudes, are very useful in assisting an observer to find the star he wants, the position of each star being fixed by its right ascension and declination. All the constellations are also entered on these star maps. On the maps † the stars are projected as we actually see them in the heavens looking at them from the earth; but on celestial globes, as seen from the outside, their positions are reversed, as the earth on which the observer is placed, is supposed to occupy the centre of the globe.

(*b*) The next three examples (Nos. 4 to 6) provided in this article have reference to sun azimuths for observations taken under conditions such as those described in Chapter XIII, Art. 94, or any others. The azimuth values derived from star observations are, from the nature of the respective objects (the star being better defined) superior to those obtained from the sun.‡ The latter should, therefore, be restricted to places where, from whatever cause, it is not convenient to observe to stars. The "Variation of the needle" given in the forms refers to the 'Meridian of origin' of the survey and *not* to the 'True meridian' of the place of observation.

(*c*) The last two examples (Nos. 7 and 8) given in this article refer to star azimuths worked out in a new form which will be found serviceable. The computations for sun azimuths can likewise be worked out in these forms, as they also can in the forms to examples 1 to 3 *ante*.

OBSERVATIONS.

* Larger ones are printed in the Trigonometrical Survey Office at Dehra Dun, which may perhaps be had on application.

† The projection of the maps is geometrical and globular. The visible universe, or celestial sphere, is of vast dimensions and is viewed from inside the vault; the cardinal points on the maps are different to those of a terrestrial atlas, that is with the north at the top, the west will be to the right of each map and the east on the left. The stars revolve round the poles of the heavens in the same time that the earth takes to rotate on its axis, in a direction contrary to the hands of a watch. The motion of the earth is from W. to E., and the apparent motions of the stars is thus from E. to W. To view the stars as they would appear in the sky from a point on the earth, hold the maps erect facing North with the corresponding point of the circumference at the bottom when the northern hemisphere will be pictured. Similarly, by facing South and the corresponding point of the circumference lowest the southern hemisphere is represented.

‡ The samples given, however, are selected ones and considerably better than the average of this description of azimuth.

Example 1.

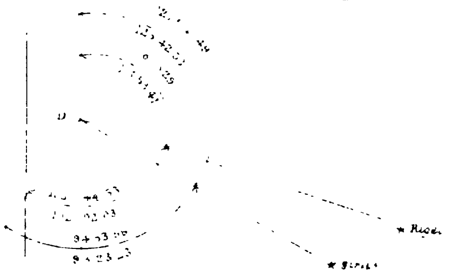
(Applicable to Stars of North or South Declination.)

14th January 1878. Observation for Azimuth at ⊙ D, the trijunction at Ghoreman, Kohi and Agency lands of Main Circuit No. 8, District Banda, Pargana Badausa; Length of Line 30·26 Chains; Latitude N. 25° 04′ 00″; Needle bearing 218° 22′ 40″; Theodolite 8-inch, No. 260, by Troughton and Simms.

Face.	Object.	Horizontal Verniers.				Angles between Star & Mark.	Obsns.	Vertical Verniers.			Refn. —	True Altitude.	Civil time.
		A.	B.	C.	Mean.			Front.	Back.	Mean.			
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark C	42 36 20	35 50	36 30	42 36 13								
	Star Rigel	298 51 30	51 00	51 30	298 51 20	103 44 53	1	29 49 30	48 50	29 49 10	1 41	29 47 29	.. ..
	Star Rigel	120 33 50	34 10	33 30	120 33 50	102 02 03	2	31 50 00	49 30	31 49 45	1 34	31 48 11	... ...
	Mark	222 35 50	35 40	36 10	222 35 53								
R	Mark	222 35 50	35 40	36 10	222 35 53								
	Star Sirius	127 41 50	41 40	42 20	127 41 57	94 53 56	3	26 28 00	28 20	26 28 10	1 56	26 26 14	... ...
	Star Sirius	309 12 50	12 10	12 50	309 12 47	93 23 23	4	28 03 10	04 00	28 03 35	1 49	28 01 46	... ...
	Mark	42 36 00	35 50	36 10	42 36 00								

Observation for check Azimuth or to (a Ursæ Minoris) Polaris.

Face.	Object.	A.	B.	C.	Mean.	Angles between Star & Mark.
L	Mark C	42 36 10	35 40	35 50	42 35 53	
	Star	180 30 00	29 50	30 10	180 30 00	222 05 53
	Star	180 30 00	29 50	30 00	180 29 57	222 00 00
	Mark	42 36 00	35 50	36 00	42 35 57	
R	Mark	222 35 40	35 10	35 50	222 35 33	
	Star	360 29 40	29 50	30 00	360 29 50	222 05 43
	Star	360 29 40	29 30	30 10	360 9 47	222 05 43
	Mark	222 35 40	35 20	35 40	222 35 30	
						222 05 50



Computation.

Star Rigel (β Orionis).

Declination S. ... ... = 8° 20′ 36″ Log. Sin. ... ... = 9·161681
Latitude N. ... ... = 25 04 00 Log. Cosec. ... ... = 10·372970
Constant Log. .. = 1̄·534651

Star Sirius (a Canis Majoris).

Declination S. ... ... = 16° 32′ 58″ Log. Sin. ... ... = 9·454605
Latitude N. ... ... = 25 04 00 Log. Cosec. ... ... = 10·372970
Constant Log. ... = 1̄·827575

	1ST OBSERVATION.	2ND OBSERVATION.	3RD OBSERVATION.	4TH OBSERVATION.	5TH OBSERVATION.	6TH OBSERVATION.
Constant Log. =	$\bar{1}$·534 651	$\bar{1}$·534 651	$\bar{1}$·827 575	1·827 575	......	......
Altitudes ... ... Log. cosec. =	10·303 780	10·278 189	10·351 429	10·327 971	......	......
1st = 29° 47′ 29″ ... ... Log. *x* =	$\bar{1}$·838 431	$\bar{1}$·812 840	0·179 004	0·155 546	......	......
2nd = 31 18 11 ... ... ∴ *x* =	− 0·6893	− 0·6500	− 1·5101	− 1·4307	......	......
3rd = 26 26 14 ... ... and (*x*—1) =	− 1·6893	− 1·6500	− 2·5101	− 2·4307	......	......
4th = 28 01 46 ... ... Log. (*x*—1) =	0·227 707	0·217 484	0·399 69.	0·385 731	......	......
Altitudes ... ... Log. tan. =	9·757 780	9·792 402	9·696 544	9·726 213	......	......
Latitude ... ... Log. tan. =	9 669 991	9·669 991	9·669 991	9·069 991	......	......
Azimuths ... ... Log. cos. of =	9·655 478	9·679 937	9·766 226	9·781 935	......	......
∴ Azimuths of Star =	116° 53′ 41″	118° 35′ 29″	125° 42′ 53″	127° 14′ 49″	...° ...′ ...″	...° ...′ ...″
Angles between Star and Mark =	103 44 53	102 02 03	94 53 56	93 23 23	... ...	... ...
Azimuths of Mark =	230 38 34	220 37 32	220 36 49	220 38 12	... ...	... ...
Mean Azimuth of Mark =				220 37 47		... ...
Convergency for 7·50 Miles W. =				+ 03 03		... ...
Variation of Needle 2° 18′ 10″ E. ... ... Working Azimuth =				220 40 50		... ...

Computation of *α Ursæ Minoris* when taken at Elongation.

Declination North = 88° 39′ 58″	Log. Cos. =	8·366 957
Latitude = 25 04 00	Log. Sec. =	10·042 900
	Azimuth Log. Sin. =	8 400 917
	∴ Azimuth of Star =	1° 28′ 21″
	Mean Angle between Star and Mark =	222 05 50
	Azimuth of Mark =	220 37 29
	Convergency for 7·50 Miles =	+ 03 03
	Working Azimuth =	220 40 32

Computation *for Latitude* (*vide* N. A. Tables).

True Altitude of Polaris	24° 58′ 10″
Deduct as per rule	— 0 01 00
Reduced Altitude	24 57 10
First Correction	+ 0 05 14
First Approximate Latitude	25 02 24
Second Correction	+ 0 00 26
Second Approximate Latitude	25 02 50
Third Correction	+ 0 01 10
Latitude of Place	25 04 00

NOTE.—Always *subtract* the reading of the Star from that of the Mark. When the Star's Declination is *North*, the sign of *x* is *plus*. When the Declination is *South*, the sign of *x* is ***minus***. The expression (*x*—1) must be treated algebraically. The Azimuth of the Star is *less* than 90° when the sign of (*x*—1) is *plus*, and *greater* than 90° when the sign of (*x*—1) is *minus*. When the Star observed is to the *East* of the Meridian, ***add together*** the Azimuth of the Star and the Angle between the Star and Mark. When the Star observed is to the *West* of the Meridian, ***subtract*** the Azimuth of the Star from the Angle between the Star and Mark. Convergency is *additive* to Azimuths observed *West* of Origin, and *subtractive* from those observed *East* of Origin.

N.B.—When *Polaris* is observed for Azimuth, a *check* observation must be taken to some *East* or *West* Star.

Example 2.

22nd November 1876. Observation for Azimuth at ⊙ O of Main Circuit No. 12, District Bannu. Length of Line 19·17 Chains..............Latitude N. 33° 01′ 25″, Needle Bearing 265° 29′ 30″. Theodolite 8-inch, No. 256, by T. Cooke and Sons.

Face.	Object.	Horizontal Verniers. A.	B.	C.	Mean.	Angles between Star & Mark.	Obsns.	Vertical Verniers. Front.	Back.	Mean.	Refn. —	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark	219 04 10	04 20	03 30	219 04 00								
	Star Capella ...	357 05 50	06 00	05 50	357 05 53	221 58 07	1	18 28 20	27 30	18 27 55	2 49	18 25 06	... ...
	Star Capella ...	357 32 30	32 40	32 20	357 32 30	221 31 33	2	19 13 20	12 30	19 12 55	2 43	19 10 12	.. ..
	Mark	219 04 10	04 30	03 30	219 04 03								
R	Mark	39 04 10	04 50	04 00	39 04 20								
	Star Capella ...	178 03 10	03 10	03 00	178 03 06	221 01 14	3	20 07 00	07 50	20 07 25	2 34	20 04 51	... ...
	Star Capella ...	178 30 40	30 40	30 20	178 30 33	220 33 50	4	20 59 00	59 50	20 59 25	2 27	20 56 58	... ...
	Mark	39 04 10	05 00	04 00	39 04 23								

Star Capella (α Aurigæ).

	° ′ ″
Declination N.	= 45 52 21
Latitude N.	= 33 01 25

Computation.

Log. Sin. ... ... = 9·855 690
Log. Cosec ... ... = 10·263 010
Constant Log. ... ... = 0·119 615

Star..............................

Declination ... ... =
Latitude ... ... =

Log. Sin. ... ... =
Log. Cosec. ... ... =
Constant Log. ... ... =

			1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
		Constant Log. =	0·119 615	0·119 615	0·119 615	0·119 615	... ..	......
Altitudes ° ′ ″	...	.. Log. Cosec. =	10·500 378	10·483 033	10·464 268	10·446 671	......	. . ....
1st = 18 25 06	...	... Log. x =	0·619 093	0·603 248	0·583 883	0·506 286	......	......
2nd = 19 10 12	...	... ∴ x =	+ 4·1086	+ 4·0110	+ 3·8360	+ 3·6837	......	......
3rd = 20 04 51	...	... and $(x-1)$ =	+ 3·1086	+ 3·0110	+ 2·8360	+ 2·6837	......	......
4th = 20 56 58	...	... Log. $(x-1)$ =	0·500 867	0·478 711	0·452 706	0·428 734	.....	......
Altitudes	...	... Log. tan. =	9·522 459	9·541 142	9·562 969	9·583 031	......	......
Latitude	...	... Log. tan. =	9·812 909	9·812 909	9·812 909	9·812 909	......	......
Azimuths	...	... Log. cos. of =	9·836 235	9·832 762	9·828 584	9·824 674	......	......
		∴ Azimuths of Star =	46° 41′ 48″	47° 07′ 31″	47° 37′ 57″	48° 05′ 57″	...° ...′ ...″	...° ...′ ...″
		Angles between Star & Mark =	221 58 07	221 31 33	221 01 14	220 33 50	... ...	... ...
		Azimuths of Mark =	268 39 55	268 39 04	268 39 11	268 39 47	... ...	... ...
					Mean Azimuth of Mark =	268 39 29		... ...
					Convergency for 19·45 Miles W. =	+ 10 38		... ...
Variation of Needle 3° 20′ 37″ E.	...	...	...	...	... Working Azimuth =	268 50 07		... ...

Example 3.

25th November 1876. Observation for Azimuth at ⊙ H of Main Circuit No. 12, District Bannu, Length of Line 80·96 Chains................Latitude N. 32° 56′ 40″. Needle Bearing 202° 16′ 10″ E. Theodolite 12-inch, No. 17, by Troughton and Simms.

Face.	Object.	Horizontal Verniers. A.	B.	C.	Mean.	Angles between Star & Mark.	Obsns.	Vertical Verniers. Front.	Back.	Mean.	Refn. —	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark	121 26 50	27 40	27 00	121 27 10								
	Star Sirius	143 45 30	46 00	45 50	143 45 46	337 41 24	1	24 49 50	48 40	24 49 15	02 03	24 47 12	... ...
	Star Sirius	145 09 50	10 20	09 30	145 09 53	330 17 20	2	23 43 00	42 30	23 42 45	02 10	23 40 35	... ...
	Mark	121 27 00	77 40	27 00	121 27 13								
R	Mark	301 27 10	27 30	26 50	301 27 10								
	Star Sirius	327 32 50	32 50	32 30	327 32 43	333 54 27	3	21 41 40	42 20	21 42 00	02 23	21 39 37	... ...
	Star Sirius ...	328 39 00	39 20	38 40	328 39 00	332 48 06	4	20 42 00	42 50	20 42 25	02 29	20 39 56	... ...
	Mark	301 27 10	27 20	26 50	301 27 06								

Star Sirius (α Canis Majoris).

		° ′ ″
Declination S.	=	16 32 41
Latitude N.	=	32 56 40

Computation.

Log. Sin. ... = 9·454485
Log. Cosec. ... = 10·264540
Sum Constant Log. ... = $\bar{1}$·719025

Star..............................

		° ′ ″
Declination ..	=	... ...
Latitude ...	=	... ...

Log. Sin. = ...
Log. Cosec. = ...
Sum Constant Log. = ...

° ′ ″			1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
	Constant Log.	=	$\bar{1}$·719 025	$\bar{1}$·719 025	$\bar{1}$·719 025	$\bar{1}$·719 025	......	......
	Log. Cosec. Altitude	=	10·377 536	10·396 238	10·432 853	10·452 334	......	......
1st = 24 47 12	... Sum = Log. x	=	0·096 561	0·115 263	0·151 878	0·171 359	......	......
2nd = 23 40 35	... ∴ x	=	— 1·2 490	— 1·3 040	— 1·4 187	— 1·4 837	......	......
3rd = 21 39 37	... and $(x-1)$	=	— 2·2 490	— 2·3 040	— 2·4 187	— 2·4 837	......	......
4th = 20 39 56	... Log. $(x-1)$	=	0·351 989	0·362 482	0·383 582	0·395 099	......	......
	Log. tan. Altitude	=	9·664 437	9·641 947	9·598 949	9·570 550	......	......
	Log. tan. Latitude	=	9·811 594	9·811 594	9·811 594	9·811 594	......	......
Azimuths ...	... Sum = Log. cos. A	=	9·828 020	9·816 023	9·794 125	9·783 243	......	......
	A (= Azimuth of Star from North)	=	132° 17′ 59″	130° 53′ 41″	128° 29′ 51″	127° 22′ 42″	...° ...′ ...″	...° ...′ ...″
	Angles between Star & Mark	=	337 41 24	336 17 20	333 54 27	332 48 06	... ...	... ...
	Azimuth of Mark from North	=	205 23 25	205 23 39	205 24 36	205 25 24	... ...	... ...

Mean Azimuth of Mark from N.	=	205 24 16	... ...
Convergency for 28·00 Miles W.	=	+ 15 19	... ...
Working Azimuth from N.	=	205 39 35	... ...

Variation of Needle 3° 23′ 25″ E. ... ...

Example 4.

APPLICABLE TO THE SUN.

6th May 1885. Observation for Azimuth at ⊙ A 623 of Main Circuit No. 13, District B. Length of line 10·00 Chians. Latitude N. 18° 44′ 00,″ Longitude 95° 56′ 30″, Needle Bearing 308° 14′ 00″, Theodolite...inch. No..................by..............

Face.	Object.	Horizontal Verniers. A.	B.	C.	Mean.	Angles between Star & Mark.	Obsns.	Vertical Verniers. Front.	Back.	Mean.	Refn. — Parlx. +	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark	0 00 00	... ...	... ...	... ...								
	Sun E.	130 25 00	... ...	... ...	... .	229 35 00	1	26 56 00	... ...	... ...	... ...	... ...	... ...
	Sun E.	131 03 00	... ...	... ...	... ...	228 57 00	2	27 50 00	... ...	... ...	... ...	... ...	... ...
	Mark	0 00 00	... .	... ...	... ...								
											{ — 1 50 + 0 08		8 A.M.
R	Mark	180 00 00	... ...	... ..	... ...								
	Sun E.	311 32 00	... ...	... ...	... ...	228 28 00	3	27 52 00	... ...	... ...	... ...	... ...	... ...
	Sun E.	311 02 00	... ...	... ...	... ...	228 58 00	4	28 36 00	... ..	... ...	... ...	... ...	... ...
	Mark	180 00 00	... ...	... ..	... ...								
					Genl. Mean.	228 59 30			Genl. Mean.	27 48 30	— 1 42	27 46 48	8 00 00 A.M.

			° ′ ″			° ′ ″			° ′ ″			° ′ ″
A	N. P. D. ...	=	73 27 43	N. P. D. ...	=	... ...	N. P. D. ...	=	... ...	N. P. D. ...	=	... ...
B	Z. D. ...	=	62 13 12	Z. D. ...	=	... ...	Z D. ...	=	... ...	Z D ...	=	... ...
C	Co. Lat. ...	=	71 16 00	Co. Lat. ...	=	... .	Co Lat. ...	=	... .	Co. Lat. ...	=	... ...
D	Sum ...	=	206 56 55	Sum ...	=	... ...	Sum ...	=	... ...	Sum ...	=	... ...
E	Half Sum ...	=	103 28 27	Half Sum ...	=	... ...	Half Sum...	=	... ...	Half Sum ...	=	.. ..

			1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
E	= 103° 28′ 27″	Cosec. =	0·012122	......	......	......	......	......
E—A	= 30 00 44	Cosec. =	0·300870	......	......	......	......	......
E—B	= 41 15 15	Sine =	9·819149	......	......	......	......	......
E—C	= 32 12 27	Sine =	9·726716	......	......	......	......	......
	Sum = Log. tan.^{2} ½A	=	19·858857	......	......	......	......	......
	Log. tan. ½A	=	9·929429	......	......	......	......	......
	½A	=	40° 21′ 55″	...° ...′ ...″	...° ...′ ...″	...° ...′ ...″	...° ...′ ...″	...° ...′ ...″
	∴ Azimuth (A) of Sun	=	80 43 50	... ...	... ...	... ...	... ...	... ...
	Angle between Sun & Mark	=	228 59 30	... ...	... ...	... ...	... ...	... ...
	Azimuth of Mark	=	309 43 20	... ...	... ...	... ...	... ...	... ...

		° ′ ″	° ′ ″
Mean Azimuth of Mark	=	309 43 20	... ...
Convergency for 28·00 Miles E.	=	— 08 15	... ...

Example 5.

Observation and Computation of Azimuths.

(Applicable to the Sun).

8th March 1892. Observation for Azimuth at ⊙ M 491 of Main Circuit No 5. District Thayetmyo, Pyalo circle. Length of Line 19·49 Chains. Latitude N. 19° 13′ 08″; Longitude E. 95° 14′ 12″. Needle Bearing 177° 02′ 30″. Theodolite 5-inch, No. 682, by Thomas Cooke and Sons.

Face.	Object	Horizontal Verniers. A.	Horizontal Verniers. B.	Horizontal Verniers. C.	Horizontal Verniers. Mean.	Angle between Star and Mark.	Observations.	Vertical Verniers. Front.	Vertical Verniers. Back.	Vertical Verniers. Mean.	Refn. — Parlx. +	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark ...	177 02 30	02 30	02 30	177 02 30								
	Sun W. ...	252 00 00	00 00	00 00	252 00 00	285 02 30	1	... ...	... ...	... ...	... ...	... ...	4 25 26
	Sun W. ...	251 41 00	41 00	41 00	251 41 00	285 21 30	2	27 30 30	30 00	27 30 15	... ...	... ...	4 27 50
	Mark ...	177 02 30	02 30	02 30	177 02 30						{ − 01 56 + 00 08 }		
R	Mark ...	357 03 00	02 30	03 00	357 02 50								
	Sun W. ...	72 55 30	55 30	55 30	72 55 30	284 07 20	3	... ...	... ...	... ...	... ...	... ...	4 33 40
	Sun W. ...	72 35 30	35 30	35 00	72 35 30	284 27 20	4	25 35 00	30 00	25 35 00	... ...	... ...	4 36 00
	Mark ...	357 03 00	02 30	03 00	357 02 50								
						284 44 40	..	... .	... ...	26 33 07	− 1 48	26 31 19	4 30 44 P.M.

Observations for additional Azimuth or to α Ursæ Minoris (Polaris).

Face.	Object	A.	B.	C.	Mean.	Angle between Star and Mark.	Observations.	Front.	Back.	Mean.	Refn. — Parlx. +	True Altitude.	Civil time.
L	Mark ...	177 02 30	02 30	02 30	177 02 30								
	Sun W. ...	253 39 30	39 30	39 30	253 39 30	283 23 00	.	.. .. ..	... ...	... .	... ..	... ...	4 40 09
	Sun W. ...	253 20 00	20 00	20 00	253 20 00	283 42 30	5	24 10 00	10 00	24 10 00	... .	.. ..	4 42 27
	Mark ...	177 02 30	02 30	02 30	177 02 30						{ − 02 11 + 00 08 }		
R	Mark ...	357 03 00	02 30	03 00	357 02 50								
	Sun W. ...	74 19 00	19 00	19 00	74 19 00	282 43 50	...	... .	...	. ... ..	... ...	.. ..	4 46 20
	Sun W. ...	73 59 30	60 00	60 00	73 59 50	283 03 00	6	22 42 00	42 00	22 42 00	... ...	... .	4 48 42
	Mark ...	357 03 00	02 30	03 00	357 02 50								
						283 13 05	...	.. ..	.. ...	23 26 00	− 2 06	23 23 54	4 44 25 P.M.

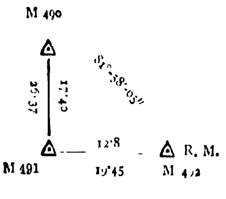
Computation.

Sun W.

		° ′ ″			
Declination S.	=	4 36 46	Log. sin.	=	8·905 371
Latitude N.	=	19 13 08	Log. cosec.	=	0·482 570
			Sum = Constant log.	=	9·387 941

Sun W.

		° ′ ″			
Declination S.	=	4 36 31	Log. sin	=	8·904 979
Latitude N.	=	19 13 08	Log. cosec	=	0·482 570
			Sum = Constant log.	=	9·387 549



		1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
Constant Log.	=	......	9·387941	......	.....	9·387549	......
Log. cosec. Altitude	=	......	0·350131	......	......	0·401077	......
Sum = Log. *x*	=	......	$\bar{1}$·738072	......	......	$\bar{1}$·788626	......
x	=	......	− 0·5471	......	......	− 0·6146	......
x—1	=	.. .	− 1·5471	.....	......	− 0·6146	......
Log. „	=	......	0·189 518	.....	......	0·208 065	......
Log. tan. Altitude	=	......	9·698 153	......	......	9·636 191	......
Log. tan. Latitude	=	......	9·542 335	......	......	9·542 335	......
Sum = Log. cos. A.	=	......	9·430 006	......	......	9·386 591	......
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
A (= Azimuth of Star from North)	=	... ...	105 36 51	... ...	... ...	104 05 47	... ...
Angle between Star and Mark	=	... ...	284 44 40	... ...	... ...	283 13 05	... ...
Azimuth of Mark from North	=	... ...	179 07 49	... ...	... ...	179 07 18	... ...

Mean Azimuth of Mark from N.	=	179 07 33	... ...
Convergency for 17·41 Miles (Table XLII*) W.	=	+ 05 16	... ...
Variation of Needle 2° 10′ 19″ ... ... Working Azimuth from N.	=	179 12 49	... ...

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* "Calculating Tables" (5th edition).

Example 6.

Observation and Computation of Azimuths.

(Applicable to the Sun.)

17th April 1892. Observation for Azimuth at ⊙ M572 of Main Circuit No. 5. District Thayetmyo, Nyaungbinzcik circle. Length of Line 14·45 Chains. Latitude N. 19° 01′ 07″. Longitude E. 95° 10′ 03″. Needle Bearing 161° 31′ 20″. Theodolite 5-inch, No. 682, by Thomas Cooke and Sons.

Face.	Object.	Horizontal Verniers. A.	Horizontal Verniers. B.	Horizontal Verniers. C.	Horizontal Verniers. Mean.	Angle between Star and Mark.	Observations.	Vertical Verniers. Front.	Vertical Verniers. Back.	Vertical Verniers. Mean.	Refn. — Parlx. +	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″		° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L.	Mark ...	161 31 30	31 00	31 30	161 31 20								
	Sun U. L. ...	83 30 10	30 00	30 00	83 30 00	78 01 20	1	... ...	... ...	... ...	... ...	... ...	7 08 56
	Sun L. L. ...	83 06 30	06 30	06 30	83 06 30	78 24 50	2	18 11 30	11 00	18 11 15	... ...	... ...	7 11 12
	Mark ...	161 31 30	31 30	31 00	161 31 20								
											{ −02 44 +00 08 }		
R.	Mark ...	341 31 00	30 30	31 00	341 30 50								
	Sun U. L. ...	264 15 30	15 00	15 30	264 15 20	77 15 30	3	... ...	... ...	... ...	... ...	... ...	7 19 15
	Sun L. L ...	263 51 30	51 30	51 30	263 51 30	77 39 20	4	20 35 00	34 30	20 34 45	... ...	... ...	7 21 31
	Mark ...	341 31 00	30 30	31 00	341 30 50								
						77 50 15	...	... ...	... ...	19 23 00	−02 36	19 20 24	7 15 14 A.M.

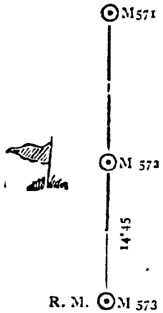
Observations for additional Azimuth or to α Ursæ Minoris (Polaris).

Face.	Object.	Horizontal Verniers. A.	Horizontal Verniers. B.	Horizontal Verniers. C.	Horizontal Verniers. Mean.	Angle between Star and Mark.	Observations.	Vertical Verniers. Front.	Vertical Verniers. Back.	Vertical Verniers. Mean.	Refn. — Parlx. +	True Altitude.	Civil time.
L.	Mark ...	161 31 30	31 00	31 30	161 31 20								
	Sun U. L. ...	84 49 00	49 00	49 30	84 49 10	76 42 10	...	... ...	... ...	... ...	... ...	... ...	7 26 41
	Sun L. L. ...	84 24 30	25 00	25 00	84 24 50	77 06 30	5	22 22 20	22 00	22 22 10	... ...	... ...	7 28 55
	Mark ...	161 31 30	31 00	31 30	161 31 20								
											{ −02 14 +00 08 }		
R.	Mark ...	341 31 00	30 30	31 00	341 30 50								
	Sun U. L. ...	265 29 00	29 00	29 30	265 29 10	76 01 40	..	... ...	... ...	... ...	... ...	... ...	7 35 40
	Sun L. L. ...	265 04 30	04 30	04 30	265 04 30	76 26 20	6	24 27 00	27 00	24 27 00	... ...	... ...	7 37 55
	Mark ...	341 31 00	30 30	31 00	341 30 50								
						76 34 10	...	... ...	... ...	23 24 35	−02 06	23 22 29	7 32 18 A.M.

Computation.

		Sun E. ° ′ ″			
Declination N. ...	=	10 34 46	Log. sin. ...	=	9·263 870
Latitude N. ...	=	19 01 07	Log. cosec. ...	=	0·486 948
			Sum = Constant log. ...	=	9·750 818

		Sun E. ° ′ ″			
Declination N. ...	=	10 34 41	Log. sin. ...	=	9·263813
Latitude N. ...	=	19 01 07	Log. cosec. ...	=	0·486948
			Sum = Constant log. ...	=	9·750761



		1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
Constant log.	=	......	9·750 818	......	9·750 761	......	......
Log. cosec. Altitude	=	......	0·479 945	......	0·401 491	......	......
Sum = Log. x	=	......	0·230 763	......	0·152 252	......	......
x	=	......	+ 1·7012	......	+ 1·4199	......	......
$x - 1$	=	......	+ 0·7012	......	+ 0·4199	......	......
Log. „	=	......	$\bar{1}$·845 842	......	$\bar{1}$·623 146	......	......
Log. tan. Altitude	=	......	9·545 281	......	9·635 700	......	......
Log. tan. Latitude	=	......	9·537 430	......	9·537 430	......	......
Sum = Log. cos. A	=	......	8,928 553	......	8·796 276	......	......
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
A (= Azimuth of Star from North)	=	... ...	85 08 01	... ...	86 24 48	... ...	... ...
Angle between Star and Mark	=	... ...	77 50 15	... ...	76 34 10	... ...	... ...
Azimuth of Mark from North	=	... ...	162 58 16	... ...	162 58 58	... ...	... ...

		° ′ ″	
Mean Azimuth of Mark from N.	=	162 58 37	... ...
Convergency for 2·00 Miles (Table XLII*) W.	=	+ 06 35	... ...
Variation of Needle 1° 33′ 52″ .. .. Working Azimuth from N.	=	163 05 12	... ...

* "Calculating Tables" (5th edition).

Example 7.

19th April 1900. Computation of Azimuth at ⊙ Y^7 of Main Circuit No. 8. District Lower Chindwin. Length of Line Y^7 X^7 = 12·29 Chains.
Latitude N. 22° 31′ 29″. Needle Bearing 87° 13′ 10″. Theodolite 6-inch, No. 1028, by Thomas Cooke and Sons.

Star E. or W.	Aldebaran W.	......	Aldebaran W.	......	Rigel W.	......	Rigel W.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mean Zenith Distance	63 26 40	... ...	64 18 50	... ...	67 12 20	... ...	67 56 00
Refraction	+ 02 56	... ...	+ 02 01	... ...	+ 02 19	... ...	+ 02 24
Corrected Z. D. ... = ζ	63 29 36	... ...	64 20 51	... ...	67 14 39	... ...	67 58 24
Co-latitude ... = γ	67 28 31	... ...	67 28 31	... ...	67 28 31	.. ..	67 28 31
N. P. Distance ... = Δ	73 41 31	... ...	73 41 31	... ...	98 19 10	... ...	98 19 10
Sum = 2s	204 39 38	... ...	205 30 53	... ...	233 02 20	... ...	233 46 05
s	102 19 49	... ...	102 45 26	... ...	116 31 10	... ...	116 53 02
s — Δ	28 38 18	... ...	29 03 55	... ...	18 12 00	... .	18 33 52
s — γ	34 51 18	... ...	35 16 55	... ...	49 02 39	... ...	49 24 31
s — ζ	38 50 13	... ...	38 24 35	... ...	49 16 31	... ...	48 54 38
Log. cosec. s	0·010 135	... ...	0·010 855	... ...	0·048 282	... ...	0·049 672
,, ,, (s — Δ)	0·319 412	... ...	0·313 537	... ...	0·505 379	... ...	0·497 066
,, sin. (s — γ)	9·757 017	... ...	9·761 628	... .	9·878 071	... ...	9·880 453
,, ,, (s — ζ)	9·797 341	... ...	9·793 288	... ...	9·879 585	... ...	9·877 190
Sum = Log. tan.² ½ A	19·883 905	... ...	19·879 308	... ...	20·311 317	... ...	20·304 381
,, tan. ½ A	9·941 953	... ...	9·939 654	... ...	10·155 659	... ...	10·152 190
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
½ A	41 10 56	... ...	41 01 55	... ...	55 03 18	. . ..	54 50 23
A	82 21 52	... .	82 03 50	... ...	110 06 36	... ...	109 40 46
Angle R. M. and Star * ...	171 25 40	... ...	171 08 00	... ...	199 10 00	... ...	198 44 40
Azimuth of R. M. from N. † ...	89 03 48	89 03 59	89 04 10	... ...	89 03 24	... ...	89 03 54

		° ′ ″
	Mean Azimuth of Mark from N. =	89 03 49
	‡ Convergency for 1,521 Chs. W. (Table XLII)§ =	+ 06 51
Variation of Needle E. 1° 57′ 30″	... Working Azimuth from N. =	89 10 40

Example 8.

24th April 1900. Computation of Azimuth at ⊙ P¹² of Main Circuit No. 8. District Lower Chindwin. Length of Line P¹² O¹² = 8·24 Chains.
Latitude N. 22° 26′ 58″. Needle Bearing 272° 28′ 00″. Theodolite 6-inch, No. 1028, by Thomas Cooke and Sons.

Star E. or W.	Betelgeux W.	......	Betelgeux W.	......	Capella W.	......	Capella W.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mean Zenith Distance	65 32 10	... ...	66 11 00	... ...	65 33 10	... ...	66 56 40
Refraction	+ 02 07	... ...	+ 02 11	... ...	+ 02 13	... ...	+ 02 18
Corrected Z. D. ... = ζ	65 34 17	... ...	66 13 11	... ...	66 35 23	... ...	66 58 58
Co-latitude ... = γ	67 33 02	... ...	67 33 02	... ...	67 33 02	... ...	67 33 02
N. P. Distance ... = △	82 36 48	... ...	82 36 48	... ...	44 06 09	... ...	44 06 09
Sum = 2*s*	215 44 07	... ...	210 23 01	... ...	178 14 34	... ...	178 38 09
s	107 52 03	... ...	108 11 30	... ...	89 07 17	... ...	89 19 04
s — △	25 15 15	... ...	25 34 42	... ...	45 01 08	... ...	45 12 55
s — γ	40 19 01	... ...	40 38 28	... ...	21 34 25	... ...	21 46 02
s — ζ	42 27 40	... ...	41 58 19	... ...	22 31 54	... ...	22 20 06
Log. cosec. *s*	0·021 479	... ...	0·022 268	... ...	0·000 051	... ...	0·000 031
" " (*s* — △)	0·369 944	... ...	0·364 773	... ...	0·150 372	... ...	0·148 889
" sin. (*s* — γ)	9·810 914	... ...	9·813 704	... ...	9·565 436	... ...	9·569 183
" " (*s* — ζ)	9·827 991	... ...	9·825 275	... ...	9·583 419	... ...	9·579 808
Sum = Log. tan.² ½ *A*	20·030 318	... ...	20·026 110	... ...	19·299 278	... ...	19·297 911
" tan. ½ *A*	10·015 159	... ...	10·013 055	... ...	9·649 639	... ...	9·648 955
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
½ *A*	45 59 59	... ...	45 51 40	... ...	24 03 06	... ...	24 01 06
A	91 59 58	... ...	91 43 20	... ...	48 06 12	... ...	48 02 12
Angle R. M. and Star * ...	6 21 30	... ...	6 05 00	... ...	322 27 40	... ...	322 24 00
Azimuth of R. M. from N. † ...	274 21 32	274 21 36	274 21 40	... ...	274 21 28	274 21 38	274 21 48

Mean Azimuth of Mark from N. = 274 21 37
‡ Convergency for 1,674 Chs. W. (Table XLII) § = + 07 31
Variation of Needle E. 2° 01′ 08″ ... ... Working Azimuth from N. = 274 29 08

* Aways *subtract* the reading of the Star from that of the R. M. to get the angle R. M. and Star.

† When the Star is $\frac{\text{east}}{\text{west}}$ of the meridian, the azimuth of the Star is to be $\frac{\text{added to}}{\text{subtracted from}}$ the angle R. M. and Star; if the result is *negative*, substract it from 360°; and if positive but greater than 360,° educt 360° from it. The result thus obtained is the azimuth of R. M reckoned from north by east.

‡ To be $\frac{\text{added to}}{\text{subtracted from}}$ azimuth observed $\frac{\text{west}}{\text{east}}$ of origin.

§ "Calculating Tables" (5th edition).

71.[*] PROBLEM XV.—*To determine the azimuth at a place by observation of Stars of North Declination only.* (Figs. 505, 506.)[†]

A specimen alternative form (which contains the solution, according to the new method, of Example 2 of Art. 70) of computation of the Azimuths of North Declination from observations of Altitude, prepared by Captain H. Lees Smith, is given below. This form is much simpler and less liable to mistakes, than the form given in (Art. 70), and in it a great deal can be worked out before the observation; also, the formula given in (Art. 70) requires frequent turning from one Log-book to another in consequence of the fact that angular and numerical values are both used in the same computation, while in the formula given below an advantage is gained by employing angular values only.

It is of little disadvantage that this formula is applicable to Stars of North Declination only, as in practice greater accuracy in Azimuth computations may be expected from Stars of Northern Declination, than from those of Southern Declination.

The following is the explanation of the formula for the computation of Azimuths of Stars of North Declination:—

When the Azimuth is less than 90° (see Fig. 505).

" " " " *greater than 90° (see* Fig. 506).

ABC is a spherical triangle, in which A is the Zenith,
B is the Pole,
C is the Star observed;
hence the sides a = co-Declination or N.P.D.,
b = co-Altitude or Z.D.,
c = co-Latitude.

BZ is an arc perpendicular to the side (b) from Pole B.
M is the middle point of (b).

Let the arc MZ = θ, then (Fig. 1) $AZ = \frac{b}{2} + \theta = \frac{ZD}{2} + \theta$,

$$CZ = \frac{b}{2} \quad \theta = \frac{ZD}{2} - \theta.$$

In the right $\angle^{d}$ spherical $\triangle$ BCZ, Cos. CZ × Cos. BZ = Cos. a ⎫ By Napier's Cir-
In do. do. ABZ, Cos. AZ × Cos. BZ = Cos. c ⎭ cular Parts.

$$\therefore \frac{\text{Cos. CZ}}{\text{Cos. AZ}} = \frac{\text{Cos.}\left(\frac{b}{2} - \theta\right)}{\text{Cos.}\left(\frac{b}{2} + \theta\right)} = \frac{\text{Cos. } a}{\text{Cos. } c}$$

$$\therefore \frac{\text{Cos.}\left(\frac{b}{2} - \theta\right) - \text{Cos.}\left(\frac{b}{2} + \theta\right)}{\text{Cos.}\left(\frac{b}{2} - \theta\right) + \text{Cos.}\left(\frac{b}{2} + \theta\right)} = \frac{\text{Cos. } a - \text{Cos. } c}{\text{Cos. } b + \text{Cos. } c}$$

$$\therefore \text{tan. } \theta \times \text{tan. } \frac{b}{2} = \text{tan. } \frac{a-c}{2} \times \text{tan. } \frac{a+c}{2}.$$

$$\therefore \text{tan. } \theta = \text{tan. } \frac{a-c}{2} \times \text{tan. } \frac{a+c}{2} \times \text{cot. } \frac{b}{2} =$$

$$\text{tan. } \frac{\text{co-Decn.} - \text{co-Lat.}}{2} \times \text{tan.}$$

$$\frac{\text{co-Decn.} + \text{co-Lat.}}{2} \times \text{cot. } \frac{ZD}{2} \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

again, in the right $\angle^{d}$ spherical $\triangle$ BZA,

OBSERVATIONS.

[*] Extracts from a Circular Order of the Survey of India.

[†] For the logarithmic calculations and Refraction and Parallax values occurring in the Examples (three in number) to the subject of this Article, consult the "Calculating Tables" (5th edition), Tables XXXI (Log. Numbers), XXXII (Log. Sines and Tangents), XXXVIII (Refraction) and XXXIX (Parallax). For the correction for Convergency of Meridian in Example I, refer to Table XCII of the same book. The Tables given in the 'Nautical Almanac' for finding the Latitude of a place by Observations of the Pole Star (α Ursæ Minoris) are founded on the following formula: $l = a - p \cos h + \frac{1}{2} \sin 1'' (p \sin h)^2 \tan a$, where l denotes the latitude, a the true altitude of the Star, p the apparent polar distance, and $h = S - a$, the hour angle of the Star; S being the local sidereal time of Observation and α the apparent right ascension of the Star. Table I contains $- p \cos h$ or the *first correction*; and Table II., $\frac{1}{2} \sin 1'' (p \sin h)^2 \tan a$ or the *second correction*, assuming $p = 71' \, 0''$, and $\alpha = 21° \, 30'$. Table III, which depends on the difference between the true and assumed values of p and α, contains the *third correction* increased by 1′ for the purpose of rendering the quantities additive. A fourth term $- \frac{1}{3} \sin^2 1'' (p \cos h) (p \sin h)^2$ is omitted, its greatest value being less than 0·5.″

$$\text{Cos. A} = \text{cot. c} \times \text{tan. AZ} = \text{cot. co-Lat.} \times (\text{tan.} \tfrac{ZD}{2} + \theta).$$

$$\therefore \text{Cos. Azimuth} = \text{tan. Lat.} \times \text{tan.} \left(\tfrac{ZD}{2} + \theta\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Equations 1 and 2 are those on which the formula depends.

N.B.—In figure 2 where the Azimuth is greater than 90°, $AZ = (\theta - \frac{ZD}{2S})$ and Cos. Azimuth $= \text{tan. Lat.} \times \text{tan.} (\theta - \frac{ZD}{2})$.

An inspection of the above figures will make clear the following rules :—

If the Declination is greater than the Latitude, the ***sum*** of the arcs θ and $\frac{ZD}{2}$ is to be used in the computation.

If the Declination is less than the Latitude, the *difference* of the arcs θ and $\frac{ZD}{2}$ is to be used.

The Azimuth is either greater than, equal to, or less than 90°, according as θ is greater than, equal to, or less than $\frac{ZD}{2}$

Example 1. (Same as Example 2 of Art. 70.)

(Applicable to Stars of North Declination only.)

22nd November 1876. Observation for Azimuth at ⊙ O of Main Circuit No 12 District Bannu. Length of Line 19 17 Chains.
Latitude N. 33° 01′ 25″. Needle Bearing 265° 29′ 30″. Theodolite 8-inch, No. 256, by Thomas Cooke and Sons.

Face.	Object	Horizontal Verniers. A.	B.	C.	Mean.	Angles between Star & Mark.	Obsns	Vertical Verniers. Front.	Back.	Mean.	Refr. —	True Altitude.	Civil time.
		° ′ ″	′ ″	′ ″	° ′ ″	′ ″		′ ″	′ ″	° ′ ″	′ ″	° ′ ″	H. M. S.
L	Mark	219 04 10	04 20	03 20	219 04 00								
	Star Capella	357 05 50	06 00	05 50	357 05 53	221 58 07	1	18 28 20	27 30	18 27 55	2 49	18 25 06	... ...
	Star Capella	357 32 30	32 40	32 20	357 32 30	221 31 33	2	19 13 20	12 30	19 12 55	2 43	19 10 12	... ...
	Mark	219 04 10	04 30	03 30	219 04 03								
R	Mark	39 04 10	04 50	04 00	39 04 20								
	Star Capella	178 03 10	03 10	03 00	178 03 06	221 01 14	3	20 07 00	07 50	20 07 25	2 34	20 04 51	... ...
	Star Capella	178 30 40	30 40	30 20	178 30 33	220 33 50	4	20 59 00	59 50	20 59 25	2 27	20 56 58	... ...
	Mark	39 04 10	05 00	04 00	39 04 23								

Additional observations, or to (a Ursæ Minoris) Polaris.

Face.	Object	A.	B.	C.	Mean.	Angles between Star & Mark.	Obsns	Front.	Back.	Mean.	Refr.	True Altitude.	Civil time.
L	Mark	... ...	... ...	... ...	... ...	... ...	5	... ...	... ...	... ...	... ...	... ...	... ...
	Star	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...
	Star	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...
	Mark	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...
R	Mark	... ...	... ...	... ...	... ...	... ...	6	... ...	... ...	... ...	... ...	... ...	... ...
	Star	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...
	Star	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...
	Mark	... ...	... ...	... ...	... ...	... ...		... ...	... ...	... ...	... ...	... ...	... ...

Star Capella (a Aurigæ). *Computation.* *Star*......................................

	° ′ ″		° ′ ″								
Dec. N. ... =	45 52 21	Co-Decn. ... =	44 07 39	L. tan. ½ Sum =	10·084 698	Dec. N. ... ... =	......	Co-Decn. =	... ...	L. tan. ½ Sum =	... ...
Lat. N. ... =	33 01 25	Co-Lat. ... =	56 58 35	L. tan. ½ Diff. =	9·051 538	Lat. N. ... ... =	......	Co-Lat. =	... ...	L. tan. ½ Diff. =	... ...
Diff. ... =	12 50 56	Sum ... =	101 06 14	Const. Log. =	$\bar{1}$·136 236	Diff. ... ... =	......	Sum =	... ...	Const. Log. =	... ...
½ Diff. ... =	6 25 28	½ Sum ... =	50 33 07			½ Diff. ... ... =	......	½ Sum =	... ...		

	1st Observation.	2nd Observation.	3rd Observation.	4th Observation.	5th Observation.	6th Observation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Co-Altitude or Z. D. ... ... =	71 34 54	70 49 48	69 55 09	69 03 02	... ...	... ...
$\frac{Z.D.}{2}$ ... ... =	35 47 27	35 24 54	34 57 35	34 31 31	... ...	... ...
Log. Cot. $\frac{Z.D.}{2}$ ... ... =	10·142 077	10·148 095	10·155 423	10·162 455	......	......
Constant Log. ... ... =	$\bar{1}$·136 236	$\bar{1}$·136 236	$\bar{1}$·136 236	$\bar{1}$·136 236	......	......
Log. tan. θ ... ... =	9·278 313	9·284 331	9·291 659	9·298 691	......	......
∴ θ =	10° 44′ 50″	10° 53′ 37″	11° 04′ 28″	11° 15′ 03″	...° ...′ ...″	...° ...′ ...″
& θ ± $\frac{Z.D.}{2}$ =	46 32 17	46 18 31	46 02 03	45 46 34	... ...	... ...
Log. tan. θ ± $\frac{Z.D.}{2}$ =	10·023 328	10·019 845	10·015 682	10·011 767	......	......
Log. tan. Lat. =	9·812 909	9·812 909	9·812 909	9·812 909	......	......
Azimuth ... ... Log. Cos. of =	9·836 237	9·832 754	9·828 591	9·824 676	......	......
∴ Azimuth of Star =	46° 41′ 48″	47° 07′ 35″	47° 37′ 54″	48° 05′ 56″	...° ...′ ...″	...° ...′ ...″
Angle between Star and Mark =	221 58 07	221 31 33	221 01 14	220 33 50	... ...	... ...
Azimuth of Mark =	268 39 55	268 39 08	268 39 08	268 39 46	... ...	... ...
Mean Azimuth of Mark ...				268 39 29		... ...
Convergency for 19·45 Miles W. ...				+ 10 38		... ..
Variation of Needle 3° 20′ 37″ E. ... ... Working Azimuth ...				268 50 07		... ...

Computation for *α Ursæ Minoris* when taken at Elongation.

Declination North ... = ...° ...′ ...″ Log. Cos. = ...
Latitude ... = ... Log. Sec. = ...
Azimuth Log. Sin. = ...
Azimuth of Star = ...° ...′ ...″
Mean Angle between Star and Mark = ...
Azimuth of Mark = ...
Convergency for.........Miles...... = ...
Working Azimuth = ...

Computation *for Latitude vide* N. A. Tables.

True Altitude of Polaris .. ..° ..′ ...″
Deduct as per rule ... ... — ... ...
Reduced Altitude ... ...
First Correction ... ...
First Approximate Latitude . ...
Second Correction .. ..
Second Approximate Latitude .. ..
Third Correction ... ...
Latitude of Place ... ...

NOTE.—Always *subtract* the reading of the Star from that of the Mark. When the Star's Declination is greater than the Latitude, the sum of θ and $\frac{Z.D.}{2}$ is to be used for Computation. When the Declination is less than the Latitude, the difference of θ and $\frac{Z.D.}{2}$ is to be used. The Azimuth is greater or less than 90, according as θ is greater or less than $\frac{Z.D.}{2}$. When the Star observed is to the *East* of the Meridian, *add together* the Azimuth of the Star and the Angle between the Star and Mark. When the Star observed is to the *West* of the Meridian, *subtract* the Azimuth of the Star from the Angle between the Star and Mark. Convergency is *additive* to Azimuths observed *West* of Origin, and *subtractive* from those observed *East* of Origin.

N.B.—When *Polaris* is observed for Azimuth, a *check* observation must be taken to some *East* or *West* Star.

Example 2 (same as Example 2 of Art. 70). *Another method.*

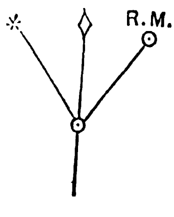
Computation of Azimuth (Horizontal and Vertical Angles observed simultaneously).

Station ... =	⊙ O of M.C. No. 12.			⊙ O of M.C. No. 12.			⊙ O of M.C. No. 12.			⊙ O of M.C. No. 12.		
Star, E. or W. =	* Capella, E.			* Capella, E.			* Capella, E.			* Capella, E.		
Date Ast. ... =	22nd November 1876.			22nd November 1876.			22nd November 1876.			22nd November 1876.		
	°	′	″	°	′	″	°	′	″	°	′	″
Mean Zenith Distance =	71	32	05	70	47	05	69	52	35	69	00	35
Refraction =	+	02	49	+	02	43	+	02	34	+	02	27
Corrected Z. D. (ζ) =	71	34	54	70	49	48	69	55	09	69	03	02
Co-latitude (γ) =	56	58	35	56	58	35	56	58	35	56	58	35
N. P. Distance (Δ) =	44	07	39	44	07	39	44	07	39	44	07	39
Sum =	172	41	08	171	56	02	171	01	23	170	09	16
Half „ (s) =	86	20	34	85	58	01	85	30	42	85	04	38
s − Δ =	42	12	55	41	50	22	41	23	03	40	56	59
s − γ =	29	21	59	28	59	26	28	32	07	28	06	03
s − ζ =	14	45	40	15	08	13	15	35	33	16	01	36
Log. cosec. s =	0·000 886			0·001 077			0·001 334			0·001 605		
„ „ s − Δ =	0·172 683			0·175 841			0·179 730			0·183 495		
Log. sin. s − γ =	9·690 544			9·685 442			9·679 155			9·673 044		
„ „ s − ζ =	9·406 182			9·416 872			9·429 419			9·441 042		
Sum = Log. tan.² ½ A =	19·270 205			19·279 215			19·289 638			19·299 186		
„ tan. ½ A =	9·635 148			9·639 607			9·644 819			9·649 593		
	°	′	″	°	′	″	°	′	″	°	′	″
½ A =	23	20	54	23	33	47	23	48	58	24	02	58
A =	46	41	48	47	07	34	47	37	56	48	05	56
Angle R. M. and Star =	221	58	07	221	31	33	221	01	14	220	33	50
Azimuth of R. M. from N. =	268	39	55	268	39	07	268	39	10	268	39	46

Example 3. (*When the Time is known*). On the 10th May 1893 (Astronomical date), in λ. 19° 26′ 02″, L. 93° 34′ 39″, observations for azimuth were taken at ⊙ B 24 of Main Circuit No. 1, District Bassein, with a 5″ theodolite by Cooke and Sons. Length of line = 13·05 chains: Needle bearing = 214° 04′ 00″. True M. T. of observations: 7*h.* 51*m.* 11*s.*, 7*h.* 54*m.* 6*s.*, 8*h.* 3*m.* 19*s.*, and 8*h.* 07*m.* 15*s.*, respectively. [Star: R. A. = 5*h.* 51*m.* 40*s.*; Dec. N. = 44° 56′ 22″.]

Computation of Azimuth from Star Observations when the Time is known.

Station ... =	⊙ B 24 of M. C. No. 1.			⊙ B 24 of M. C. No. 1.		
Star, E. or W. =	* β Aurigæ, E.			* β Aurigæ, E.		
Date Ast. ... =	10th May 1893.			10th May 1893.		
Local S. T. of Obsn.	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*
(θ). =	11	01	32	11	04	28
R. A. of Star (α). =	5	51	40	5	51	40
θ − α (t). ... =	5	09	52	5	12	48
t in Arc ... =	77°	28′	00″	78°	12′	00″
Co-latitude (γ) =	70	33	58	70	33	58
N. P. D. (Δ) =	45	03	38	45	03	38
Log. tan. Δ =	10·000 838			10·000 838		
„ cos. t =	9·336 475			9·310 685		
Sum = Log. tan. φ =	9·337 313			9·311 523		
φ =	12°	16′	00″	11°	34′	45″
γ − φ =	58	17	58	58	59	13
Log. sin. (γ − φ) =	9·929 830			9·933 007		
„ cot. t =	9·346 949			9·319 961		
„ cosec. φ =	0·672 719			0·697 405		
Sum = Log. cot. A =	9·949 498			9·950 373		



		°	′	″	°	′	″
A	=	48	19	47	48	15	59
Angle R. M. and Star	=	72	34	40	72	31	40
Azimuth of R. M. from N.	=	24	14	53	24	15	41

72. PROBLEM XVI.—*To determine the direction of the meridian at a place, by observations of a star at equal altitudes.*

Rule: Plant the theodolite over the station of observation, at which the meridian is desired to be laid down; put it carefully through its temporary adjustments. Select a bright star in the eastern hemisphere, which will describe a good-sized arc during the night. Set vernier A of the horizontal plate to zero, and by means of the lower horizontal and vertical tangent screws, intersect the star. Now release the upper horizontal clamp, and turn the telescope to the western hemisphere, where allow it to remain until the same star descends to the same altitude on the other side of the meridian, when, getting into the field of the telescope, you follow it by the upper horizontal tangent screw until again intersected. Next, read vernier A of the horizontal plate, which will be the arc the star has described between the eastern and western observations. Now set vernier A to half the above reading, and the telescope will point in the direction of the true meridian.*

73. PROBLEM XVII.—*To find the latitude of a place, by observations to a celestial object; the longitude of the place and the meridian altitude of the object being known.*

Rule: Correct the observed meridian altitude of the object (by Prob. XI or XII, Arts. 67—68). Subtract its true meridian altitude thus found from 90° for its zenith distance, which mark N. or S. according as the zenith is north or south of the celestial object. Next look up for the object's declination N. or S. (from the 'Nautical Almanac'), correcting it for longitude in case the sun is the object observed. Then, if the zenith distance and declination be both north or both south, their *sum* will be the latitude sought, and of the same name; but if one be north and the other south, their *difference* will be the latitude, of the same name as the greater.†

Example (1): At a place in longitude 85° 35′ E., the meridian altitude of the sun's centre ‡ on the 31st December 1880 was observed to be S. 57° 22′ 20″; find the latitude of the place.

$$\begin{array}{lr}
\text{Observed altitude of }\odot\text{'s centre} \quad \ldots & = 57^\circ\ 22'\ 20''\ \text{S.}\\
\left.\begin{array}{l}\text{Correction for refraction} \;=\; -\ 37''\\ \text{,,} \qquad \text{,, parallax} \;=\; +\ 5''\end{array}\right\} & = -\ \ 00\ \ 32\\
\text{True altitude of sun} \quad \ldots & = \underline{57\ \ 21\ \ 48\ \ \text{S.}}\\
\text{Zenith distance} \;=\; (90^\circ - 57^\circ\ 21'\ 48'') & = 32^\circ\ 38'\ 12''\ \text{N.}\\
\left.\begin{array}{l}\text{Sun's dec. 31st Decr. 1880}\\ \quad\text{(N.A., p. II)} \;=\; \text{S. } 23\ \ 03\ \ 25\\ \text{Correction for long. E. } 85^\circ\ 35'\\ \quad(= 5h.\ 42m.\ 20s.) = + \qquad 01\ \ 04\ \S\end{array}\right\} & = 23\ \ 04\ \ 29\ \ \text{S.}\\
\text{Latitude required} & = \overline{9\ \ 33\ \ 43\ \ \text{N.}}
\end{array}$$

OBSERVATIONS.

* The above method for finding the meridian is simple, but very tedious, as the observer may have to sit up a great part of the night waiting for the descent of the star.

† Conversely, if the latitude of a place and the declination of a celestial object are given, the meredian altitude may be found thus: Altitude (reckoned from S.) = co-latitude — South Declination.
= co-latitude + North Declination.
If the result is in excess of 90°, subtract it from 180° to obtain the altitude from the N. side.
Or, Altitude (reckoned from N.) = 90° + latitude — North Declination.
= 90° + latitude + South Declination.

‡ If the upper or lower limb of the sun was only observed, the correction for semi-diameter would also have to be applied to the observed altitude.

§ Dec. 30th Decr. = 23° 07′ 54″
,, 31st ,, = 23 03 25

Diff. in 24 hours = 04′ 29″ = 269″
∴ Diff. for 5*h*. 42*m*. 20*s*. (corresponding to long. 85° 35′) = 64″.

Example (2): At a place in longitude 90° 30′ E., the meridian altitude of Sirius (α Canis Majoris), on the 15th May, 1880, was observed to be S. 24° 56′ 30″; find the latitude of the place of observation.

Observed altitude of star ... ... = 24° 56′ 30″ S.
Correction for refraction ... ... = − 02 04
True altitude of star = 24 54 26 S.
Zenith distance = (90°—24° 54′ 26″) = 65 05 34 N.
Star's Dec. 15th May 1880 (N.A. App. places) = 16 33 19 S.
Latitude required = 48 32 15 N.

Note.—To find the latitude by observations of the two meridian altitudes of a circumpolar star. (See Fig. 480). Let Z be the zenith, P the pole, HO the horizon, and α the altitude BO of a star at its upper transit, β the altitude DO of the same star at its lower transit; then, PO $= \frac{1}{2}$ (BO + DO), or, *lat.* $= \frac{1}{2} (\alpha + \beta)$. In this method no previous knowledge of the star's declination is required. The observed altitudes must be corrected for refraction.

74. PROBLEM XVIII.—*To find the latitude of a place by observations of the pole star (Polaris) out of the meridian; the longitude of the place, the time of observation, and the altitude of the star being known.**

Rule: *1st.* From the corrected altitude (*i.e.*, the observed altitude corrected for refraction, by Prob. XII, Art. 68) subtract 1′.

2nd. Reduce the mean time of observation at the place to the corresponding sidereal time (by Prob. V, Art. 61).

3rd. With this sidereal time take out the *first correction* from Table I of the N.A., Tables of corrections) with its proper sign. If the sign be +, the correction must be *added* to the reduced altitude; but if it be −, it must be *subtracted*; the result will give an *approximate latitude.*

4th. With this approximate latitude and sidereal time of observation, take out (from Table II of the N.A., Tables of corrections) the *second correction*; and with the day of the month and the same sidereal time, take out (from Table III of the N.A., Tables of corrections) the *third correction.* These two corrections *added* to the approximate latitude, will give the latitude of the place.

Example (1): On 6th March 1880, in longitude 37° W., at 7*h*. 43*m*. 35*s*. P.M. mean civil time, the altitude of the Pole Star (*Polaris*) when corrected for refraction, is 46° 17′ 28″. Required the latitude of the place.

Mean time = 7*h*. 43*m*. 35*s*.
Diff. long. W. (37°) in time = + 2 28 00
Greenwich mean time = 10 11 35

Sidereal time at Greenwich mean noon (N. A., Table II). = 22*h*. 58*m*. 23*s*.
Sidereal time corresponding to mean time at place = 7*h*. 43*m*. 35*s*. (mean time) + 1*m*. 41*s*. (acceleration for 10*h*. 12*m*.) = 7 45 16
Sidereal time of observation = 6 43 39

° ′ ″
Corrected altitude ... ... = 46 17 28
Subtract ... ... = − 01 00
Reduced altitude ... = 46 16 28
With *Argument* † 6*h*. 43*m*. 39*s*., *First correction* = − 0 10 54

OBSERVATIONS.

* This is of course only applicable to northern latitudes. In the southern hemisphere there is no star sufficiently near to the south pole to be made available in thus determining the latitude. If the altitude of the pole star can be taken when on the meridian, its polar distance added to the altitude, if the star is below the pole, or subtracted therefrom, if above the pole, will give at once the latitude of the place of observation.

† The *argument* of a table is the known quantity which serves to determine the value of some dependent function.

			°	′	″
	Approximate latitude ...	=	46	05	34
Arguments, 46° 17′ 6*h*. 44*m*.	*Second correction*	= +		00	58
Arguments, 6th March 1880 6*h*. 44*m*.	*Third correction*	= +		01	22
	Latitude of the place ...	=	46	07	54 N.

Example (2): Observations for latitude taken at ⊙ D 12, in Labutta circle, Bassein District, on 13th January 1893, with Troughton and Simms' 6″ Transit, No. 817.

Computation of Latitude from Observations of Polaris out of the Meridian

Station	=	⊙ D 12		
Date Ast.	=	13th January 1893.		
		°	′	″
Long. in Arc	=	94	50	00
		h	*m*	*s*
Mean of Chr. Times	=	7	44	53
Error of Chronometer	=	—	10	21
Local Mean Time (L.M.T.)	=	7	34	32
Long. in Time	=	— 6	19	20
Greenwich Mean Time (G.M.T.) (*a*) ...	=	1	15	12
S. T. at Gr. Mean Noon	=	19	32	45
Local Mean Time	=	7	34	32
Acceleration for (*a*)	=		00	12
S. T. of Observation	=	3	07	29
		°	′	″
Mean Altitude	=	17	19	11
Refraction	=	—	03	05
(Constant)	=	—	01	00
Reduced Altitude	=	17	15	06
First Correction	=	— 1	07	38
Second Correction	=	+	00	02
Third Correction	=	+	01	27
Latitude	=	16	07	57

75. PROBLEM XIX.—*To find the true mean time at a place by a meridian observation of a celestial object*; * *the latitude and longitude of the place, and the altitude of the object being known.*

Rule: *By a star*: Choose a bright star for observation and calculate the mean civil time of transit by (Prob. IX, Art. 65); also the meridian altitude (by the formula: " Alt.= co-lat. $\begin{smallmatrix} - \text{ S. Dec.} \\ + \text{ N. Dec.} \end{smallmatrix}$", at the footnote to Prob. XVII, Art. 73). Set the telescope of the theodolite at the computed altitude in the plane of the meridian, and note the watch time at the instant the star crosses the vertical wire of the telescope. The difference of the watch time, if any, from the computed time of transit is its error, *fast* or *slow*, according as it reads in *excess* or *defect* of transit time.

By the sun: About a quarter of an hour before noon set the telescope in the meridian of the place, and observe the sun; noting the watch time at the instant the western limb touches, and the eastern limb leaves, the vertical wire of the telescope.† The mean of these two readings is the *apparent* time of the sun's

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OBSERVATIONS.

* When a celestial object is on the meridian of the observer, its Right Ascension is the same as the sidereal time of its transit.

† In (Fig. 437) are shown the various positions, in observations for time and azimuth, that the sun would occupy within a telescope provided with an inverting eye-piece. L and L′ represent the *lower* limb in contact with the horizontal wire, and the *left* and *right* limbs respectively in contact with the cross-wires. Similarly, U and U′ show the contacts of the upper limb with the horizontal wire, and the *right* and *left* limbs respectively with the cross-wires. In observing for time, the lower and upper limbs of the sun are

transit, which must be converted into *mean* time, by applying to it the equation of time (given in Table XX, "Calculating Tables," 5th edition).

Example: On the 17th June 1880, at a place in latitude 29° 52′ N., longitude 29° 52′ E., the R.A. of *Arcturus* (*α* Bootis) was = $14^{h.}\ 10^{m.}\ 14^{s.}$, declination = 19° 48′ 20″ N., and sidereal time of mean noon = $5^{h.}\ 44^{m.}\ 28^{s.}$; find the mean civil time of transit.

R. A. of star (N.A., App. places) ... = $14^{h.}\ 10^{m.}\ 14^{s.}$
Sidereal time of mean noon, 17th June 1880 (N.A., p. II) = $5^{h.}\ 44^{m.}\ 28^{s.}$ Correctn. for long. E. = 29·87° × $0^{s.}$ —657 = — 20 } = 5 44 08

Sidereal time of star's transit = 8 26 06
∴ Corresponding mean civil time of ditto. { by Table XXXVII, "Calculating Tables," 5th edition. } = $8^{h.}\ 24^{m.}\ 43^{s.}$ P.M.
Co-lat. ... ... = 60° 08′ 00″
Dec. N. ... ... = + 19 48 20

Altitude of star S. of zenith = 79 56 20

Now set the telescope in the plane of the meridian of the place looking south, at an altitude of 79° 56′ 20″, and suppose that at the instant the star crossed the vertical wire of the telescope the watch indicated $8^{h.}\ 39^{m.}\ 16^{s.}$ P.M.; then the error of the watch would be thus found:—

Calculated civil time of star's transit ... = $8^{h.}\ 24^{m.}\ 43^{s.}$
Time shown by watch at „ „ ... = 8 39 16

Error of watch, *fast* ... = 0 14 33

76. PROBLEM XX.—*To find the true mean time at a place by observation of a celestial object out of the meridian; the latitude and longitude of the place, approximate time of observation, and altitude of the object being known.* (Fig. 437.)

Rule: *By the sun*:—*1st.* Observe, on both faces of the theodolite, the altitude of the sun's upper limb first and then the lower, if the sun is on the east of the meridian and ascending; but if on the west and descending, observe first the lower limb and then the upper.* Note the precise watch times of observation. Then take the mean of all the observations for the mean altitude of the sun's centre, and the mean of all the recorded times for the mean time of observation, as shown by the watch.

2nd. Correct the observed altitude for refraction and parallax (given in Tables XXXVIII and XXXIX, "Calculating Tables," 5th edition) for the true altitude, which subtract from 90° to get the zenith distance.

3rd. Obtain the sun's declination at Greenwich (from the 'Nautical Almanac') for the given date, correcting for difference of longitude and time of observation. From this corrected declination obtain the north polar distance (by Prob. X, Art. 66.)

OBSERVATIONS.

taken, and for azimuth the left and right limbs; in the latter case, if in ascent, the left limb was first observed then the right; in descent the right limb would be first taken then the left, and *vice versâ*; if only one of the limbs and not the other were observed to, whether in ascent or descent, an angular error, equal to the sun's semi-diameter, would be engendered. Where the observations are for both time and azimuth, by means of the vertical and horizontal tangent-screws place the disc of the sun, whether ascending or descending, in each of the opposite quadrants of the diaphragm so that its limb may touch the vertical and horizontal wires; then read and record the horizontal and vertical verniers, noting the times of contact with a watch; next change face and repeat the observations; the mean of the horizontal readings of the sun would represent the mean bearing (where zero was set to north) of its centre and the mean of the vertical readings the mean elevation of the centre The Moon is similarly observed to the sun, but in place of the dark glass used for the latter to protect the eye against its rays, a bull's-eye lamp is employed to illuminate the wires of the telescope (by directing the ray of light on the object glass at an angle to the optical axis so as to give sufficient illumination without obstructing the line of sight or obscuring the celestial object), which would not otherwise be visible. The same method of observation is employed in the case of a star, except that, as it has no limb or semi-diameter owing to its distance, it can be at once placed in the intersection of the wires.

* *See* † on previous page.

4th. Subtract the given latitude of the place from 90°, for the co-latitude.

5th. Add the zenith distance, north polar distance, and the co-latitude together, and take half the sum, of which find the log. co-secant. Diminish the half sum.

1st, by the zenith distance, and find its log. co-secant;
2nd, by the north polar distance, and find its log. sine;
3rd, by the co-latitude, and find its log. sine.

6th. Add the two log. co-secants (omitting 10 from the index of each), and the two log sines above mentioned together, and divide the sum by 2 for the log. tangent of an arc, which being found and doubled, will furnish the sun's hour angle at the time of observation.

7th. Convert the hour angle * which is in space into time (by Table XXXIV, "Calculating Tables," 5th edition), and to this result apply the equation of time (taken from Table XX, "Calculating Tables," 5th edition, or from the N.A., p. I) for the mean time of observation.

8th. The watch time of observation must now be compared with the mean computed time, and the difference will be the error of the watch, *fast* or *slow*.

By a star :—*1st.* Observe, on both faces of the theodolite, the altitude of the star, whether it be on the east or west of the meridian. Note the precise watch times of observation. Then take the mean of all the observations for the mean altitude of the star, and the mean of all the recorded times for the mean time of observation as shown by the watch.

2nd. Correct the observed altitude for refraction (given in Table XXXVIII, "Calculating Tables," 5th edition), for the true altitude, which subtract from 90° for the zenith distance.

3rd. Obtain the star's declination (from the 'Nautical Almanac') for the given date; and from it obtain the north polar distance (by Prob. X, Art. 66).

4th, 5th and *6th.* (Same as for sun.)

7th. Convert the hour angle † which is in space into time (by Table XXXIV, "Calculating Tables," 5th edition), and *add* the result to the star's right ascension, if the observation be made to the *west* of the meridian, or *subtract* it from the star's right ascension (increased by 24 hours, if necessary), if the observation be made to the *east* of the meridian. This will give the sidereal time of the observation, which converted into mean solar time (by Prob. VI, Art. 62) will be the mean time of the observation.

8th. (Same as for sun.)

SPECIMEN OF FIELD-BOOK.

Example (1) : Afternoon angles taken at Taungshun with T and S.'s 7″ Theod., on the 22nd January 1880, by J. C. C.

Lat. = 14° 14′ 57″ N.
Long. (arc) = 98 09 36 E.
„ (time) = 6ʰ· 32ᵐ· 38ˢ·

Aspect of object observed. } Sun—W. of meridian.

Object observed.		Face.	Vertical Verniers.								Observed watch times.		
			A.			B.		Mean.					
			°	′	″	′	″	°	′	″	H.	M.	S.
⊙'s Lower Limb	...	L	26	23	20	22	50	26	23	05	4	28	25
⊙'s Upper „	...	...	26	23	20	22	50	26	23	05	4	30	55
⊙'s Lower „	...	R	25	32	20	31	50	25	32	05	4	33	16
⊙'s Upper „	...	...	25	32	20	31	50	25	32	05	4	35	45

OBSERVATIONS.

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* When the sun is the object observed, the hour angle is the apparent time for apparent noon at the place of observation.

† When a star is the object observed, the hour angle is its distance in time from the meridian.

Mean observed altitude	... =	25° 57′ 35″
,, ,, time	... =	4*h*. 32*m*. 05*s*.

Computation.

Mean observed altitude ...	... =	25° 57′ 35″	
Refraction and parallax = —01′ 59″ + 08″	=	— 01 51	
True altitude	... =	25 55 44	
Zenith distance = (90° — 25° 55′ 44″)	=	64° 04′ 16″	
North polar distance = (90 + 19 46 03)*	=	109 46 03	
Co-latitude = (90 — 14 14 57)	=	75 45 03	
Sum	... =	249 35 22	
Half sum	... =	124 47 41	, Cosec. = 0·085 550
Half sum—zenith distance	... =	60° 43′ 35″	, Cosec. = 0·059 348
Half sum—north polar distance	... =	15 01 38	, Sin. = 9·413 765
Half sum—co-latitude ...	... =	49 02 38	, Sin. = 9·878 069
			2)19·436 732
Half the hour angle ...	... =	27° 36′ 08″	, Tan. = 9·718 366
		2	
Hour angle in space ...	... =	55 12 16	
∴ Hour angle in (apparent) time	... =	3*h*. 40*m*. 49*s*.	
Mean time of apparent noon (N.A., p. I, corrected)	... =	+ 11 44	
Mean time of observation ...	... =	3 52 33	
Watch ,, ,, ,, ...	... =	4 32 05	
∴ Error of watch, *fast* ...	... =	0 39 32	

Example (2): (*Another method*).—Observations for Time taken at Labutta station, Bassein District, on the 13th January 1893. (Star's R.A.=6$^{h.}$ 40$^{m.}$ 27$^{s.}$ and Dec. S.=16° 34′ 09″, *vide* N.A.)

Computation of Time from Single Altitudes.

				°	′	″
Station of Observation = α ⊙ Labutta (Bassein District) ...	Latitude N. ...	=	16	10	30
	Longitude (arc) ...	=	94	50	00
			h.	*m.*	*s.*
	,, (time) ...	=	6	19	20
Date of Observation = 13th January 1893	Approximate time of Observation ...	=	7	23	17
Object observed = α Canis Majoris (Sirius)	Aspect of Object observed	=	E. of Meridian.		

Instruments used : T. and S.'s Theod. (Transit) No. 812, Chronometer No. (Watch) 720.

		°	′	″
Observed Apparent Zenith Distance	=	65	02	50
Correction for Refraction (and Parallax in case f othe Sun)	=	+	02	05
True Zenith Distance or (a)	=	65	04	55

OBSERVATION.

* Given watch time = 4*h*. 32*m*. 05*s*. P.M., 22nd January.
Longitude E., in time = 6 32 38

Greenwich time = 2*h*. 00*m*. 33*s*. before noon = 21*h*. 59*m*. 27*s*., 21st January, for which declination has been calculated.

		°	′	″
Zenith Distance ...	(a) =	65	04	55
Co-latitude ...	(b) =	73	49	30
N. P. Distance ...	(c) =	106	34	09
Sum ...	(2p) =	245	28	34
½ Sum ...	(p) =	122	44	17
	(p—b) =	48	54	47
	(p—c) =	16	10	08
P = Hour Angle				
½ P in arc...	=	28	31	20
P in arc...	=	57	02	40

Log. Sin. (b) ...	=	9·982 459
Log. Sin. (c) ..	=	9·981 581
Log. Sum ...	=	$\bar{1}$·964 040
Ar. Co. Log. Sum ...	=	0·035 960
Log. Sin. (p—b) ...	=	9·877 206
Log. Sin. (p—c) ...	=	9·444 778
Sum = Log. Sin.² ½ P.	=	19·357 944
Log. Sin. ½ P. ...	=	9·678 972
		h. m. s.
In time	=	*—3 48 11

		h.	m.	s.
Star's Right Ascension (or, in case of the Sun, Equation of Time)	=	6	40	27
True Local Sidereal Time of Observation	=	2	52	16
True Sidereal Time at Local Mean Noon	=	19	31	43
Sidereal Interval from Mean Noon	=	7	20	33
Corresponding Mean Interval, *i.e.*, True M. T. of Observation	=	7	19	21
Chronometer Time of Observation	=	7	23	17
Chronometer Error {Fast + / Slow −}	=	+	03	56
Daily rate brought up from ... ... {Gaining + / Losing —}		...	...	...

NOTE.—P in time is ∓ when *Star* is East / West of the Meridian; in case of *Sun* when observed E. of the Meridian 12 *h*—P in time must be entered at.*

Example (3): (The subject of Example 2 computed by the method in Example 1.)

Computation.

		°	′	″		
Z. D.	(a) =	65	04	55		
N. P. D.	(c) =	106	34	09		
Co-lat.	(b) =	73	49	30		
Sum	(2p) =	245	28	34		
½ Sum	(p) =	122	44	17	, Cosec. ... ... =	0·075 126
	(p—a) =	57	39	22	, Cosec. ... ... =	0·073 219
	(p—c) =	16	10	08	, Sin. ... ... =	9·444 778
	(p—b) =	48	54	47	, Sin. ... ... =	9·877 206
						2) 19·470 329
½ Hour ∠	=	28	31	20	, Tan. ... ... =	9·735 104
				2		
Hour ∠ in space ...	=	57	02	40		
		h.	m.	s.		
∴ Hour ∠ in time ...	=	—3	48	11		
Star's R. A. ...	=	6	40	27		
Sidereal time of Observation	=	2	52	16		
Corresponding Mean solar time of Observation	=	7	19	21†		
Chronometer Time of Observation	=	7	23	17		
∴ Chronometer error, *fast*	=	+	03	56		

OBSERVATION.

		h.	m.	s.
† M. T. at *preceding* sidereal noon, namely, Jan. 13	=	4	26	31·1
Correction for long. E. = 94·83″ × 0·6551 = 62·1″	=	+	01	02·1
Sum ...	=	4	27	33·2
For 2 h. 52 m. 16 s. Sidereal time, equivalent mean solar time	=	2	51	47·8
Mean solar time required	=	7	19	21·0

77. PROBLEM XXI.—*To find the time and the latitude simultaneously by ex-meridian observations.**

First Method.

(i) *Latitude*—By simultaneous altitudes of two known stars.

[*Note.*—When there is only one observer he may proceed as follows:—

Take the altitude of the first Star, S_1, then the altitude of the second Star, S_2, and again the altitude of S_1, noting the corresponding times. During the few minutes that this will occupy the changes of altitude may be supposed uniform, and therefore the altitude of S^1 corresponding to the instant of the observed altiude of S_2 can be found.]

Let ζ_1 and ζ_2 be the true zenith distances of the two stars.

Δ_1 and Δ_2 be the true north polar „ „ „

d „ „ difference of right ascension,

and put $\tan.\ x = \tan.\ \Delta_1 \cos.\ d,$

$$\cos.\ y = \frac{\cos.\ \Delta_1 \cos.\ (\Delta_2 - x)}{\cos.\ x.}$$

$$\tan.\ \tfrac{1}{2} z = \sqrt{\frac{\sin.\ \tfrac{1}{2}(\Delta_2 + \Delta_1 - y).\ \sin.\ \tfrac{1}{2}(\Delta_2 + y - \Delta_1)}{\sin.\ \tfrac{1}{2}(\Delta_2 + \Delta_1 - y).\ \sin.\ \tfrac{1}{2}(\Delta_1 + y - \Delta_2)}}$$

$$\tan.\ \tfrac{1}{2} w = \sqrt{\frac{\sin.\ \tfrac{1}{2}(\zeta_2 + \zeta_1 - y).\ \sin.\ \tfrac{1}{2}(\zeta_2 + y - \zeta_1)}{\sin.\ \tfrac{1}{2}(\zeta_2 + \zeta_1 + y).\ \sin.\ \tfrac{1}{2}(\zeta_1 + y - \zeta_2)}}$$

$$u = z - w;$$

then $$\sin.\ \lambda. = \frac{\cos.\ \Delta_1 \cos.\ (\zeta_1 - \theta)}{\cos.\ \theta}$$

where $$\tan.\ \theta = \tan.\ \Delta_1 \cos.\ u;$$

and $$\sin.\ (\text{hour angle} = t) = \frac{\sin.\ \zeta_1 \sin.\ u.}{\cos.\ \lambda.}$$

The values of the quantities y and z may be tabulated for certain pairs of stars and much labour saved.

The most favourable condition of observation will be when the verticals of the two bodies are on the same side of the meridian and equally inclined to the prime vertical, the difference of the azimuths being near 90° if possible.

Example:—The altitudes of α Persei and β Orionis (at a place in longitude 77° 54′ E.) on the 11th April 1893, were found to be respectively 29° 41′ 51″ and 28° 11′ 50,″ both stars being W. of meridian, and the clock time of observation was 8*h*. 29*m*. 33*s*.

	α Persei.		β Orionis.
Observed altitude	29° 41′ 51″		28° 11′ 50″
Refraction (corrected for Bar. and Ther.)	— 01 28		— 01 34
True altitude	29 40 23		28 10 16
True zenith distance = ζ_1 =	60 19 37	ζ_2 =	61 49 44
	h. *m*. *s*.		*h*. *m*. *s*.
R. A.	3 16 39		5 09 23
N.P.D. (90° ∓ Dec. N.)—*see* N. A. $= \Delta_1$ ... ... =	40° 31′ 05″	Δ_2 =	98° 19′ 33″

h. *m*. *s*.

Difference of R. A. ... ... $= d =$ 1 52 44

„ „ „ ... ... in arc = 28° 11′ 00″

Log. $\tan.\ \Delta_1 = \bar{1}{\cdot}931\ 776$ Log. $\cos.\ \Delta_1 = \bar{1}{\cdot}880\ 929$

„ $\cos.\ d = \bar{1}{\cdot}945\ 193$ „ $\cos.\ (\Delta_2 - x) = \bar{1}{\cdot}681\ 190$

„ $\sec.\ x = 0{\cdot}097\ 599$

„ $\tan.\ x = \bar{1}\ 876\ 969$

$x = 36°\ 59'\ 27''$ „ $\cos.\ y = \bar{1}{\cdot}659\ 718$

$\Delta_2 - x = 61\ 19\ 06$ $y = 62°\ 49'\ 11''$

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* Re-worked from "Auxiliary Tables."

Δ_2	= 98° 19′ 33″		ζ_2 =	61° 49′ 44″
Δ_1	= 40 31 05		ζ_1 =	60 19 37
y	= 62 49 11		y =	62 49 11
Sum = 2s	= 201 39 49		Sum = 2s =	184 58 32

	° ′ ″				° ′ ″		
s	= 100 49 55	L. cosec.	0·007 808	s	= 92 29 16	L. cosec.	0·000 409
$s-\Delta_2$	= 2 30 22	„	$\bar{1}$·359 263	$s-\zeta_2$	= 30 39 32	„	0·292 493
$s-\Delta_1$	= 60 18 50	L. sin.	$\bar{1}$·938 896	$s-\zeta_1$	= 32 09 39	L. sin.	$\bar{1}$·726 145
$s-y$	= 38 00 44	„	$\bar{1}$·789 460	$s-y$	= 29 40 05	„	$\bar{1}$·694 585

Sum = L. tan.² $\frac{1}{2}z$ ...	= 1·095 427	L. tan.² $\frac{1}{2}w$ =	$\bar{1}$·713 632
L. tan. $\frac{1}{2}z$ ...	= 0·547 714	L. tan. $\frac{1}{2}w$ =	$\bar{1}$·856 816
$\frac{1}{2}z$ ...	= 74° 10′ 52″	$\frac{1}{2}w$ =	35° 43′ 18″
z ...	= 148° 21′ 44″	w =	71° 26′ 36″

$u = z - w = 76° 55′ 08″$

L. tan. Δ_1	... $\bar{1}$·931 776
„ cos. u	... $\bar{1}$·354 742
„ tan. θ	... $\bar{1}$·286 518
θ	... 10° 56′ 51″
$\zeta_1 - \theta$	... 49 22 46
L. cos. $(\zeta_1 - \theta)$	... $\bar{1}$·813 612
„ cos. Δ_1	... $\bar{1}$·880 929
„ sec. θ	... 0·007 976
Sum = L. sin. λ	... $\bar{1}$·702 517
λ	... 30° 16′ 18″
(ii) *Time*—	
L. sin. ζ_1	... $\bar{1}$·938 952
„ sin. u	... $\bar{1}$·988 582
„ sec. λ	... 0·063 665
Sum = L. sin. t.	... $\bar{1}$·991 199
t in arc	... 78° 30′ 14″
	h. m. s.
t in time	... 5 14 01
R. A. of α Peresi	... 3 16 39
S. T. of observation	... 8 30 40
Clock time of „	... 8 29 33
Error of clock	— 0 01 07

Second Method.

By two observations of the altitude of a star and the elapsed time. This is the same problem as the foregoing: the elapsed S. T. reduced to arc will give d

Here $\sin.\frac{y}{2} = \sin.\Delta \sin.\frac{d}{2}$,

$\cot.\frac{1}{2}z = \cos.\Delta \tan.\frac{d}{2}$;

and the remainder of the solution will be as before.

The same solution will apply to two altitudes of the sun and the elapsed time, provided this elapsed interval be reckoned in *mean* time.* The declination of the sun may be considered constant.

OBSERVATION.

* Though this is not rigorous, it is sufficiently correct for all practical purposes, the change in interval due to equation of time being too small to be worth notice. It should be noticed that the formulæ above given are applicable to north latitude only.

78. LONGITUDE * EXPLAINED, WITH METHODS OF DETERMINATION. (Figs. 477, 488—489*a*). A variety of methods are employed for determining the longitude of a place, but almost all of them depend upon one general principle—namely, the comparison of the relative times under two different meridians; so that, if the time under a given meridian be known, and also the time under any other meridian, the difference of these times turned into degrees and minutes, in the proportion of 15 degrees to 1 hour, will be the difference of longitude between the two meridians. For, as the sun *apparently* moves round the earth, from east to west, in 24 hours, or over an arc of 15 degrees of the equator in one hour of time, all places lying to the eastward of any meridian, will have noon sooner—or, if to the westward, later—by as much time as the sun takes to pass from the meridian of one place to the meridian of the other; hence, if the time at the meridian of Greenwich (from whence the longitude is reckoned), and of any other place, at the same moment of absolute time, be given, its longitude from Greenwich may be inferred by reducing the difference of times into degrees and minutes, in the proportion of 15 degrees of longitude to 1 hour of time: moreover, if the time at the place be greater than that at Greenwich, its longitude will be east; but if less, it will be west. Thus, suppose it is ascertained that the time at Greenwich, is two hours past noon, when it is just noon at a certain place, it will thence appear that the longitude of the place is 30 degrees west of the meridian of Greenwich, because the sun passes over 30 degrees of the equator in two hours of time; and having left the meridian of Greenwich two hours since, the said place must consequently be to the westward of that meridian. If the time at the place in question be assumed to be four hours past noon at 2 P.M., G. T., its longitude would be 30 degrees east of Greenwich; for the sun, in this case, would have passed the meridian of the place two hours before it passed that of Greenwich.

Now, the time at any given meridian may be easily computed by an altitude of the sun or a star, taken when distant from the meridian, or from observations of the sun when at equal altitudes; and the time at Greenwich may be ascertained by means of a time-keeper, such as a good watch or chronometer whose error and rate are known, or by various astronomical observations. With respect to the first of these, it is obvious that, if a clock or watch could be so constructed as to go uniformly in all seasons and at all places, such a machine being once set to the time at Greenwich would always show the real time at Greenwich, in whatever part of the earth it might be; and, therefore, when the time under any other meridian was found, and compared with that shown by the time-keeper, the longitude of the place from Greenwich would be readily obtained. To effect this purpose several ingenious artists have exerted their abilities, and have brought time-keepers to an astonishing degree of perfection; whereby they have become a valuable acquisition in determining the difference of longitude made in short periods: however, considering the delicacy of their construction, and the various accidents to which they are liable, an implicit confidence ought not to be placed in them, particularly during long journeys; but recourse should be had to astronomical observations, whenever opportunities present themselves. The various astronomical methods of determining the longitude depend likewise upon the above-mentioned general principle; for, by observing the time at the meridian of a given place when any celestial appearance happens, and comparing the same with the time at Greenwich, shown by the 'Nautical Almanac,'† their difference, reduced to degrees and minutes, gives the longitude as before. Suppose, for instance, that an eclipse of the moon should be observed at a certain place to begin at midnight, and that by the Almanac the time at Greenwich, when the eclipse commenced, was three hours past midnight; now, as the commencement of the eclipse must be seen at the same moment of absolute time in all parts where it is visible, the difference between the time at the place of the observer and that of Greenwich, which is three hours, and answers to

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* Partly from Norie's "Navigation," and partly from the "Hints to Travellers."

† This excellent and most useful work, so essential to the astronomer, surveyor, and navigator, is published annually, by order of the Lords Commissioners of the Admiralty, generally four years forward, and is computed for the meridian of the Royal Observatory at Greenwich. It was orginally constructed and issued (in 1769) under the immediate supervision of the then Astronomer Royal, Dr. Maskelyne, but in recent years it has been much improved and enlarged.

45 degrees, must be the longitude of the place; and it is evidently west, because the time at the place is less than the time at Greenwich. Upon the same principle the eclipses of Jupiter's satellites will give the longitude. But eclipses of the moon happen too seldom to be of much use to the ordinary observer and the satellites of Jupiter are visible only through a telescope of considerable magnifying power. The most practical method of finding the longitude, by celestial observations, is that of measuring the angular distance between the moon and sun, or the moon and certain stars, near the ecliptic, usually called a *lunar observation*. The moon describes every month an orbit round the earth from west to east, at the distance of about 240,000 miles; in appearance it moves on the celestial concave in a curved line, near the ecliptic or apparent orbit of the sun. It moves faster than the sun from west to east, by about a degree and a half in three hours. Supposing it to be at the distance of from about 30° to 120° from the sun, either west or east of it, the two bodies are frequently enabled to be seen above the horizon at the same time. When this is the case, the distance of two bodies may be observed with a sextant and thence the correct distance of their centres, as seen from the earth's centre, which is called their true distance, may be computed. And from this true distance, with the help of the 'Nautical Almanac,'* wherein the true distance (within certain limits) is put down for every third hour, Greenwich apparent time may be easily determined. In favourable weather distances may be taken at all times, when the objects are more than four or five degrees above the horizon, except about the time of new moon; and as the moon's daily motion is about 13 degrees, or at the rate of 1 minute of a degree in 2 minutes of time, if her true angular distance from the sun or a star can be ascertained within 30 seconds of a degree, the corresponding time at Greenwich will be known within 1 minute of time, and hence the longitude within 15 minutes of a degree. To facilitate this important problem, the true angular distances of the moon from the sun, or a fixed star,† are set down in the 'Nautical Almanac' for the beginning of every third hour of Greenwich time; and the time answering to any intermediate distance may be found by proportional parts: hence, the distance between these objects being taken with a sextant and the corresponding time at Greenwich found by the Almanac, and compared with the time at the place, their difference will be the longitude of the place of observation. But before the observed distance is compared with those in the Almanac, it must be corrected, in order to find the true distance; for, by the combined effects of parallax and refraction, the moon is always seen lower than its true place,‡ and the sun or star higher: hence the true distance is almost always greater or less than the observed.

With these preliminary remarks, the manner of determining the longitude by the following three methods only may be proceeded with, namely: (i) by meridian distance, (ii) by the occultation of a star, and (iii) by lunar observations.

(i) BY MERIDIAN DISTANCE.

As before stated, the difference of longitude of two places is the difference of time between them at the same instant. If, therefore, the time at one place can

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* *N.B.*—The tables of Lunar Distances are omitted in Nautical Almanacs from 1907; but examples of the method of calculating Lunar Distances are given, *seq.*, under (iii) Lunar Observations in Art. 78.

† The stars used in the 'Nautical Almanac' (pages XIII to XVIII for the above purposes are, α Arietis (R. A $=2^h\ 1^m$; Dec. $=22°\ 59'$ N.) α, Tauri (Aldebaran: R. A. $=4^h\ 30^m$; Dec. $=16°\ 18'$ N.), β Geminorum (Pollux: R. A. $=7^h\ 39^m$; Dec. $28°\ 16'$ N) α Leonis (Regulus: R A $=10^h\ 03^m$; Dec. $12°\ 28'$ N.), α Virginis (Spica: $13^h\ 20^m$; Dec. $10°\ 38'$ S.) α Scorpii (Antares: R. A. $=16^h\ 23^m$; Dec. $26°\ 12'$ S.) α Aquilæ (Altair: R.A. $=19^h\ 46^m$; Dec. $8°\ 36'$ N.), α Piscis Australis (Fomalhaut: R. A. $=22^h\ 52^m$, Dec. $30°\ 10'$ S.), and α Pegasi (Markab: R. A. $23^h\ 00^m$; Dec. $14°\ 40'$ N). As a knowledge of these stars is of great importance to the observer, their positions in the celestial maps (*see* Figs. 488 and 489) containing them should be studied, and in order that they may be the more readily known, the maps should likewise be compared with the heavens.

‡ This is due to the moon's horizontal parallax being considerable: it averages about 57′, while that of the sun averages only about 9″; the mean atmospherical refraction at the horizon is about 34′; the correction for $\frac{\text{parallax}}{\text{refraction}}$ is always $\frac{\text{additive}}{\text{subtractive}}$, in respect to the altitude; thus, the correction for the $\frac{\text{moon}}{\text{sun}}$ would be $\frac{+(\text{parallax}-\text{refraction})}{-(\text{refraction}-\text{parallax})}$. Fixed stars have practically no sensible parallax, though that of several stars has been ascertained but is small, the largest not even amounting to 1″. The average horizontal parallaxes of the planets Mercury, Venus, Mars, Jupiter and Saturn are about 10″, 19″, 13″, 2″, and 1″, respectively.

be transported, by means of a watch, to another place, and the true time obtained at that second place, the difference of those times is the difference of longitude between the two places. This is accomplished in practice, by finding the errors of the watch or chronometer at the two places, either by absolute, or equal altitudes, and the rate, in any case at one of them, though it is better to find it at both, and take the mean. The difference of longitude is then thus found:—

		H.	M.	S.	
Error of Watch at Mombassa, 8 A.M., 14th of July	...	2	18	32	slow.
” ” ” 9 A.M., 20th ”	...	2	17	14	”
Interval 6·04 days. Difference	=		01	18	
	6·04)			78	
Daily rate	=			12·91	gaining.

		H.	M.	S.	
Error of Watch at Taveta, 4 P.M., July 25th	...	2	08	05	slow.
” ” ” 8 A.M., July 30th	...	2	06	48	”
Interval 4·67 days. Difference	=		01	17	
	4·67)			77	
Daily rate	=			16·5	gaining.
Former daily rate	=			12·9	”
			2)	29·4	
Mean daily rate	...			14·7	”

		H.	M.	S.		°	′	″
Error of Watch at Mombassa, July 20th, 9 A.M.	=	2	17	14	slow.			
5·3 days' mean rate	=		01	18	gaining.			
Error of Watch at Mombassa, July 25th, 4 P.M.	=	2	15	56	slow.			
” ” Taveta, ” ” ...	=	2	08	05				
Meridian distance, or difference of longitude between Mombassa and Taveta.	=		07	51	=	1	57	45

and as the watch is less slow at Taveta than at Mombassa, Taveta is west of Mombassa.

		°	′	″	
The longitude of Mombassa being ...	...	39	40	00	E.
Meridian distance, west	...	1	57	45	W.
Longitude of Taveta	=	37	42	15	E.

Here the rate has been supposed to be obtained at both places. If, however, it was only ascertained at one end, that rate would have to be used. In the case supposed the result would be a difference of 10 seconds in the determination of the longitude of Taveta, or 2′ 30″ of longitude. This method can be used at *any part of a journey* to measure the differences of longitude between two places. If the longitude of one of the places has been fixed by lunars, or other absolute methods, the longitude of the other is known at once. If not, the longitude of either of the places may be fixed hereafter, and the longitudes of the places whose meridian distances have been measured will be in connection with it, and not be scattered about with large individual errors, as would be the case were they determined separately by one or two lunars.

(ii) BY THE OCCULTATION OF A STAR.

This is the best of the absolute methods of finding longitude, when a sextant or other convenient astronomical instrument is available for ascertaining the local time. The following describes the manner in which the observation is taken:—The moon in its monthly revolutions round the earth frequently passes between the earth and a fixed star so as to intercept a spectator's view of the latter; the disappearance of a star from this cause is called an *immersion*, and its re-appearance from behind the moon is called an *emersion*. A list of these phenomena is given in the 'Nautical Almanac,' with the limits in latitude beyond which a star cannot be occulted by the moon. As the elements refer to the moon and star as they would be seen from the earth's centre, they serve equally for all places on the earth's surface. This observation is much easier, and more certain in its results, than the lunar observation; as the instrument (the sextant or telescope) is one that every person can use, and is not liable to any error, all that is required is that the observer shall be certain that one instant he does see the star and that the next instant he does not (with an emersion the exact contrary is the case). Neither is there much difficulty in recognizing the star, its position with reference to the moon being clearly pointed out in the 'Nautical Almanac,' and as the moon only moves its own diameter among the stars in an hour, there is ample time after the star and moon are in, apparent, close proximity to make sure of the star. Before, or immediately after this observation, a set of sights should be taken to find the error of the watch on apparent or mean time at place. When it has been decided to observe an occultation, the observer should, during the day, find the local time of that phenomenon, by applying the assumed longitude in time to the G.M.T. of conjunction in R.A. of the moon and star, which he will find among the elements of occultations in the 'Nautical Almanac,' *adding* the longitude in time if it be *East*, and *subtracting* if it be *West*. An hour before the time so found, he should point his telescope to that limb of the moon by which the star will be occulted; it is necessary to take this precaution as his assumed longitude, and therefore his time, may be considerably in error. The moon will be seen to approach the star from west to east, until its eastern limb will reach the star and occult it; note the instant when this takes place. After a certain interval the star will re-appear on the other side of the moon; note this time also. Either of these observations are sufficient to determine the G.M.T., and thence the longitude, in the manner shown in the example. When the star is occulted by the moon's dark limb, the observation will afford most decisive results. At or near full moon a star occulted by the bright limb is not so easy an observation. The telescope suitable for this observation is one having a magnifying power of not less than 40. The example given is computed by Raper's rule and tables. It will be observed that several of the logs can be taken at one opening of the book, and as only four places of decimals are used, the log. sines, cosines, etc., can be taken at sight to the nearest 30″; this is not, however, the case with the proportional logs; where they occur the strictest accuracy must be observed, and the decimals of seconds must not be neglected. This remark also applies to the Moon's Declination, Right Ascension, Horizontal Parallax, and Semi-diameter.

Longitude by the Occultation of a Star.

July 3rd, 1892.—*Uranus*, immersion at Daboja, Lat. 9° 31′ 26″ N. Approx. Long. 1° 43′ W. Watch 17 min. 36 sec. fast of local time. ☽ West of the Meridian.

	H.	M.	S.
Time by Watch of immersion ...	8	50	30
Error of Watch	−	17	36
	8	32	54
West Long. in Time	+	06	52
Corresponding G. D., July 3	8	39	46

	′	″
☽'s Semid. p. III, N.A. Noon	14	48·2
☽'s Semid. p. III, N.A. Midt.	14	48·6
	00	00·4
Noon ...	14	48·2
Correction ...	+	0·3
Corr. Semid. ...	14	48·5

	°	′	″
Latitude ...	9	31	26
Reduction (Table LXX—D*)	0	03	43
Geocentric Lat. ...	9	27	43

	′	″
☽'s Hor. Par. Noon ...	54	14·2
☽'s Hor. Par. Midt. ...	54	15·4
12 Hourly Diff. =	00	01·2
Corr. by 12 Hourly Diff. for G.D.... =	+	00·87
☽'s Hor. Par. Noon =	54	14·20
☽'s Hor. Par. corr. for G. D.	54	15·07
Correction for Lat. (Table LXXC*) −		00·2
☽'s Reduced Hor. Par. =	54	14·87

OBSERVATION.

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* "Calculating Tables" (5th edition).

	M. D.		H.	M.	S.
Sidereal Time, July 3 (p. II, N. A.)			6	47	53·17
Accel. (Table XXXVI)*	8 H.	=		01	18·85
	39 M.	=			06·40
	46 S.	=			00·13
Reduced Sidl. Time ...		...	6	49	18·55
Mean Time at Place ...		... +	8	32	54
R. A. of Meridian ...		... =	15	22	12·55
*'s Right Ascension ...		... −	13	59	57·27
*'s Hour Angle ...		... =	1	22	15·28

	°	′	″
☽'s Declination, July 3rd, at 8 hours	10	42	57·8S.
*'s Variation of Decln. in 1 m = ... 12·6″ × 39·8 min. of G. D † ...	+	08	21·5
Reduced Declination =	10	51	19·3S.

† The variation in 1 m. is found by removing the decimal point in the difference of Declination in 10 minutes (given in the Nautical Almanac) one figure to the left hand.

*'s Declination ... ... 11° 42′ 44″·9S.

☽'s Reduced Hor. Par. Pro. Log. ... 0·5209 ... ... 0·5209 ... ... 0·5209
Geocentric Latitude. Cosec. ... 0·7841 ... Sec. ... 0·0059 ... Sec. ... 0·0059
*'s Declination. Sec. ... 0·0091 ... Cosec. ... 0·6925 *'s Hour ∠ Cosec. ... 0·4544

Arc A = 8′ 44″ = Pro. Log. = 1·3141 *'s Hour ∠ Sec. ... 0·0286 ‖ Sum of 3 Logs. = 0·9812
Arc C − 0 00 × 2

(A−C) = 8 44 Arc B = 10 10″·3 = Pro. Log. = 1·2479 1·9624

Constant ... + 1·5820
*'s Declination Cotan ... + 0·6834

Arc C = 00′ 00″ = Pro. Log. = 4·2278

	°	′	″
*'s Declination	11	42	44·9S.
(A−C) + when Lat. and Decl. same name ... − when Lat. and Decl. different name ...	−	08	44
	11	34	00·9
B.—when Hour ∠ is *less* than 6 hours; + when *more* −	0	10	10·3
Prepared Declination ... =	11	23	50·6

Part I, for ☽'s Parallax in R.A. *Part II.*

	°	′	″		
Prepared Declination ..	11	23	50·6	Cos. ...	9·9913
☽'s Declination ...	10	51	19·3	Constant	1·1761
Difference ...		32	31·3		
☽'s Semi-diameter ...		14	48·5		
Difference ..		17	42·8	½ Pro. Log.	0·5035
Sum ...		47	19·8	½ Pro. Log.	0·2011

m. sec.
Part I. = 1 58·15 = Pro. Log. = ... 1·9610

... ... Cos. ... 9·9913
... ... Constant ... 1·1761
‖ Sum of 3 logs. used to find C ... 0·9812
Part II, 1 m. 16·7 s. = Pro. Log. ... 2·1486

		H.	M.	S.
	*'s R.A.	13	59	57·27
Part I.	If Immersion − / If Emersion +	−	01	58·15
		13	57	59·12
Part II	When ☽ E. of Merid. / When ☽ W. of Merid. } +	+	01	16·7
	☽'s Right Ascension =	13	59	15·82

	H.	M.	S.		
(1) ☽'s R.A. (thus found) ...	13	59	15·82		
(2) ☽'s R.A. preceding hour, viii h.	13	58	00·84		
(3) ☽'s R.A. following hour ...	13	59	50·05	Constant ...	0·4771
Diff. between (1) and (2) ...		01	14·98	Pro. Log. ...	2·1586
Diff. between (2) and (3) ...		01	49·21	Ar. Co. Pro. Log. ...	8·0048
	0	41	11	= Pro. Log. ...	0·6405
Hour of (2) ... ...+	8	00	00		
†† G. M. T.	8	41	11		
Mean Time at Place	8	32	54		
Longtitude in Time	0	08	17	=	2° 04′ 15″ W.

†† For extreme accuracy, re-compute Part I with this G.M.T., and the result will be the true G.M T., possibly some seconds different from the first obtained.

OBSERVATION.

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* "Calculating Tables" (5th edition).

(iii) LUNAR OBSERVATIONS.

In this observation the observed distance is not only liable to errors caused by a defect of parallelism in the telescope, which always makes the observed distance too great, but to all other instrumental errors, some of which may very possibly be unknown to the observer, and as an error in the distance, of whatever kind, produces about thirty times its amount in longtitude, it will be readily understood that but little value can be attached to the results obtained from a single set of lunar distances, even when the observation has been taken by a competent person, as making the contact slightly above or below the centre of the field, combined with the effects of irradiation, may very well cause an error of 20″ in the observed distance, the effect of which would be, in average cases, 600″ or 10′ error of longtitude. For these reasons lunar observations cannot be recommended to any person who has not acquired a perfect knowledge of the use of the sextant or other suitable astronomical instrument, its errors and adjustments; or who is unable to remain at one place long enough to take a series of distances east and west of the moon.

To Measure the Angular Distance between the Moon and Sun.—As the enlightened limb of the moon is always nearest to the sun, the angular distance measured is always that of the near limbs; but since, on account of her comparatively feeble light, it is necessary to observe the moon by direct vision, and since the sun at the time of observation may be either to the east or the west of the moon, the sextant (where this instrument is used) has to be held with its face up or down as the case may require. In *north latitude*, when the sun is to the west of the moon, the instrument is held with its face upwards; but when the sun is to the east of the moon, it must be held with its face downwards. In *south latitude* the *opposite* of this rule must be followed. This is often much easier if the observer can hold the sextant in his left hand; the position of the hand and wrist may otherwise be cramped and almost painful. Before taking an observation, look at the sun through the dark shades, and select those which reduce its brightness in the greatest degree compatible with good definition; put these down before the index glass; see that the inverting telescope is adjusted to focus; set the index to zero (0°); and hold the instrument with its plane parallel to a line joining the sun and moon; look at the moon through the telescope collar and horizon glass and move the index slowly forward until the sun's reflected image makes a rough contact with the moon, seen by direct vision through the unsilvered part of the horizon glass, clamp the index, screw in the telescope, and make the contact perfect in the centre of the field with the tangent screw, moving the sextant slowly round the axis of the telescope, by which means the reflected image of the sun will appear to pass the moon, and the accuracy of the contact can be tested.

Between the Moon and Star or Planet.—The angular distance between a star or planet and the moon is always measured to the moon's enlightened limb, which is often the farthest from the star or planet. When this is the case, the moon must be brought by reflection past the star or planet before the contact can be made; in other respects the observation is precisely similar to that already described, when the angular distance of the sun is taken. In observations of this class, the utmost attention must be paid to accuracy, and a faulty habit of observation in making contacts of the moon's limb with a star is not necessarily eliminated, as is very generally supposed, and frequently stated, by taking distances east and west of the moon. For example, if it is an observer's habit, in making a contact, to place the star within the moon's disc, M. (*see* Fig. 477), as at S′, the distance S″ S′ is too small, and the distance S‴ S′ too great; but supposing the moon to be moving in the direction from S′ to S‴, each distance will give too early a Greenwich time, for each will give the time when the moon's limb was actually at S′. When, however, the sun is the object observed east and west of the moon, errors of this sort in observation, *if constant*, will be eliminated, since, as the moon's enlightened limb is always turned towards the sun, such errors would increase both distances and produce errors of an opposite description in the Greenwich time. A single observation is of little value; distances should always be observed in sets, with stars east and west of the moon, and as nearly equidistant from it as possible; the observer should also note which limb of the moon has been observed, and whether the star was east or west of it. The more nearly the two bodies approach the same horizontal plane, the easier will be the observation to take, and distances between 45° and 90° will be least liable to errors in observation. The thermometer and the barometer (or its equivalent, a boiling-point thermometer) should be noted, and the refraction corrected accordingly; because, if thermometric and barometric corrections be omitted, in observations made on a high and heated plateau, there may be serious errors in the results. A complete pair of lunars, made wholly by one person, consists of the following observations, *in addition to those for latitude.* An hour before beginning to observe, get everything in perfect order; see that the lamp is well trimmed, its air-holes free, and that it is filled with oil. Also rehearse the expected observations with the sextant, that no hitch may occur after they have commenced. Then let the hand and eye have ample time to repose, and go on as follows:—

(1) Read Thermometer in air.
(2) Adjust horizon-glass, if necessary.
(3) Two pairs of observations for index error.
(4) Three altitudes for time, star E.
(5) Three altitudes for time, star W.
(6) Three altitudes of moon.
(7) Five lunar distances, star E. of moon.
(8) Five lunar distances, star W. of moon.
(9) Three altitudes of moon.
(10) Three altitutes for time, star W.
(11) Three altitudes for time, star E.

Clearing the Lunar Distance by Raper's Rigorous Method.—As this is one of the shortest, and at the same time a strictly accurate method of clearing the Lunar Distance, it is here given for the benefit of those who may not have Raper's work in their possession. Having found the Greenwich date with the assumed longitude in time, and the mean time at place by a watch or chronometer, the error of which on local time has been found by previous observation, reduce thereto the moon's horizontal parallax and semidiameter, and if the sun be one of the objects observed, take its semidiameter from the 'Nautical Almanac' (page II). From the observed altitudes get the apparent and true altitudes; from the observed distance get the apparent distance. Add to, or subtract from, the apparent altitudes as many seconds as are necessary to bring them to odd or even minutes, then add them together and subtract their sum from 180°, and the remainder will be the sum of the Apparent Zenith Distances. Increase or diminish the True Altitudes by the same number of seconds as were added to or subtracted from their respective Apparent Altitudes; add them together and subtract their sum from 180°, and the remainder will be the sum of the True Zenith Distances. Add together the Log-secants of the Apparent Altitudes and the Log.-cosines of the True Altitudes; the sum, rejecting tens in the index, will be the Logarithmic Difference. Increase or diminish the Apparent Distance by any quantity of seconds necessary to bring it to an odd or even minute (noting the number of seconds); to this add the sum of the Apparent Zenith Distances; take Half the sum, and from this Half Sum subtract the Apparent Distance—call this Remainder. To the Log.-sines of the Half Sum and Remainder add the Logarithmic Difference, and the sum, rejecting tens in the index, will be the Log.-sine square of the auxiliary arc x. Arc x may also be found without any special table of Log.-sines square in the following manner:—When the sum of these three logs. has for an index a number above 20, reject 10 from such index, and then divide the sum by 2; this will give ½ the Log.-sine of the arc, which multiplied by 2 will give auxiliary arc x; *this, of course, applies to all cases where a Log.-sine square is mentioned.* Under x put the sum of the True Zenith Distances, take their sum and difference and their Half Sum and Half Difference, add together the Log-sines of the Half Sum and Half Difference, and their sum is the Log.-sine square of an arc, to which apply the same number of seconds by which the Apparent Distance was increased or diminished to bring it to an odd or even minute, subtracting them if the Apparent Distance was increased, but adding them if diminished, and the result will be the true distance nearly. Take the difference between the proportional logs. in the 'Nautical Almanac'* against the two distances between which the computed true distance falls. With this difference and the portion of time just found, enter the table of corrections for second differences (Nautical Almanac or table 57 Raper), and take out the seconds. When the proportional logs. in the 'Nautical Almanac' are *increasing*, *subtract* these seconds from the True Dist., nearly; when they are *decreasing*, *add* them; the result will be the M. T. at Greenwich.

Lunar (Raper's Rigorous Method).

Latitude ...	... 51° 31′ 11″ N.
Thermometer	... 49°
Barometer ...	... 30 inches.

Date: Nov. 22nd, 1879, P.M. at place of observation, Mars and ☽'s. Mars East of Meridian.

	H.	M.	S.
Supposed. Time by Watch ...	7	46	33
Accumulated Rate of Watch ...		–	29
	7	46	04
Error of Watch ...		–	08
G.M T., Nov. 22nd...	7	45	56

	°	′	″
☽'s Semid. (Table 40, Raper)		15	02·5
Augmentation (Table LXX—E*)			09·2
☽'s Aug. Semid. ...		15	11·7
Mars' Declination ...	17	28	23 N.
	90	00	00
Mars' Polar Dist. ...	72	31	37 N.
	H.	M.	S.
Mars' R. A. ...	2	56	15

	′	″
☽'s Hor. Par. Noon	55	28·9
Mid.	55	11·2
Variation in 12 hours	00	17·7
Correction (Table 21, Raper)	00	11·4
Hor. Par. Noon	55	28·9
Hor. Par. corr. for G. D. ... =	55	17·5
Corr. for Lat. (Table LXX—C†)	–	06·7
Reduced Hor. Par. ... ... =	55	10·8

Art. Hor.	°	′	″
Mars' Observed Alt.	80	21	30
Index error ...	+		03
	2)80	21	33
App. Alt. =	40	10	46·5
Refraction—1′ 9·11″ Par. in Alt. + 13″	–	00	56·1
Mars' True Alt. =	40	09	50·4

Art. Hor.	°	′	″
Observed Alt. ...	78	30	54
Index error ...	+		03
	2)78	30	57
	39	15	28·5
Augt. Semid. ...	+	15	11·7
☽'s App. Alt. =	39	30	40·2
☽'s corr. in Alt. (Table 39, Raper)	+	41	25 0
☽'s True Alt. =	40	12	05 2

Day.	H.	M.	S.
Sidereal Time at Mean Noon, Nov. 22nd ...	16	04	24·24
Acceleration for G. D. (Table XXXVI) 7 hours ...		01	09·00
45 min. ...			07·19
55 sec. ...			00·15
Mean ⊙'s R. A. ... ... =	16	05	40·78

	°	′	″
Observed Distance F. L.	53	30	10
Index error	+	2	00
	53	32	10
☽'s Semid. Augmented ... ... =		15	11·7
☽'s Apparent Dist. ... ...=	53	16	58·3

App. Alts.	°	′	″
Mars' (+ 13·15″) ...	40	11	00
☽'s (+ 19·8″) ...	39	31	00
Sum ... =	79	42	00
	180	00	00
Apparent Zenith Dists.	100	18	00

True Alts.	°	′	″
Mars' (+13·5″) ...	40	10	03·9
☽'s (+19 8″) ...	40	12	25·00
Sum	80	22	28·9
	180	00	00
True Zenith Dists.	99	37	31·1

	°	′	″		
App. Dist. ...	53	17	00	(Note 1·7″ added.)	
App. Zenith Dists. ...	100	18	00		
Sum ... ...=	2)153	35	00		
½ Sum ... .. =	76	47	30	Sin.	9·988 356
½ Sum – App Dist. =	23	30	30	Sin.	9 600 815
Logarithmic difference				=	9·905 731
Arc *x*	75	38	32·0	= {Sin. sq.}	= 9·584 932
True Zenith Dists ...	99	37	31·1		
Sum	176	16	23·1		
Difference	22	58	39·1		
½ Sum	88	03	11·5	Sin.	9·999 770
½ Difference	11	29	19·5	Sin.	9·299 238
Approx. True Dist. ...	52°	59′	49″	= {Sin. ½ l.}	= 9·299 008
Seconds added ...	–		01·7		

Mars App. Alt.	Sec.	0·115 916
☽'s App. Alt.	Sec.	0 112 698
Mars' True Alt.	Cos.	9·883 184
☽'s True Alt.	Cos.	9·882 933
Logarithmic Difference... ... =		9·905 731

	°	′	″		
Mars' True Alt.	40	09	50		
Lat.	51	31	11	Sec.	0·206 039
N. P. D.	72	31	37	Cosec.	0·020 516
Sum ... ...2)	164	12	38		
½ Sum	82	06	19	Cosin.	9·137 839
½ Sum—Alt.	41	56	29	Sin.	9·825 017
	H.	M.	S.		
Hour L.	3	05	16	= Log. Sin.sq.	9·189 411
Mars' R. A. + 24 hours ...	26	56	15		
	23	50	59	= R. A. of Merid.	
	16	05	41	Mean Sun's R. A.	
	7	45	18	= Mean Time at place.	
	7	45	51	= G. M. T. by Lunar.	
	00	00	33	= Long. in Time =	° ′ ″ 00 08 15 West.

	°	′	″	
True Dist. ... =	52	50	47·3	
N.A. at VI hours ...	53	55	19	Pro., Log. N. A. 2805
Difference		55	31·7	Pro. Log. (Table 74, Raper) 5108
	H.	M.	S.	
Time of Dist. by N. A. ...	6	00	00	
	1	45	55	= Pro. Log. = 2303
	7	45	55	
Correction for 2nd difference (Table 57, Raper)			04	
G. M. T. by Lunar ...	7	45	51	
Time by Watch ...	7	46	33	
Watch fast of G. M. T. =	0	00	42	

NOTE.—*To compute the altitude of a Fixed Star.*‡—When a Lunar Distance is observed, it is frequently convenient to compute the Altitude of one or both the objects, and to this end it is necessary that the Longitude and the Time at place should be known with tolerable accuracy. The following example shows in detail the order and method of making the calculation when the object is a Fixed Star.

OBSERVATIONS.

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*N.B.—The tables of Lunar Distances are omitted in 'Nautical Almanacs' from 1907; but examples of the method of Calculating Lunar Distances are given *seq.*, under (iii) Lunar Observations, in Art. 78.

† "Calculating Tables" (5th edition).

‡ Adapted from Rosser's Book of the Stars.

Rule: Obtain the corresponding M. T. at Greenwich and the P. D. of the Star. Then work out the S. T. at Place (which is the R. A. of the Meridian) and subtract it from the R. A. of the Star for its Meridian distance (W. or E.) or H. A. Next find Arc I, by adding together the Log. cos. of the objects H. A. and the Log. cot. of the Lat.; the sum of which (rejecting 10 in the Index) will be the Log. tan. of Arc I. Then find the True Alt., by adding together the Log. sin. of the Lat., the Log. sec of Arc I, and the Log. cos. of the difference * (Arc II) of Arc I and the P. D.; the sum of which will be the Log. sin of the True Alt Lastly, apply the correction for Refraction to the True Alt. to obtain the App. Alt. of the object. Thus, required the true and apparent altitudes of α Virginis (*Spica*) on the 20th January 1898 (Civil date) at 4*h*. 10*m*. M. T., in Lat. 48° 30′ N. and Long. 29° 30′ W.

		H.	M.	S.
M. T. (Astron.) at place, 19th January	=	16	10	00
Long. of place, in time, W. (Tab. XXXIV, "Cal. Tables," 5th edition)	=+	1	58	00
(By Prob. VII) M. T. at Greenwich, January 19 ...	=	18	08	00

		°	′	″
*'s Dec. S. (N. A., App. places)	=	10	37	53
(By Prob. X) P. D.	=	100	37	53

		H.	M.	S.
S. D. at Greenwich M. N. (N. A., p. II). Jany. 19	=	19	55	36·02
Correction for Long. W. = 29·5° × 0·6571′′ ...	=	+		19·38
Sum	=	19	55	55·40
For 16*h*. 10*m*. solar time, equivalent sidereal time (Tab. XXXVI, "Cal. Tables," 5th edition)	=	16	12	39·30
(By Prob. V.) S. T. at place	=	12	08	34·70
*'s R. A. (N. A., App. places)	=	13	19	50·41
*'s E. Mer. Dist. or H. A.	=	1	11	15

		H.	M.	S.		°	′	″	
*'s H. A.	=	1	11	15	Cos. 9·978665	(17	48	45)	
λ	=	48°	30′	00″	Cot. 9·946808	..	...		Sin. 9·874 456
Arc I	=	40	06	29	Tan. 9·925 473	...	...		Sec. 0·116 434
*'s P. D.	=	100	37	53					
Arc II	=	60	31	24	... ...	...	..		Cos. 9·692 026
*'s T. Alt.						28	48	24	Sin. 9·682 916
Refracn. (Tab. XXXVIII, "Cal. Tables," 5th edition)	=					+	01	45	
*'s App. Alt.						28	50	09	

Lunar Distances.—Lunar distances which have heretofore been given in the 'Nautical Almanac' are omitted from that publication from 1907. The following examples (from the 'Nautical Almanac') are therefore inserted to show how Lunar Distances may be calculated.

Calculation of a Lunar Distance.

The true distance between the moon and any other heavenly body can be ascertained by working the spherical triangle of which the two sides are the polar distances of the moon and the other heavenly body, and the angle contained is the difference of the right ascension of the moon and the heavenly body.

OBSERVATION.

* Unless the Hour Angle be more than 6*h*. or 90°; in which take the Log. cos. of the *sum* of Arc I and the Polar Distance.

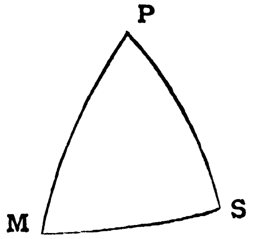
Star and Moon.

On September 12, 1907, at noon, Greenwich Mean Time, from the 'Nautical Almanac,'

	H.	M.	S.
R.A. of moon =	14	54	2·9
R.A. of Spica =	13	20	17·7

		°	′	″
Declination of Moon =	S.	11	43	34
S.P.D. of Moon =		78	16	26
Declination of Spica =	S.	10	40	34
S.P.D. of Spica =		79	19	26

Then in



	°	′	″
PM =	78	16	26
PS =	79	19	26
PS + PM =	157	35	52
PS − PM =	1	3	0
½ (PS + PM) =	78	47	56
½ (PS − PM) =	0	31	30

	H.	M.	S.
Moon's R.A. =	14	54	2·9
Star's R.A. =	13	20	17·7
Difference =	1	33	45·2

Difference = 1 33 45·2 = 23° 26′ 18″ = P. ½ P = 11° 43′ 9″

Then, by Napier's Analogies,

$$\tan.\ \tfrac{1}{2}\,(M+S)=\cos.\ \tfrac{1}{2}\,(PS-PM)\times\sec.\ \tfrac{1}{2}\,(PS+PM)\times\cot.\ \frac{P}{2}$$

$$\tan.\ \tfrac{1}{2}\,(M-S)=\sin.\ \tfrac{1}{2}\,(PS-PM)\times\text{cosec.}\ \tfrac{1}{2}\,(PS+PM)\times\cot.\ \frac{P}{2}$$

			°	′	″		
+0·683110	cot.	½ P	11	43	9	cot.	+0·683110
+0·711632	sec.	½ (PS + PM)	78	47	56	cosec.	+0·008353
+9·999982	cos.	½ (PS − PM)	0	31	30	sin.	+7·962031
+1·394724	tan.	½ (M + S)	87	41	33		
		½ (M − S)	2	34	42	tan.	+8·653494

angle at M = 90 16 15
angle at S = 85 6 51

sin. M : sin. PS : : sin. P : sin. MS (true distance)

sin. P	9·599623
sin. PS	9·992416
cosec. M	0·000005
sin. MS	9·592044

MS 23° 0′ 33″ = true distance.

Therefore 23° 0′ 33″ is the true distance between the Moon and Spica at noon on September 12, 1907.

Sun and Moon.

On August 16, 1907, at 4*h*., Greenwich Mean Time, from the 'Nautical Almanac,'

	H.	M.	S.
R.A. of Moon at 4*h*. =	15	14	59·1
R.A. of Sun at noon (p. II) =	9	39	15·66
* Var. at 2*h*. × 4 =			+37·46
R.A. of Sun at 4*h*. =	9	39	53·1

	°	′	″
Dec. of Sun at noon (p. II.) =	+14°	2′	1·6″
* Var. at 2*h*. × 4 =		−3	7·8
Declination of Sun at 4*h*. =	+13	58	54

		°	′	″
Declination of Moon at 4*h*. =	S.	13	13	37
N.P.D. of Moon =		103	13	37
Declination of Sun at 4*h*. =	N.	13	58	54
N.P.D. of Sun =		76	1	6

	°	′	″
PM =	103	13	37
PS =	76	1	6
PM + PS =	179	14	43
PM − PS =	27	12	31
½ (PM + PS) =	89	37	22
½ (PM − PS) =	13	36	15

OBSERVATION.

* It may be assumed that the variations for Mean Noon are the same as for Apparent Noon (pageI).

		H.	M.	S.
Moon's R.A.	=	15	14	59·1
Sun's R.A.	=	9	39	53·1
Difference	=	5	35	6·0 = 83° 46′ 30″ = P ½ P = 41° 53′ 15″

Then, by Napier's Analogies,

$$\tan.\ \tfrac{1}{2}\ (S + M) = \cos.\ \tfrac{1}{2}\ (PM - PS) \times \sec.\ \tfrac{1}{2}(PM + PS) \times \cot.\ \frac{P}{2}$$

$$\tan.\ \tfrac{1}{2}\ (S - M) = \sin.\ \tfrac{1}{2}\ (PM - PS) \times \mathrm{cosec.}\ \tfrac{1}{2}\ (PM + PS) \times \cot.\ \frac{P}{2}$$

		° ′ ″		
+ 0·047278 cot. … … … … …	½ P =	41 53 15	… … …	cot. + 0·047278
+ 2·181530 sec. … … …	½ (PM + PS) =	89 37 22	… … …	cosec. + 0·000009
+ 9·987641 cos. … … …	½ (PM − PS) =	13 36 15	… … …	sin. + 9 371461
+ 2·216449 tan. … … …	½ (S + M) =	89 39 7		
	½ (S − M) =	14 41 46	… … …	tan. + 9·418748
	S =	104 20 53		
	M =	74 57 21		

sin. M : sin. PS :: sin. P : sin. MS (true distance)

sin. P	9·997432
sin. PS	9·986939
cosec. M	0·015146
sin. MS	9·999517
MS	87° 17′ 50″ = true distance.

Therefore 87° 17′ 50″ is the true distance between the Moon and the Sun at 4*h.* on August 16, 1907.

It is presumed that the Greenwich Time is always known within an hour or so.

By calculating the distance at the even hour before, and after, the approximate Greenwich Time, and taking out the proportional logarithm between the two distances so calculated, then, the observed Lunar Distance having been cleared, as in the example given *ante* under (iii) Lunar Observations in Art. 78.

*By observation on the Moon and a N. A. Star.**—To find the gain in R.A. between the moon and a N. A. star, observe the difference in time of the transits of the moon and star and compare the same with the difference in time of the transits of the moon and the star at Greenwich. The difference of these two differences will be the gain in R.A. of the moon for the difference of longitude, and from the recorded variation for one hour in longitude the difference of longitude can be calculated.

Example.—To determine the longitude of a place R. by observations on the transits of the moon's bright limb, and α Scorpii (Antares) on the 18th June 1864.

Transit of ∗ α Scorpii. *Observed times.* H.	M.	S.		Transit of ☾'s bright limb. *Observed times.* H.	M.	S.
10	17	21·5		10	36	23·0
		42·0				43·5
	18	02·5			37	04·0
		22·5				24·5
		42·0				44·0
10	18	02·01	Times of transit over mean wire	10	37	03·8
				10	18	02·1

∴ Difference at R. in mean solar equivalents … …= 0 19 01·7

and difference at R. in sidereal equivalents (*i.e.*, sidereal interval between transits of ☾'s bright limb at R.) … …= 0 19 04·82

R A. of ☾ I.U. at transit at Greenwich on 18th June …= 16 53 38·16

R.A. of α Scorpii … … … …= 16 21 08·46

OBSERVATION.

* Adopted from the Roorkee Treatise on Surveying.

		H.	M.	S.
Sidereal interval between transits of ☾'s bright limb and ✱ at G.	=	0	32	29·70
Do. do. do. at R.	=	0	19	04·82
Change in R A. of bright limb of ☾ due to longitude of R.	=	0	13	24·88

		M.	S.
But variation of ☾'s R.A in 1 hour of longitude at upper transit at G., 18th June	=	+2	35·71
Do. do. 12 hours previous ...	=	+2	32·71
Change of variation in 12 hours	=	+	3·00
∴ Change of variation in 5*h.* 10*m.* approximate longitude E	=	−	1·29
∴ Variation of R.A of ☾'s bright limb in 1 hour at R.	=	+2	34·42
Do. do at G.	=	+2	35·71
Do. do. in 1 hour half way between	=	+2	35·06

But total change ... = 13 24·88

$$\therefore \text{Longitude of place (R.)} = \frac{13m.\ 24{\cdot}88s.}{2\quad 35{\cdot}06} = \frac{804{\cdot}88}{155{\cdot}06}\ \text{(hours)} = 5h.\ 11m.\ 26s.,\ \text{E}$$

CELESTIAL SPHERE.

79. **Names and Positions of the Principal Stars.*** (Figs. 488—489*a*).†

The stars, for the purpose of finding any one more readily, are divided into parcels called *Constellations* These, in order to assist the imagination, are supposed to be circumscribed with some known figure, as that of a *man, woman, ship, sextant,* etc.; and those stars which lie between constellations are called *Unformed Stars*. As it would be an endless task to give a proper name to each star, it has therefore been customary to mark the stars of each constellation with the letters of the Greek alphabet,‡ in such a manner, that the first letter is prefixed to the brightest star, the second letter to the next in brightness, and so on. Many of the brightest of the fixed stars have also proper names—thus, α Boötis, is also called *Arcturus*; β Virginis, is called *Vindemiatrix*, etc. Besides this system of notation, it has been found necessary to denote the fainter stars by numbers—as, 20 Ceti, B.A.C. (British Association Catalogue), 5140, etc.

The celestial sphere is divided into three parts, the *Zodiac*, and the *Northern* and *Southern Hemispheres*

The zodiac extends to about 8° on each side of the ecliptic, and contains the orbits of all the planets; there are 12 constellations in the zodiac.§ According to the ancients, there were 21 constellations in the Northern Hemisphere and 15 in the Southern; and consequently, 48 constellations in the zodiac and both hemispheres. Modern astronomers, however, by curtailing several of the ancient constellations of some of their stars, which they formed into new constellations, and by forming into constellations the unformed stars, or those which lay between the ancient ones, have increased the number of constellations in the Northern Hemisphere to about 30, and those in the Southern to about 46; and consequently

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OBSERVATIONS.

* From "Mackay on Longitude" and "Drew's Manual of Astronomy."

† Reduced from the maps of Colonel James, R. E., F.R S., etc.

‡ The following are the letters of the Greek alphabet, in order, with their respective names:—α (Alpha) β (Beta), γ (Gamma), δ (Delta), ε (Epsilon), ζ (Zēta), η (Ēta), θ (Thēta), ι (Iōta), κ (Kappa), λ (Lambda) μ (Mu), ν (Nu), ξ (Xi), ο (Omicron), π (Pi), ρ (Rho), σ (Sigma), τ (Tau), υ (Upsilon), φ (Phi) χ (Chi), ψ (Psi) ω (Omĕga).

§ The effect of precession, which throws back the place of the equinox among the stars about 50″ a year, has caused a displacement of the signs of the zodiac with respect to corresponding constellations. The sign Taurus, at present occupies the constellation Aries, the sign Gemini the constellation Taurus, and so on, the signs having retreated among the stars about 30° since the present division of the zodiac was adopted. In consequence, therefore, of this retrograde movement of the "first point of Aries" along the ecliptic, the *constellations of the zodiac* should be distinguished from the *signs of the zodiac* bearing the same names. The N. and S. poles of heavens $\frac{\text{were}}{\text{will be}}$ in $\frac{\text{11684 B.C.}}{\text{11784 A.D.}}$ at the points shown in Figs. 488—489*a* of R.A. XVIII and Dec. N. 40 18′, and R. A. VI and Dec. S. 40° 18′, respectively; *vide*, also, the 2nd footnote to Art. 22, Chap. XVI, *ante*.

there are about 88 constellations in all. The names of these constellations are as follows :—

*Zodiacal Constellations.**

1.	Aries	The Ram	Northern.	7	Libra	The Balance	Southern.
2.	Taurus	„ Bull		8.	Scorpius	„ Scorpion	
3.	Gemini	„ Twins		9.	Sagittarius	„ Archer	
4.	Cancer	„ Crab		10.	Capricornus	„ Goat	
5.	Leo	„ Lion		11.	Aquarius	„ Water-bearer	
6.	Virgo	„ Virgin		12.	Pisces	„ Fishes	

Northern Constellations.

1.	Ursa Minor	The Little Bear.	18.	Delphinus	The Dolphin.
2.	Ursa Major	„ Great Bear (The Plough).	19.	Equuleus	„ Horse Head (or Littles).
3.	Draco	„ Dragon	20.	Pegasus	„ Flying (or Winged) Horse.
4.	Cepheus	Cepheus (The Monarch).			
5.	Boötes	Boötes (The Herdsman).	21.	Andromeda	„ Andromeda (The Chained Lady).
6.	Corona Borealis	The Northern Crown.			
7.	Hercules	Hercules.	22.	Triangulum (Borealis)	„ The Northern Triangle.
8.	Lyra	The Harp (or Lyre).			
9.	Cygnus	„ Swan.	23.	Coma Berenices	„ Berenice's Hair.
10.	Cassiopea	„ Cassiopea (The Lady's chair).	24.	Camelopardus	„ Camelopard (or Giraffe).
11.	Perseus	„ Perseus(The Rescuer).	25.	Monoceros	„ Unicorn.
12.	Auriga	„ Waggoner.	26.	Lynx	„ Lynx.
13.	Serpentarius	Serpentarius.	27.	Leo Minor	„ Little Lion.
14.	Ophiuchus	The Serpent-bearer.	28.	Canes Venatici	„ Greyhounds (or Hunters).
15.	Serpens	„ Serpent.			
16.	Sagitta	„ Arrow.	29.	Vulpecula	„ Fox.
17.	Aquila	„ Eagle.	30.	Lacerta	„ Lizard.

Southern Constellations.

1.	Cetus	The Whale (or Sea-Monster.	26.	Chameleon	The Chameleon.
			27.	Triangulum Australis.	„ Southern Triangle.
2.	Orion	Orion (The Giant Hunter).	28.	Pisces Volans	„ Flying Fish.
3.	Eridanus	The River Eridanus.	29.	Dorado	„ Sword Fish.
4.	Lepus	„ Hare.	30.	Toucan	„ American Goose (or Toucan).
5.	Canis Major	„ Great Dog.			
6.	Canis Minor	„ Little Dog.	31.	Hydrus	„ Water-Snake.
7.	Puppis Argo Navis	„ Stern of the Ship Argo.	32.	Sextans	„ Sextant.
8.	Vela „ „	„ Sails „ „	33.	Apparatus Sculptoris	„ Apparatus of the Carver (or the Sculptor's Tool).
9.	Carina „ „	„ Keel „ „			
10.	Hydra	„ Sea Serpent.	34.	Fornax Chemica	„ Chemical Furnace.
11.	Crater	„ Cup.	35.	Horologium	„ Clock.
12.	Corvus	„ Crow.	36	Reticulum	„ Rhomboid (or Net).
13.	Centaurus	„ Centaur.	37.	Cœlum Sculptoris	„ Graving Tool (or the Engraver).
14.	Lupus	„ Wolf.			
15.	Ara	„ Altar.	38.	Equuleus Pictorius	„ Painter's Easel.
16.	Corona Australis	„ Southern Crown	39.	Pixis Nautica	„ Mariner's Compass.
17.	Pisces Australis	„ Southern Fish.	40.	Antlia Pneumatica	„ Air-Pump.
18.	Columba Noachi	Noah's Dove.	41.	Octans	„ Octant or Hadley's Quadrant.
19.	Grus	The Crane			
20.	Phœnix	„ Phœnix.	42.	Circinus	„ Compass.
21.	Indus	„ Indian.	43.	Norma	„ Square and Rule.
22.	Pavo	„ Peacock.	44	Telescopium	„ Telescope.
23.	Apus	„ Bird of Paradise	45.	Microscopium	„ Microscope.
24.	Musca	„ Fly (or Bee).	46.	Mons Mensæ	„ Table Mountain.
25.	Crux	„ Cross.			

The number of stars of the first magnitude in the zodiac and in both hemisperes do not amount to twenty.

The following method of tracing the relative position of the constellations will facilitate an acquaintance with the principal stars, and will be found useful in connection either with maps of the stars, as given in the Figures, or the celestial globe:—

We will suppose the day of observation to be January the 1st, at 8 o'clock in the evening; though the directions given will apply equally well on any other day

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OBSERVATION.

* In English rhyme these are as under :—

" The Ram, the Bull, the Heavenly Twins,
And next the Crab, the Lion shines,
The Virgin and the Scales,
The Scorpion, Archer, and Sea-goat,
The Man that bears the watering-pot,
And Fish with glittering tails."

of the year, to such of the constellations as may at that time be above the horizon, if it be borne in mind that they will then be differently circumstanced with regard to that circle.

The most remarkable constellation in the northern hemisphere is the *Great Bear*. Occupying, as it does, a large space in the heavens, and in our latitude being always visible, it may be discovered without difficulty, by remarking its form on the globe; and by means of it, when once known, we may gain acquaintance with the other constellations.

Four of the seven principal stars composing the *Great Bear* are in the form of a trapezium. Two of them α and β, called the *Pointers*, point directly to a star in the *Little Bear*, which is so near the north pole that it has obtained the name of *Polaris* (or Al-ruccabah), or the *Pole Star*. On the 1st of January, at 8 o'clock, the *Great Bear* will be midway between the horizon and the north *Polar Star*, in the direction N.N.E. The *Pole Star* is in the extremity of the tail of the *Little Bear*, which constellation resembles the *Great Bear* in form, but is inferior to it both in the space it occupies in the heavens and in the magnitude of its component stars.

Between the *Great* and *Little Bear* lies part of the tail of *Draco*; which constellation half surrounds the *Little Bear*, and returns into itself with a serpentine curve. The two bright stars in the head, β and γ *Draconis*, are about as far from *Polaris*, the other side of the meridian, as the *Pointers* are on the east.

Continue the line formed by the last two stars in the tail of the *Great Bear* about five times their distance, and it will pass near a bright red star (which will be below the horizon, however, at the time we are supposing): this is *Arcturus* in *Bootes*.

The bright star called *Cor Caroli* may be found in line with α in the tail of *Draco*, and ζ in the tail of *Ursa Major*: at the time we are supposing, it will be just above the horizon in the N.N.E.

Cassiopea will be near the zenith directly opposite to the *Great Bear*, the *Pole Star* being nearly equidistant from both.

The line which is formed by the *Pointers* and *Polaris* being continued, will run between *Cepheus* and *Cassiopea*; the former adjoining *Draco*.

The same line continued between *Cassiopea* will pass β and α *Pegasi*; these two, with α *Andromeda* and γ *Pegasi*, form an irregular square. The most northerly forms the head of *Andromeda*; the opposite one is called *Markab*; the other two γ and β *Pegasi* are *Algenib* and *Scheat*. The diagonal of this square being continued towards the north, will meet β and γ *Andromeda* and α *Persei*; this last, at the time we have supposed, being very near the Zenith. These seven stars, it will be remarked, are not unlike the constellation of the *Great Bear* on a large scale, and in a reversed order.

Join α and γ *Ursæ Majoris*: when prolonged, this line will reach α *Persei*; and further on, *Algol*, in *Medusa's* head, a star rendered singular from its constantly varying its brilliancy every three days.

Produce a line formed by joining γ and β *Pegasi* towards the north, it will pass through the centre of *Cygnus*. The bright star *Vega*, or α *Lyræ*, will be below *Cygnus*, towards the north; on the other side, lower down towards the horizon, *Delphinus* will be setting due west.

Gemini will be found by joining γ and β *Ursæ Majoris*, and producing it south. The two principal stars are *Castor* and *Pollux*.

Facing the S.E., on the day and hour in question, you will have before you the most splendid constellation in the heavens, *Orion*, with his belt and sword; the former being three bright stars at equal distances. The line of these three continued some distance to the south will point to α *Canis Majoris*, or *Sirius*, the brightest star in the heavens.

Above *Orion*, to the westward, will be the red star *Aldebaran*, in the eye of *Taurus*; in the neighbourhood of which are the two beautiful clusters, the *Hyades* and *Pleiades*: these latter have retained, till the present time, the names by which they were known to the poet Homer 950 years before the Christian era.

LATITUDES AND LONGITUDES OF PLACES.

80. LATITUDES, LONGITUDES AND HEIGHTS OF PLACES OF NOTE. These values refer to places almost exclusively in India and Burma, while the positions of some of the other principal public and private observatories are also given. The longitudes are all reckoned from the meridian of Greenwich.

Note.—(1) All the former Longitudes of the Indian and Burma stations marked * were referrible to the old value of the Madras Observatory, 80° 17′ 21″, to which a correction of—03′ 01·5″ was required to reduce to the value given in the 'Nautical Almanac' for 1862 and subsequently, but the values now given are free from this discrepancy, and, further, have reference to the most recent value of Madras, namely, L. 80° 14′ 47″ E.

(2) (I) stands for India and (B) for Burma.

(3) Places marked with an asterisk (*), denote that their values have been derived from the records of the Great Trigonometrical Survey of India.

(4) Standard Time † was introduced into India and Burma from the 1st July 1905. Indian Standard Time is 5*h.* 30*m.* (corresponding to longitude 82° 30′ E.) and Burma Standard Time is 6*h.* 30*m.* (corresponding to longitude 97° 30′E.) in advance of Greenwich Time. The former is therefore ahead of Madras Time (= 5*h.* 21*m.* 00*s.* E., corresponding to longitude 80° 14′ 54″ E.) by 09*m.* 00*s.* and the latter of Rangoon Time (= 6*h.* 24*m.* 40*s.* E.), corresponding to longitude 96° 10′ E.) by 5*m.* 20*s.*

The following is a list of Standard Times that have been adopted in various places for railway and other purposes, referred to the Meridian of Greenwich :—East-Europe, 2h. E. ; Mid-Europe, 1h. E. ; South Africa, 2h. E. ; Japan, 9h. E. ; America : Intercolonial, 4h. W. ; Eastern, 5h. W. ; Central, 6h. W. ; Mountain, 7h. W. ; Pacific, 8h. W. ; Australia : West Australia, 8h. E. ; South Australia, 9½h E. ; Victoria, 10h. E. New South Wales, 10h. E., Queensland, 10h. E. ; Tasmania, 10h. E. ; New Zealand, 11½h. E.

OBSERVATIONS.

* *See* footnote at page 258 *seq.* The *net* correction thus applied to the old value of Madras = —02′ 33·9″ (*i.e.*, 80° 17′ 21″—80° 14′ 47·1″) instead of the former corection of — 03′ 1·5″.

† Standard time is now adopted in most civilised countries, and is referred to the meridian of Greenwich. The globe is supposed to be divided into "hour-zones" extending 7° 30′ on each side of the central meridians, and the local mean time (standard) is used for all places within that zone. The central lines of the zones are thus 0°, 15°, 30°, 45°, and so forth, or 15° apart E. or W. of the common standard. But in Cape Colony and some of the other South African Colonies, New Zealand, India and Burma, for reasons of practical convenience, the central meridians have been fixed at 1½ hours, 11½ hours, 5½ hours and 6½ hours, respectively, in advance of Greenwich time ; that is, 22° 30′, 172° 30′, 82° 30′ and 97° 30′ E., respectively, of the common meridian. On the meridian of 180° the time differs by 12 hours, east or west, from that of Greenwich, and provision has consequently to be made here for the change of date + or — according as the movement is eastward or westward. To the east of the line the date prevails which has come round gradually retreating by America, and to the west of the line the date that has come round gradually advancing by Europe. Thus, if on the "date line" the date brought *viâ* America were the 15th, that brought *viâ* Europe would be the 14th ; and the change would be made to the 14th if moving west, and to the 15th if moving east of the line of division, respectively.

Place.	Kingdom, Province or State.	Latitude.	Longitude.		Height above M. Sea Level.
			In Arc.	In time.	
		° ′ ″	° ′ ″	h. m. s.	Feet.
Adelaide	Australia	34 55 39 S.	138 35 05 E.	9 14 20 E.	141
Aden Light-house, Marshag	Arabia	12 45 43 N.	45 00 46 E.	3 00 15 E.	...
Agra Fort *	United Provinces(I)	27 10 53 N.	78 01 12 E.	5 12 05 E.	551
Ahmedabad City Tower Clock *	Bombay (I)	23 01 26 N.	72 34 47 E.	4 50 19 E.	...
Ahmednagar City	„ „	19 05 55 N.	74 44 23 E.	4 58 59 E.	...
Ajmere, Taragarh Fort *	Rajputana (I)	26 26 30 N.	74 36 57 E.	4 58 28 E.	...
Akyab Point, G. T. S. Station*	Arakan (B)	20 06 45 N.	92 53 50 E.	6 11 36 E.	10
„ G. T. S Station, on old Temple *	„ „	20 08 53 N.	92 52 36 E.	6 11 31 E.	72
Allanmyo, G. T. S. Station, on Hill *	Minbu (B)	19 22 27 N	95 13 26 E.	6 20 54 E.	281
Alguada Light-house *	Irrawaddy (B)	15 42 14 N.	94 12 03 E.	6 16 48 E.	4
Allahabad Fort Flag Staff *	United Provinces(I)	25 26 00 N.	81 52 41 E.	5 27 31 E.	...
Amherst, G. T. S. Station *	Tenasserim (B)	16 04 53 N.	97 33 51 E.	6 30 16 E	14
„ Pagoda *	„ „	16 04 14 N.	97 33 38 E.	6 30 15 E.	...
Amritsar *	Punjab (I)	31 37 12 N	74 42 37 E.	4 58 51 E.	...
Arcot Fort, Nabob's House *	Madras (I)	12 54 16 N.	79 19 57 E.	5 17 20 E.	...
Ballasore Spire *	Bengal (I)	21 30 12 N.	86 55 42 E.	5 47 43 E.	...
Bangalore, Commissioner's Flag Staff *	Mysore (I)	12 58 52 N.	77 35 30 E.	5 10 22 E.	3,131
Bangkok, British Consul-General's Flag Staff.*	Siam	13 44 33 N.	100 32 28 E.	6 42 10 E.	†22
Bangkok, Phukhacthong Station *	„	13 45 10 N.	100 30 29 E.	6 42 02 E.	137
„ H. M. King's Flag Staff *	„	13 45 02 N.	100 29 32 E.	6 41 58 E.	...
Bareilly Church *	United Provinces(I)	28 22 05 N.	79 25 31 E.	5 17 42 E.	560
Baroda Clock Tower *	Bombay (I)	22 17 59 N.	73 12 31 E.	4 52 50 E.	185
Bassein Jail *	Irrawaddy (B)	16 46 44 N.	94 44 18 E.	6 18 55 E.	40
Bellary Fort *	Madras (I)	15 08 57 N.	76 51 38 E.	5 07 39 E.	1,976
Benares Hindoo Observatory *	United Provinces(I)	25 18 26 N.	83 00 38 E.	5 32 03 E.	...
Berlin	Germany	52 30 17 N.	13 23 43 E.	0 53 35 E.	112
Bhamo *	Mandalay (B)	24 15 27 N.	96 20 36 E.	6 25 22 E.	391
Bhawalpore City *	Bhawalpore (I)	29 23 45 N.	71 40 32 E.	4 46 42 E.	...
Bhiling Hill *	Tenasserim (B)	17 13 59 N.	97 14 22 E.	6 28 55 E.	193
Bhopal Palace	Central India (I)	23 15 35 N.	77 23 22 E.	5 09 34 E.	...
Bikanir *	Rajputana (I)	28 01 19 N.	73 20 20 E.	4 53 20 E.	...
Blue Mountain *	Arakan (B)	21 21 30 N.	93 03 15 E.	6 12 13 E.	...
Bombay (Colaba) *	Bombay (I)	18 53 45 N.	72 48 56 E.	4 51 16 E.	114
Brisbane	Australia	27 28 00 S.	153 01 36 E.	10 12 06 E.	...
Burdwan Town	Bengal (I)	23 15 50 N.	87 51 07 E.	5 51 25 E.	...
Cachar Church *	Assam (I)	24 49 42 N.	92 48 14 E.	6 11 13 E.	...
Cachar Tomb *	„ „	24 49 30 N.	92 48 02 E.	6 11 12 E.	...
Calcutta, Fort William, Semaphore Tower *	Bengal (I)	22 33 25 N.	88 20 07 E.	5 06 41 E.	...
„ St. Xavier's College Observatory	„ „	22 32 51 N.	88 21 21 E.	5 53 25 E.	...
Calicut, Flag Staff *	Madras (I)	11 15 09 N.	75 46 14 E.	5 03 05 E.	...
Cambridge	England	52 12 52 N.	0 05 41 E.	0 00 23 E.	92
Cannanore, Flag Staff *	Madras (I)	11 51 12 N.	75 22 10 E.	5 01 29 E.	...
Cape Comerin *	Travancore (I)	08 04 00 N.	77 33 07 E.	5 10 13 E.	...
Cape of Good Hope	Africa	33 56 04 S.	18 28 41 E.	1 13 55 E.	52
Cawnpore Christ Church	United Provinces(I)	26 28 15 N.	80 21 11 E.	5 21 25 E.	408
Chaito Town Pagoda *	Tenasserim (B)	17 18 50 N.	97 01 48 E.	6 28 07 E.	...
China Bakeer New Light-house	Pegu (B)	16 16 33 N.	96 11 37 E.	6 24 47 E.	29
Chittagong Judge's Court *	Bengal (I)	22 21 03 N.	91 50 10 E.	6 07 21 E.	...
Cochin Flag Staff *	Cochin (I)	9 58 07 N.	76 14 26 E.	5 04 58 E.	...
„ Light-house	„ „	9 57 47 N.	76 11 11 E.	5 04 45 E.	...
Colombo	Ceylon	6 56 06 N.	79 46 26 E.	5 19 18 E.	...
Coorg, Mercara Fort *	Coorg (I)	12 25 11 N.	75 44 19 E.	5 02 55 E.	3,804
Cutch, Bhooj Town Temple	Bombay (I)	23 14 52 N.	69 40 47 E.	4 38 39 E.	...
Cuttack Fort *	Bengal (I)	20 29 04 N.	85 52 55 E.	5 43 28 E.	132
Dacca City	„ „	23 42 30 N.	90 23 26 E.	6 01 34 E.	...
Darjeeling Church *	„ „	27 02 48 N.	88 16 02 E.	5 53 04 E	...
Dehra Dun (New Observatory)	United Provinces(I)	30 19 29 N.	78 02 54 E.	5 12 12 E,	2,255
„ West End Base *	„ „	30 19 36 N.	77 51 35 E.	5 11 26 E.	1,958
Delhi Fort *	Punjab (I)	28 38 58 N.	77 13 56 E.	5 08 56 E.	...
Dholpore *	Rajputana (I)	26 41 40 N.	77 50 48 E.	5 11 23 E.	...
Diamond Island, Flag Staff *	Irrawaddy (B)	15 51 48 N.	94 16 48 E.	6 17 07 E.	124
Dibrugarh Church *	Assam (I)	27 29 13 N.	94 54 28 E.	6 19 38 E.	394
Dinajpoor Station *	Bengal (I)	25 36 34 N.	88 37 38 E.	5 54 31 E.	...
Double Island Light-house *	Tenasserim (B)	15 52 26 N.	97 35 12 E.	6 30 21 E	89
Dublin (Dunsink)	Ireland	53 23 13 N.	6 20 17 W.	0 25 21 W.	157
Eastern Grove Light	Pegu (B)	16 30 01 N.	96 23 15 E.	6 25 33 E.	124
Edinburgh (Blackford Hill)	Scotland	55 55 28 N.	3 11 03 W.	0 12 44 W.	348
Elephant Point Column *	Pegu (B)	16 28 43 N.	96 20 42 E.	6 25 19 E.	142
Fyzabad, Sooja Dowla's Tomb *	United Provinces(I)	26 46 45 N.	82 09 26 E.	5 28 38 E.	332
Ganjam Fort *	Madras (I)	9 22 30 N.	85 03 25 E.	5 40 14 E.	...
Garhwal, Badrinath Hill *	United Provinces(I)	30 44 16 N.	79 17 46 E.	5 17 07 E.	23,210
Gauhati Church *	Assam (I)	26 11 17 N.	91 44 53 E.	6 07 00 E.	268
Glasgow	Scotland	55 52 43 N.	4 17 38 W.	0 17 11 W.	...
Goa, St. Dennis' Church *	Portuguese Territory (I).	15 21 24 N	73 53 56 E	4 55 36 E.	...

OBSERVATION.

† Ground level of Bangkok.

P. 264, line 43, column 5, *for* "5 06 41 E." "5 53 20 E."

Place.	Kingdom, Province or State.	Latitude.	Longitude.		Height above M. Sea Level.
			In Arc.	In Time.	
		° ′ ″	° ′ ″	*h. m. s.*	Feet.
Goalpara Hill over City *	Assam (I)	26 10 53 N.	90 37 58 E.	6 02 30 E.	399
Gorakhpur Jail *	United Provinces (I)	26 44 44 N.	83 21 03 E.	5 33 24 E.	225
Greenwich (Royal Observatory)	England	51 28 38 N.	0 00 00	0 00 00	154
Gwalior Hill Temple *	Central India (I)	26 13 10 N.	78 09 54 E.	5 12 40 E.	...
Henzada Jail *	Irrawaddy (B)	17 39 00 N.	95 27 43 E.	6 21 51 E.	44
Hongkong	China	22 18 13 N.	114 20 28 E.	7 36 42 E.	...
Htantabeng G. T. S. Station *	Pegu (B)	16 30 37 N.	96 12 19 E.	6 24 49 E.	21
Hyderabad, Residency Flag Staff *	Hyderabad (I)	17 23 14 N.	78 27 59 E.	5 13 52 E.	1,977
,, Morning Gun	Sind (I)	25 23 05 N.	68 22 17 E.	4 33 29 E.	66
Indore Palace *	Central India (I)	22 43 06 N.	75 51 15 E.	5 03 25 E.	...
Jabalpore, Deputy Commissioner's Office *.	Central Provinces (I)	23 09 58 N.	79 56 56 E.	5 19 48 E.	1,306
Jesalmir Fort *	Rajputana (I)	26 54 40 N.	70 54 43 E.	4 43 39 E	959
Jeypore *	,, ,,	26 55 28 N.	71 49 12 E.	4 47 17 E.	...
Jeypur Town	Jeypur (I)	18 51 27 N.	82 34 06 E.	5 30 17 E.	...
Jodhpore Fort *	Rajputana (I)	26 17 43 N.	73 01 02 E.	4 52 04 E.	...
Juggurnath Great Temple	Bengal (I)	19 48 17 N.	85 40 05 E.	5 43 16 E.	...
Kabul	Afghanistan	34 30 00 N	69 05 28 E.	4 36 22 E.	6,400
Kalagauk Island *	Tenasserim (B)	15 33 55 N.	97 39 27 E	6 30 38 E.	...
Kalianpur * (Origin of G. T. S.)	Central India (I)	24 07 11 N.	77 39 18 E.	5 10 37 E.	1,765
Kashmere, Hari Parbat, S. Bastion *	Kashmere (I)	34 06 19 N.	74 48 53 E.	4 59 16 E.	...
Katha Town	Mandalay (B)	24 09 28 N.	96 20 26 E.	6 25 22 E.	307
Katmandu	Nepaul (I)	27 41 00 N.	85 18 28 E.	5 41 14 E.	...
Kew	England	51 28 06 N.	0 18 47 W.	0 01 15 W.	33
Khelat	Beluchistan	28 53 00 N.	66 28 28 E.	4 25 54 E.	6,700
Kota Fort *	Rajputana (I)	25 10 27 N.	75 49 46 E	5 03 19 E.	...
Kurnool, Judge's Court *	Madras (I)	15 49 32 N.	78 01 59 E.	5 12 08 E.	941
Kurrachee Observatory *	Sind (I)	24 49 49 N.	67 01 29 E.	4 28 06 E.	35
Kutubdia Light-house *	Arakan (B)	21 52 30 N.	91 50 26 E.	6 07 22 E.	146
Kyaukpyu, Westernmost Bungalow *	,, ,,	19 26 02 N.	93 32 05 E.	6 14 08 E.	...
Kyauktan Pagoda *	Pegu (B)	16 37 52 N.	96 19 33 E.	6 25 18 E.	122
Lahore, Badshahi Masjid *	Punjab (I)	31 35 18 N.	74 18 30 E.	4 57 14 E.	...
Landour, Catholic Chapel *	United Provinces (I)	30 27 50 N.	78 05 43 E.	5 12 23 E.	7,559
Legu, Court-house *	Pegu (B)	17 05 47 N.	96 13 13 E.	6 25 05 E.	...
Leh Fort *	Kashmere (I)	34 09 30 N.	77 34 09 E	5 10 17 E.	11,280
Lemeina Stone Pagoda*	Pegu (B)	17 36 02 N.	95 08 36 E.	6 20 35 E.	80
Lhassa	Thibet	29 39 17 N.	90 57 09 E.	6 03 49 E.	11,707
Liverpool (Bidston, Birkenhead)	England	53 24 05 N.	3 04 20 W.	0 12 17 W.	200
Lucknow Observatory *	United Provinces (I)	26 51 10 N.	80 56 23 E.	5 23 46 E.	...
Madras Observatory * †	Madras (I)	13 04 08 N.	80 14 47 E.	5 20 59 E.	23
Madura, S. Guparam of Temple *	,, ,,	9 55 04 N.	78 07 08 E.	5 12 29 E.	...
Malacca, Flag Staff	Malacca	2 10 30 N.	102 11 38 E.	6 48 47 E.	...
Mandalay, Mandalay Hill *	Mandalay (B)	22 00 50 N.	96 06 32 E.	6 24 26 E.	770
,, King's Palace	,, ,,	21 59 34 N.	96 05 50 E.	6 24 23 E.	253‡
Manipur, west of Raja's Palace *	Manipur State (I)	24 48 24 N.	93 56 25 E.	6 15 46 E.	2,568
Martaban Pagoda on Hill, near Circuit-house	Tenasserim (B)	16 31 35 N.	97 36 54 E.	6 30 28 E.	273
Masulipatam Fort Church	Madras (I)	16 09 10 N.	81 08 55 E.	5 24 36 E.	...
Mawdeng Great Pagoda *	Irrawaddy (B)	15 57 15 N.	94 14 46 E.	6 16 59 E.	148
Meerut Church *	United Provinces (I)	29 00 41 N.	77 42 29 E.	5 10 50 E.	735
Meiktila, Yuba Hill *	Meiktila (B)	20 53 30 N.	96 14 15 E.	6 24 59 E.	3,500
Melbourne New Observatory	Australia	37 49 53 S.	144 58 32 E.	9 39 54 E.	92
Mergui, Court-house *	Tenasserim (B)	12 26 16 N.	98 35 55 E	6 34 44 E.	123
Middle Moscos, Northern Island *	,, ,,	14 10 19 N.	97 48 22 E.	6 31 14 E.	1,176
,, Southern ,, *	,, ,,	14 07 28 N.	97 48 30 E.	6 31 14 E.	1,007
Midnapore Park-house *	Bengal (I)	22 24 48 N.	87 18 38 E.	5 49 14 E.	...
Mirzapore Court-house *	United Provinces (I)	25 09 19 N.	82 34 49 E.	5 30 19 E.	...
Mõnghsat, old Pagoda *	Tributary Shan States (B).	20 32 14 N.	99 15 20 E.	6 37 02 E.	1,944
Mooltan Railway Station *	Punjab (I)	30 11 58 N.	71 28 13 E.	4 45 53 E.	408
Moorshedabad City	Bengal (I)	24 11 05 N.	88 16 16 E.	5 53 05 E.	...
Mopun Pagoda *	Tenasserim (B)	16 26 17 N.	97 37 02 E.	6 30 28 E.	...
Moulmein G. T. S. Station, near Hill Flag Staff. *	,, ,,	16 28 46 N.	97 37 49 E.	6 30 31 E.	288
Moulmein, Master Attendant's Flag Staff	,, ,,	16 28 58 N.	97 37 13 E.	6 30 29 E.	...
Mount Abu *	Rajputana (I)	24 35 37 N.	72 42 42 E.	4 50 51 E.	...
Mount Everest Peak *	Nepaul (I)	27 59 17 N.	86 55 32 E	5 47 42 E.	29,002
Mount Hamilton (Lick) Observatory	America U. S.	37 20 25 N.	121 38 43 W.	8 06 35 W.	4,209
Mussoorie Church *	United Provinces (I)	30 27 36 N.	78 04 03 E.	5 12 16 E.	6,647
Myanaung G. T. S. Station, in hospital compound. *	Irrawaddy (B)	18 17 53 N.	95 18 58 E.	6 21 14 E.	74
Myinmati, Fort Steadman *	Southern Shan States (B).	20 36 21 N.	97 01 24 E.	6 28 06 E.	6,458

OBSERVATIONS.

† The Burrard-Conyngham telegraphic determination of the longitude of Madras as obtained in 1895-96, is 5*h.* 20*m* 59·137*s.* (in time) = 80° 14′ 47·055″ (in arc.); a value which is 0·284*s.* = 4·260″ less than that which has hitherto been regarded as the most accurate available, *viz.*, the Campbell-Heaviside electro-telegraphic determination in 1878 of 5*h.* 20*m.* 59·421*s.* (in time) = 80° 14′ 51·315″ (in arc). Since then (*vide* N. A. for 1909), the longitude, in time, has been fixed at 5*h.* 20*m.* 59·613*s.* (= 80° 14′ 54·20″) which is the most recent *geodetic* value as distinguished from the former *astronomical* value. Kalianpur is the astronomical origin of latitude and longitude of the Indian G. T. Survey. The astronomical values for Kalianpur and Madras are 77° 39′ 17·57″ and 80° 14′ 47·06″, respectively: the difference thus is 2° 35′ 29·49″. But as measured by the *revisionary* triangulation the difference in longitude between the two places is 2° 35′ 36·63″, which is 7·14″ *more* than the astronomical determination, as against 6·76″ *more* by the *old* triangulation when the difference in longitude was put at 2° 35′ 36·25″. The error in the value of longitude adopted for the triangulation of the G. T. Survey is thus + 2′ 26·80″ (*i.e.*, 80° 17′ 21″ − 80° 14′ 47·06″ − 7·14″) by the *new* angular measurements, as against + 2′ 27·18″ (*i.e.*, 80° 17′ 21″ − 80° 14′ 47·06″ − 6·76″) by the *old* angular measurements. If therefore the astronomical value for the longitude of Kalianpur were adopted as the basis, the longitude of Madras calculated from the *revised* triangulation would be 80° 14′ 51·20″ (*i.e.*, 77° 39′ 17·57″ + 2° 35′ 36·63″). Accordingly, for all Indian maps the longitudes are now referrible to the Greenwich meridian taking the longitude of Madras—

(i) for the Map of India on the scale of 1 : 1,000,000, as 80° 14′ 54″ E. (Gore, 1900), the most recent geodetic value;
(ii) for the new Atlas sheets, as 80° 18′ 30″E. (Lambton, 1805); these require a correction of—1′ 9″ to make them accord with the G. T. Survey and a further correction of −2′ 27″ to make them accord with the most recent value of the geodetic longitude of Madras Observatory, *viz.*, 80° 14′ 54″;
(iii) for standard sheets and all mapping based on the longitude of the G. T. Survey, as 80° 17′ 21″ E. (Warren, 1815); these require a correction of—2′ 27″ to make them accord with the most recent value of the geodetic longitude of Madras Observatory, *viz.*, 80° 14′ 54″.

The latest values of the latitude and longitude of Madras are (N. A., 1,910): λ. 13° 04′ 08·0″ N., L. 5*h.* 20*m.* 59·62*s.* E. (= 80° 14′ 54·30″ E.); and of Kalianpur are: λ. 24° 7′ 11·26″ N., L. 77° 39′ 17·57″ E. (= 5*h.* 10*m.* 37·168*s.* E.).

‡ Ground level (deducted) of palace.

Place.	Kingdom, Province or State.	Latitude.	Longitude		Height above M. Sea Level.
			In Arc.	In Time	
		° ′ ″	° ′ ″	h. m. s.	Feet.
Myo Telegraph bungalow *	Arakan (B)	20 16 38 N.	92 44 35 E.	6 10 58 E.	...
Mysore Palace *	Mysore (I)	12 18 17 N.	76 39 14 E.	5 06 37 E.	...
Nagpur Town	Central Provinces (I)	21 09 10 N.	79 06 20 E.	5 16 26 E.	...
Nanthakun, or Fowl Island, Central Hill ...	Arakan (B)	18 03 51 N.	94 05 35 E.	6 16 22 E.	...
Nellore Church Tower *	Madras (I)	14 20 38 N.	79 58 54 E.	5 19 56 E.	...
New York, Columbia College	America U. S.	40 45 23 N.	73 58 25 W.	4 55 54 W.	...
Nowgong Jail *	Assam (I)	26 20 33 N.	92 40 56 E.	6 10 44 E.	218
Nuguan Island, Tree *	Arakan (B)	18 26 07 N.	93 54 29 E.	6 15 58 E.	...
Oodepore *	Rajputana (I)	24 35 19 N.	73 40 49 E.	6 54 43 E.	...
Oxford, Radcliffe Observatory	England	51 45 35 N.	1 25 39 W	0 05 03 W.	213
,, University Observatory ...	,,	51 45 34 N.	1 25 06 W.	0 05 00 W.	210
Ox's Hump, Nwalabo Peak *	Tenasserim (B)	24 00 37 N.	98 25 31 E.	6 33 42 E.	5,063
Panakha Town	Bootan (I)	27 32 10 N.	89 50 20 E.	5 59 22 E.	4,534
Paris Observatory	France	48 50 11 N.	2 20 14 E.	0 09 21 E.	194
Patna Gola *	Bengal (I)	25 37 12 N.	85 08 23 E.	5 40 34 E.	172
Pegu Pagoda *	Pegu (B)	17 20 10 N.	90 29 54 E.	6 26 80 E.	378
Penang, George Town, Fort Cornwallis Flag Staff.	Penang	2 25 05 N.	00 20 10 E.	6 41 21 E.	...
Peshawar *	Punjab (I)	34 01 47 N.	71 34 07 E.	4 46 17 E.	1,165
Point de Galle	Ceylon	6 01 48 N.	80 13 28 E.	5 20 54 E.	...
Pondicherry, Flag Staff *	French Territory (I)	11 55 57 N.	79 49 59 E.	5 19 20 E.	...
Poona, St. Mary's Church *	Bombay (I)	18 30 23 N.	73 52 59 E.	4 55 32 E.	...
Port Blair *	Andaman Islands	11 41 13 N.	92 40 10 E.	6 10 41 E.	73
Prome G. T. S. Station, on Hill * ...	Pegu (B)	18 50 01 N.	95 10 54 E.	6 20 44 E	900
,, Pagoda on hill, near Palm Trees * .	,, ,,	18 47 53 N.	95 15 49 E.	6 21 03 E.	...
Pulkowa	Russia	59 46 19 N.	30 20 07 E.	2 01 21 E.	246
Purian Point *	Irrawaddy (B)	15 50 06 N.	94 23 51 E.	6 17 36 E.	79
Quoin Island *	Tenasserim (B)	14 16 38 N.	97 48 26 E.	6 31 14 E.	1,187
Rangoon Great Pagoda *	Pegu (B)	16 47 51 N.	96 09 03 E.	6 24 36 E.	483
,, Port Trust Observatory * ...	,, ,,	16 46 00 N.	96 10 28 E.	6 24 42 E.	...
,, Secretariat (old) *	,, ,,	16 40 13 N.	96 09 31 E.	6 24 38 E.	81
Rome, Roman College	Italy	41 53 54 N.	12 28 53 E.	0 49 56 E.	104
Sagaing, Mingun hill *	Sagaing (B)	22 03 10 N.	95 59 35 E.	6 23 58 E.	1,328
Sandoway, Nando Temple *	Arakan (B)	18 28 09 N.	94 21 47 E	6 17 27 E.	...
Savage Island Light-house	,, ,,	20 05 07 N.	92 54 05 E	6 11 36 E.	114
Seringapatam, Tipoo Saib's Tomb * ...	Mysore (I)	12 24 35 N.	76 42 48 E.	5 06 51 E.	2,282
Shikarpore house	Sind (I)	27 57 13 N.	68 17 52 E.	4 34 32 E.	194
Shillong, Court-house	Assam (I)	25 34 18 N.	91 53 09 E.	6 07 33 E.	4,951
Shwegyin G. T. S. Station *	Tenasserim (B)	17 55 30 N.	96 52 58 E.	6 23 45 E.	157
Shwesandaw Pagoda, near Twante ..	Pegu (B)	16 42 00 N.	95 56 09 E.	6 27 32 E.	128
Silhet Church *	Assam (I)	24 53 23 N.	91 52 06 E.	6 07 29 E.	...
Simla Church *	Punjab (I)	31 06 13 N.	77 10 15 E.	5 08 41 E.	7,159
Singapore Battery	Singapore	1 17 00 N.	103 50 28 E.	6 55 22 E.	...
Sittang Pagoda *	Tenasserim (B)	17 27 06 N.	96 52 59 E.	6 27 32 E.	138
Southern Moscos, Northern Island * ...	,, ,,	13 55 24 N.	97 54 53 E.	6 31 40 E.	871
,, ,, Southern ,, * ...	,, ,,	13 50 01 N.	97 54 59 E.	6 31 40 E.	1,186
South Kensington, London, S. W. ...	England	51 29 49 N.	0 10 23 W.	0 00 42 W.	...
Stonyhurst	,,	53 50 40 N.	2 28 10 W.	0 09 53 W.	...
Surat Minaret Adrusah *	Bombay (I)	21 12 19 N.	72 49 23 E.	4 51 18 E.	160
Sydney	Australia	33 51 41 S.	151 12 23 E.	10 04 50 E.	144
Syriam Pagoda *	Pegu (B)	16 43 40 N.	96 16 20 E.	6 25 05 E.	325
Tanjore, Great Pagoda *	Madras (I)	11 01 37 N.	79 51 10 E.	5 19 25 E.	...
Taunggyi, Loi Phui Hill *	Southern Shan States (B).	20 13 07 N.	97 04 46 E.	6 28 19 E.	6,122
Tavoy, Court-house	Tenasserim (B)	14 03 00 N.	98 14 27 E.	6 32 58 E.	69
,, Point, Shengmo *	,, ,,	13 32 44 N.	98 10 04 E.	6 32 40 E.	1,131
Teheran	Persia	34 42 00 N.	51 25 28 E.	3 25 42 E.	3,810
Tezpur Church *	Assam (I)	26 37 08 N.	92 47 48 E.	6 11 11 E.	303
Thatôn Town Pagoda *	Tenasserim (B)	16 54 51 N.	97 22 16 E.	6 29 29 E.	...
Thayetmyo Jail, Flag Staff *	Minbu (B)	19 19 17 N.	95 14 10 E.	6 20 57 E.	86
Three Pagodas *	Tenasserim (B)	15 17 59 N.	98 23 13 E.	6 33 33 E.	848
Toungoo, Court-house *	,, ,,	18 56 32 N.	96 27 44 E.	6 25 47 E.	177
,, old Fort *	,, ,,	18 56 02 N.	96 25 49 E.	6 25 43 E.	182
Trichinopoly G. T. S. Station	Madras (I)	10 49 45 N.	78 41 37 E.	5 14 47 E.	...
Trivandrum Great Pagoda *	Travancore (I)	8 29 03 N.	76 56 35 E.	5 07 46 E.	...
Tumlong Town	Sikkim	27 23 50 N.	88 35 01 E.	5 54 20 E.	6,000
Twante Eastern Tree *	Pegu (B)	16 32 48 N.	95 58 28 E.	6 23 54 E.	286
,, Western ,, *	,, ,,	16 28 12 N.	95 57 13 E.	6 23 49 E.	156
Vienna Imperial Observatory	Austria	48 13 55 N.	16 30 21 E.	1 05 21 E.	...
Vizagapatam Fort	Madras (I)	17 41 34 N.	83 17 38 E.	5 33 11 E.	...
Washington Naval Observatory ...	America U. S.	38 53 39 N.	77 03 01 W.	5 08 12 E.	...
Wun Fort	Berar (I)	20 03 07 N.	78 57 16 E.	5 15 49 E.	863
Yarkand	E. Turkestan	38 20 00 N.	77 30 28 E.	5 10 02 E.	3,923
Yeh G. T. S. Station *	Tenasserim (B)	15 14 45 N.	97 51 14 E.	6 31 25 E.	86
Yunnan-fu *	China	25 02 38 N.	102 42 32 E.	6 50 50 E.	...

OBSERVATION.

[1] Burma Standard Time (=6*h*. 30*m*. E., corresponding to longitude 97° 30′ E.) is 05*m*. 20*s*. in advance of Rangoon Time (=6*h*. 24*m*. 40*s*. corresponding to longitude 96° 10′ E.)

THE NAUTICAL ALMANAC.

81. NAUTICAL ALMANAC AND ASTRONOMICAL EPHEMERIS. The following* explanation of some of the more important articles contained in the Nautical Almanac and Astronomical Ephemeris will prove useful.

All the articles of the Ephemeris are computed for Greenwich MEAN solar time; and where they are given for apparent solar or sidereal time, it is chiefly for the convenience of astronomers. A *day* is the interval of time between the departure of any meridian from a heavenly body and its succeeding return to it, and derives its name from the body with which the motion of the meridian is compared. The interval between the departure and return of a meridian to the Sun is called a *solar* day; in the case of the Moon, the interval is called a *lunar* day; and in that of a Star, a *sidereal* day. The rotation of the Earth on its axis is always performed in the same time; and if the heavenly bodies preserved the same positions with respect to each other, the intervals between the departure and return of a meridian to each would be the same, and all days, consequently, of equal length. The Sun (or more strictly, the Earth in its orbit), the Moon, and the Planets are, however, in continual motion; and with velocities not only different from each other, but varying in each particular body; the length of a day, as determined by any of these bodies, is therefore a variable quantity.

Astronomers, with a view of obtaining a convenient and uniform measure of time, have recourse to a *mean solar day*, the length of which is equal to the mean or average of all the apparent solar days in a year. An imaginary Sun, called the *mean* Sun, is conceived to move uniformly in the Equator with the real Sun's *mean* motion in Right Ascension, and the interval between the departure of any meridian from the *mean* Sun and its succeeding return to it, is the duration of the *mean solar day*. Clocks and chronometers are adjusted to mean solar time; so that a complete revolution (through 24 hours) of the hour hand of one of these machines should be performed in exactly the same interval as the rotation of the Earth on its axis with respect to the mean Sun. If the mean Sun could be observed on the meridian at the instant that the clock indicated $0^{h}\,0^{m}\,0^{s}$, it would again be observed there when the hour hand returned to the same position. As the time deduced from observation of the *true* Sun is called *true* or *apparent* time, so the time deduced from the *mean* Sun, or indicated by the machines which represent its motion, is denominated *mean* time.

Mean time cannot be obtained from observation; but it may readily be deduced from an observation of the true Sun, with the aid of the equation of time, which is the angular distance in time between the mean and the true Sun. Suppose the true Sun to be observed on the meridian of Greenwich, January 1, 1907; it would then be apparent noon at the meridian, the equation of time at this instant is $3^{m}\,18^{s}{\cdot}18$, and by the precept at the head of the column, it is "*to be added to apparent time*;" hence it appears that the corresponding mean time is January 1, $0^{h}\,3^{m}\,18^{s}{\cdot}18$, or that the mean Sun had passed the meridian previously to the true Sun, and that at the instant of observation the mean time clock ought to indicate this time.

Let a person be supposed to be under a meridian 15° West of Greenwich. The positions of the heavenly bodies, as referred to the centre of the Earth, are independent of meridians, and are the same for all places at the same absolute instant; but the relative times at Greenwich and the assumed meridian would be different. If it were 1^{h} from mean noon at the one place, it could not be 1^{h} from mean noon at the other; for time, as regards a visible phenomenon, is considered as the hour angle of the Sun *westward* from a given meridian, and at the same absolute moment of time the Sun cannot be at the same hour angle (*reckoning westward*) from two meridians which are 15° distant from each other. Before making use of the Ephemeris, it is therefore necessary to ascertain, in every instance, the distance of the Sun (*in time*) from the meridian of Greenwich, or what is commonly called the corresponding Greenwich time; and this is evidently equal to the given time under the assumed, meridian, *increased* or *diminished* by the difference (*in time*) of the two meridians, according as the assumed meridian is to the *Westward* or *Eastward* of Greenwich. In a mean solar day or 24

OBSERVATION.

* From the "Nautical Almanac" for 1907. .

mean solar hours, the Earth by its rotation from West to East, has caused every meridian in succession from East to West to pass the mean Sun; and since the motion is uniform, all the meridians 15° distant from each other will have passed the mean Sun at intervals of one mean hour; the meridian to the eastward passing first, or being, as compared with the Sun, always one mean hour in advance of the westerly meridian. When it is 6^h after mean noon at a place 15° West of Greenwich, it is therefore 7^h after mean noon at Greenwich; and it is for this Greenwich time that we must deduce the quantities (meridian passages, or times of transit excepted) required from the Ephemeris.

The day of the month in the Ephemeris (Page I excepted, being for *apparent* noon) is assumed to begin at mean noon of the corresponding civil day, or at the instant when a clock shows $0^h\,0^m\,0^s$, Greenwich mean time, and is continued through the 24 hours to the following mean noon; it may therefore be called the *mean astronomical day*, although, in practice, astronomers begin the day at the moment the true Sun's centre is on their meridian.

In the civil, or common, method of reckoning, the day commences at the *preceding* midnight, and is counted only to 12 hours or noon, when the 12 hours are reckoned over again to the next midnight. The civil reckoning is therefore always 12^h in advance of the astronomical reckoning; hence the well known rule for determining the latter from the former, *viz.*:—In civil time, for P.M. make no change, for A.M. diminish the day of the month by 1 and add 12 to the hours.

To each month there are devoted twelve pages, distinguished by the Roman numerals I to XII.

Page I contains for every 10th day of the year 1907—

Nutation. The *true* place of the equinox at any time differs from its *mean* place by a quantity termed the Nutation, to be applied with the proper sign to a longitude reckoned from the *mean* equinox to obtain the value with respect to the *true* equinox. The Nutation in Right Ascension similarly serves to find the *apparent* point of intersection of the Ecliptic *on the Equator.*

The Sun's Horizontal Parallax, or the angle under which the equatorial semidiameter of the Earth would appear at the Sun's centre.

The Sun's Aberration, or the difference between the *true* longitude and the *apparent* longitude of the Sun. If the *true* longitude of the Sun be required, the aberration must be added to the *apparent* longitude.

The Sun's Mean Longitude. These quantities are affected with aberration.
The Moon's Mean Longitude.
The Mean Longitude of the Moon's Ascending Node.
The Mean Longitude of the Moon's Perigee.

Page I of each month.

The contents of this page are adapted to *apparent noon*, or the instant when the Sun's centre is on the meridian of Greenwich. The *Sun's Right Ascension*, here given, is *affected with aberration*, and reckoned from the true equinox; it is therefore the sidereal time at apparent noon, or the time which should be shown by a sidereal clock at that instant. The *Sun's Apparent Declination* is the angular distance of the Sun's centre from the Equator of the date, measured on the meridian.

The *Var. in 1 hour* is intended to facilitate the reduction of the quantities from apparent noon to any other time. It is the variation at *noon*, and requires to be reduced to midway between noon and the time at which the R.A. Dec. or Equation of Time is required.

The *Sidereal Time of the Sun's Semidiameter passing the Meridian* is useful for reducing a transit observation of either limb of the Sun, when one only has been observed, to the transit of the centre.

The *Equation of Time* is the difference between apparent and mean time, and therefore serves for the conversion of either time into the other. The numbers here given show, for Greenwich apparent noon, the distance of the mean Sun from the meridian, or the portion of time to be *added to*, or *subtracted from* (according to the precept at the head of the column), Greenwich apparent noon to obtain the corresponding mean time at the same meridian, or the time which should be shown by a mean time clock.

Where time is deduced from observations of the Sun, the *immediate* result is *apparent* time; to convert it into mean time the equation of time is necessary.

Page II of each Month.

The *Sun's Apparent Right Ascension* and *Declination* at mean noon denote the *apparent* position of the true Sun reckoned from the true equinox, at the instant the Greenwich mean time clock indicates $0^h\,0^m\,0^s$, or when the hour angle of the true Sun is equal to the equation of time.

The *Semidiameter of the Sun* is the angle at the centre of the Earth subtended by the Sun's semidiameter, and is required for reducing observations of the limb to the centre, as in measuring the altitude of the Sun's upper or lower limb, or the distance of the Moon from the Sun.

The *Equation of Time* at mean noon serves more particularly to convert *mean* into *apparent* time by applying the equation according to the precept at the head of the column.

The *Sidereal Time at Mean Noon* is the angular distance of the first point of Aries, or the true vernal equinox, from the meridian, at the instant of mean noon: it is therefore the Right Ascension of the mean Sun, or the time shown by a sidereal clock at Greenwich, when the mean time clock indicates $0^h\,0^m\,0^s$.

The sidereal time here given is that in common use among astronomers, and expresses the actual hour angle from the meridian, westward, of the true equinoctial point at the moment of observation. It is therefore affected with the equation of the equinoxes; and is not, strictly speaking, a *mean* or uniformly increasing quantity. It ought, therefore, to be termed *apparent sidereal time* in the same manner as apparent solar time reckons from the actual arrival of the Sun's centre on the meridian; and in like manner, as mean solar time is reckoned from the arrival of an imaginary sun, moving uniformly with its mean velocity, so *mean sidereal* time or $\frac{\odot\text{'s mean longitude}}{15}$ would be reckoned frem the transit of, not the *true*, but the *mean* equinoctial point. The smallness of the fluctuations to which a clock, regulated to *apparent* sidereal time compared with one regulated to *mean* sidereal time, is subject (being at the utmost only $2^s.3$ in a period of nineteen years), has prevented the practical inconvenience of this from being felt, no clock being sufficiently perfect to go during so long a period without frequent re-adjustment; and as the corrections applied by astronomers to the observed right ascensions of all objects are adapted to this supposed irregularity in the rate of the clock, the mean right ascensions thence deduced will be correct.

The sidereal time at mean noon is useful when mean solar time is to be deduced from observation, and for the reduction of sidereal to mean solar time, or *vice versâ*, by a Table of Acceleration of Sidereal on Mean solar time, and the corresponding Table of Retardation of Mean on Sidereal time (*see* Tables XXXVI (*b*) and XXXVII (*b*), "Calculating Tables," 5th edition).

The conversion of mean solar to sidereal time, and *vice versâ*, may, however, be performed with, perhaps, less liability to error, by the Tables of Time Equivalents [*see* Tables XXXVI (*a*) and XXXVII (*a*)," Calculating Tables," 5th edition]. These Tables differ from Tables of Acceleration and Retardation in containing the *values* of intervals of each species of time, expressed in terms of the other, instead of the *corrections*.

Page III of each Month.

The *Sun's Longitude*, here given, is *affected with aberration*, and reckoned from the *true* equinox: it is therefore the apparent longitude of the Sun at the instant of mean noon; or it is (if R denotes the Radius Vector) the *true* Longitude of the Sun at the Time $0^h - 498^s.5\,R$, aberration causing the Sun to appear behind its true place in the Ecliptic.

To obtain the Sun's true longitude referred to the Mean Equinox of the beginning of the year it is necessary to add the aberration, to subtract the precession, and to apply with proper sign the nutation, corresponding to the date, to the longitude here given.

The *Sun's Latitude* is the angular distance of the Sun's centre from the plane of the Ecliptic of the date, measured on a great circle perpendicular to that plane.

The Sun's Latitude referred to the Ecliptic of the beginning of the year may be found by applying to the latitude given on these pages the correction $+\,0''\cdot471\,.\,t\sin(\odot + 186^{\circ}\cdot0)$, where t is the fraction of the year corresponding to date the and $\odot$ the Sun's true longitude.

The *Logarithm of the Radius Vector of the Earth* is the logarithm of the distance between the centre of the Earth and the true place of the centre of the Sun at mean noon, the mean distance, or the semi-axis major of the orbit being considered unity.

The *Mean Time of Transit of the First Point of Aries* is the distance of the *mean* Sun from the meridian, at the instant when the *true* point of intersection of the ecliptic and equator (called the first point of Aries) is on the meridian of Greenwich; and as the distance of the first point of Aries from the meridian, at the instant the mean Sun is on the meridian, is denominated sidereal time at mean noon, this may, by analogy, be termed the *mean time* at *sidereal noon.* It is the time by a mean time clock adjusted to the Greenwich meridian at the moment that the sidereal clock indicates exactly $0^h\,0^m\,0^s$. The use of this column is to facilitate the reduction of sidereal to mean solar time, with the help of the Table of Time Equivalents (*see* Table XXXVII (*a*), "Calculating Tables," 5th edition).

If the place of observation be not on the meridian of Greenwich, the mean time must be corrected by the *subtraction* of $9^s\cdot8296$ for each hour (and proportional parts for the minutes and seconds) of longitude, if the place be to the West of Greenwich; but by its *addition*, if to the East.

The *Moon's Semidiameter* is the angle under which her semidiameter would appear if viewed from the centre of the Earth; and the *Horizontal Parallax* is the *greatest* angle under which the Earth's equatorial semidiameter would appear if seen from the centre of the Moon. The former is requisite to obtain the position of the centre from an observation of the Moon's *limb*, as in all cases of altitudes or lunar distances; the latter, to compute the horizontal parallax of the Moon at any given Latitude on the Earth, *considered as a spheroid*; also to find the parallax in altitude, right ascension, etc., for reducing an observation of the Moon to what it would be if made at the centre of the Earth.

Page IV of each Month.

The *Moon's Longitude and Latitude* at mean noon and midnight indicate the position of the Moon at these respective times, referred to the true equinox and the Ecliptic of the date, as it would be seen from the centre of the Earth.

The *Moon's Age* at mean noon is the mean time elapsed since the Moon's ecliptic conjunction with the Sun, or since the Sun and Moon had the same Longitude.

The *Moon's Meridian Passage* is the Greenwich mean astronomical time at which the Moon's centre is on the *upper* or *lower* meridian of Greenwich. The asterisks indicate the day or which the Moon does not pass the upper or lower meridian at Greenwich. This occurs in every lunation, in consequence of the lunar day being longer than the mean solar day, and including it within its limits.

The mean astronomical time of upper transit under any other meridian may be obtained by subtracting or adding the proportional part due to the longitude, of its *preceding* or *following* difference, according as the longitude is *East* or *West* of Greenwich.

Pages V to XII of each Month.

The *Moon's Right Ascension and Declination* for every hour of the day, with the *Variation of Right Ascension and Declination in 10 minutes.* The numbers represent the position of the Moon, with respect to the true equinox and the equator, as it would appear from the centre of the Earth at the respective Greenwich times.

The *Phases of the Moon.* These are given at page XII. The numbers denote the Greenwich mean astronomical time at which the difference of Longitude between the Sun and the Moon is 0°, 90°, 180° or 270°, being 0° at New Moon, 90° at First Quarter, 180° at Full Moon, 270° at Last Quarter.

The *Moon's Apogee and Perigee.* The numbers here given indicate, to the nearest tenth of an hour, the Greenwich mean time at which the Moon is respectively at her greatest and least distance from the Earth.

Planetary Ephemerides at Mean Noon.

These pages contain the Geocentric and Heliocentric places of the Planets Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune.

The Geocentric places are the places of the centres of the planets, as they would appear from the centre of the Earth; the Heliocentric, such as they would appear from the centre of the Sun at the corresponding date. The Geocentric Right Ascensions and Declinations *are affected with aberration,* and are therefore *apparent* positions.

The Heliocentric Longitudes are reckoned from the mean equinox of the date.

The Sidereal Time of the Semidiameter passing the Meridian, or in case of Jupiter and Saturn—Sidereal Time of the Equatorial Semidiameter passing the Meridian serves to reduce an observation of the Right Ascension of the limb to that of the centre, the Semidiameter, or in the case of Jupiter and Saturn—the Polar Semidiameter, a similar purpose for the declination, and the Horizontal Parallax to reduce an observation to the centre of the Earth.

The "Meridian Passage" is the mean time of transit of the Planet's centre over the meridian of Greenwich. As in the instance of the Moon before noticed the days on which Mercury and Venus do not pass the meridian are indicated by asterisks, and for a similar reason, *viz.*, that the planetary day is longer than the mean solar day, and includes it within its limits.

Stars.

In these pages are the Mean Right Ascensions and Declinations of the more important Stars and their precessional and proper motions for the year. The Annual Precession together with the Annual Proper Motion constitutes the Annual Variation, which is the actual change in the Right Ascension or Declination of a star during the year, due to the combined effect of precession and proper motion. North Declination is considered positive and South Declination negative, and must be so taken in any reductions or logarithmic values for which they are used. In the column headed Mag. the adopted unit of brightness is designated 1·0. The magnitudes of stars are determined to tenths of a magnitude with reference to this adopted unit. The magnitudes of the ten stars brighter than the unit are indicated by figures less than 1·0: —thus the value 0·3 for Arcturus indicates that that star is seven-tenths of a magnitude brighter than the unit; the value — 1·4 for Sirius that it is 2·4 magnitudes brighter than the unit.

Moon-culminating Stars.

Those Stars are denominated Moon-culminating Stars which, being near the Moon's parallel of Declination, and not differing much from her in Right Ascension, are proper to be observed with the Moon, in order to determine differences of meridians. This is effected by comparing the differences of the observed Right Ascensions of such a Star and the Moon's bright limb at any two meridians. If the Moon had no motion, the difference of her Right Ascension from that of the Star would be constant at all meridians; but in the interval of her transit over two different meridians, her Right Ascension will have varied, and the difference between the two compared differences will exhibit the amount of this variation, which added to the difference of the meridians, shows the angle through which the westerly meridian must revolve before it comes up with the Moon; hence, and knowing the rate of her increase in Right Ascension, the difference of Longitude may be easily obtained.

The geocentric *Right Ascension of the Moon's bright limb and Declination of her centre,* at the instant of their respective transits at Greenwich, are given for the lower, as well as the Upper Culmination, L. denoting the Lower Culmination, and U. the Upper Culmination; the Roman numerals indicate the *bright* limb of the Moon. The Moon's age at the upper transit is inserted in the column containing the magnitudes of the Stars.

The numbers in the column "Var. of ☾'s R.A. in 1 hour of Long." represent the Variation in Right Ascension of the Moon's Limb during the interval of her transit over two meridians, equidistant from that of Greenwich, and *one* hour distant from each other. They have been deduced from the Right Ascensions of the *bright limb*, and therefore include the effect produced by the change of the semidiameter. They serve to determine the Longitude where the difference of meridians is not great; but where this difference is considerable, that variation in Right Ascension should be used which corresponds to the middle of the interval between the observations. They also serve to determine approximately Right Ascension of the bright limb at its transit over any other meridian.

In a similar manner the Declination may be approximately determined at transit over any meridian not far distant from Greenwich, bearing in mind that South Declinations and East Longitudes are to be considered as *negative.*

The numbers in the column entitled Sidereal Time of Semidiameter passing Meridian, express the sidereal intervals which the Moon's geocentric Semidiameter, at the time of transit at Greenwich, takes in passing the meridian, and therefore serve to determine the Transit of the centre from an observed Transit of either limb, the Semidiameter, a similar purpose for the Declination, and Horizontal Parallax, to reduce an observation to the centre of the Earth.

Eclipses.

In these pages are given the particulars necessary for indicating the times, places, etc., on the Earth, at which the Eclipses of the Sun and Moon will be visible. The elements are given for the instant of Conjunction (in cases of solar Eclipses) and of Opposition (in cases of Lunar Eclipses) of the Sun and Moon in right ascension. The subsequent results in the case of Solar Eclipses are not, however, computed from these elements unchanged, but from accurate values as interpolated for the times corresponding to each result.

It should be noted that in reckoning the position-angles of the points of Contact from the Vertex in Solar Eclipses the expression "towards the East" is to be taken as indicating the positive *direction* of measurement of position-angles, *i.e.*, N.E.S.W.; and does not necessarily indicate that the angles are to be measured from the vertex towards the east *point* of the Sun's limb.

In computing the circumstances of Lunar Eclipses the geometrical diameter of the Earth's shadow is increased by $\frac{1}{50}$th of its amount.

The fundamental plane is the plane passing through the centre of the Earth perpendicular to the axis of the Moon's shadow, *i.e.*, to the right line joining the centres of the Sun and Moon. The intersection of the fundamental plane with the Earth's equator is taken as the axis of x, and the axis of y is perpendicular to it and directed towards the North, the Earth's centre being the origin of co-ordinates; so that x and y are the co-ordinates of the point in which the axis of the shadow intersects the fundamental plane. The angle d is the declination of the point in which the axis of the shadow (in the direction Earth, Moon, Sun) intersects the celestial sphere. The angle μ is the Greenwich hour-angle of this same point.

The quantities l, l' are the radii of the shadow-cones upon the fundamental plane, l corresponding to the penumbra and l' to the umbra. The latter is regarded as positive for an annular, and negative for a total Eclipse.

The angles f and f' are the semi-angles of the shadow-cones of the penumbra and umbra respectively.

The remaining quantities x', y' and μ' are, respectively, the changes of x, y and μ in one minute of mean time.

The Besselian Elements are useful for determining the circumstances of a Solar Eclipse at any place on the Earth's surface.

Mean Places of Occultation Stars, etc.

This is a catalogue for the year of Stars (other than those appearing in the section "Mean Places of Stars,") of not less than 6·5 magnitude (with a few exceptions), the occultations of which will be visible at some part of the Earth; it also contains a few other Stars to be observed with the Moon at transit, and with Mars near its opposition.

Occultations.

These pages give the greatest difference of declination of Moon and Star, for each Greenwich Noon and Midnight throughout the year, for which an occultation is possible at some position on the Earth's surface.

Elements of Occultations.

These are :—1. The reductions to apparent place (including the effect of proper motion) at Greenwich Mean Midnight, of the selected Stars.

2. The Greenwich Mean Time at which the Moon would, if viewed from the centre of the Earth, appear to have the same Right Ascension as the Star; and the Star's Greenwich hour-angle at this time.

3. The quantities q_o, p', q', where

$$q = \frac{\delta - D}{\pi}, \qquad p' = \frac{\Delta a \cos \delta}{\pi},$$

In these expressions—

δ is the geocentric declination of the Moon at the time of geocentric conjunction of Moon and Star in Right Ascension;

π is the equatorial horizontal parallax of the Moon at the same time;

D is the declination of the Star;

Δa (in arc), $\Delta \delta$ are the variations of the Moon's Right Ascension and Declination in one hour of mean time, applicable to the time of geocentric conjunction.

4. The Limits of Latitude *beyond* which the Star *cannot* be occulted by the Moon. They also indicate whether, at a given conjunction of a Star with the Moon, the positions are likely to produce an occultation in a given Latitude.

As a precaution against the possible accumulation of neglected quantities altering the limit by 1°, it is advisable for an observer, when making a selection of stars liable to occultation, to extend the given limits by 1°.

These Elements (the hour-angle excepted), being referred to the Moon and Star as seen from the centre of the Earth, are independent of geographical position, and serve equally for all places. It is only necessary to apply the difference of longitude from Greenwich to the Greenwich Mean Time of conjunction to find the Local mean time of conjunction at any other meridian.

Suppose an observer situate at a Star, and having the Moon between him and the Earth, and that he could see the Moon projected on the Earth's disc; he would observe it moving across the disc from West to East, covering a zone whose breadth would be equal to the apparent diameter of the Moon. Now it is only within the limits of this zone that the Occultation of a Star by the Moon can take place. To all the places through which the boundary lines pass, the Star will appear just to touch the Moon's limb; and that projected parallel of latitude, to which one of the boundary lines is a tangent, is one of the limiting parallels, while the intersection of the other boundary line with the circumference of the Earth's disc determines the other limiting parallel.

The quantities q_o, p', q' are useful for determining the circumstances of the Occultation at any place on the Earth's surface.

Phenomena.

In these pages are given the approximate times of the conjunctions in Right Ascension of the Planets with the Moon and with each other, the conjunctions in Right Ascension of the Planets with certain Stars, and the times when the Planets appear to be stationary in Right Ascension; also the times of the conjunctions, etc., in longitude of the Planets with the Sun; and other notices, of use chiefly to the astronomer.

The conjunction of planet with planet is given only when the difference of declination does not exceed 3°; that of planet with star when the difference does not exceed 10′.

In computing the time of greatest brilliancy of Venus it is assumed that the brilliancy varies as $\frac{(r + \Delta + R)(r + \Delta - R)}{r^3 \Delta^3}$, where r and R are the radii vectors of Venus and of the Earth respectively, and Δ is the distance of Venus from the Earth.

Astronomical Symbols and Abreviations.

☉	The Sun.	♂	Mars.	☌	Conjunction.
☽	The Moon.	♃	Jupiter.	□	Quadrature.
☿	Mercury.	♄	Saturn.	☍	Opposition.
♀	Venus.	♅	Uranus.	☊	Ascending Node.
O or ♁	The Earth.	♆	Neptune.	☋	Descending Node.
^{h}	Hours.	$^\circ$	Degrees.	N.	North. S. South.
^{m}	Minutes of Time.	′	Minutes of Arc.	E.	East. W. West.
^{s}	Seconds of Time.	″	Seconds of Arc.		

Signs of the Zodiac.

			°				°				°
0.	♈	Aries	0	IV.	♌	Leo	120	VIII.	♐	Sagittarius	240
I.	♉	Taurus	30	V.	♍	Virgo	150	IX.	♑	Capricornus	270
II.	♊	Gemini	60	VI.	♎	Libra	180	X.	♒	Aquarius	300
III.	♋	Cancer	90	VII.	♏	Scorpio	210	XI.	♓	Pisces	330

Principal Articles of the Calendar, etc., for 1907.

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The Year 5668 of the Jewish Era commences on September 9.
The Year 1325 of the Mohammedan Era commences on February 14.

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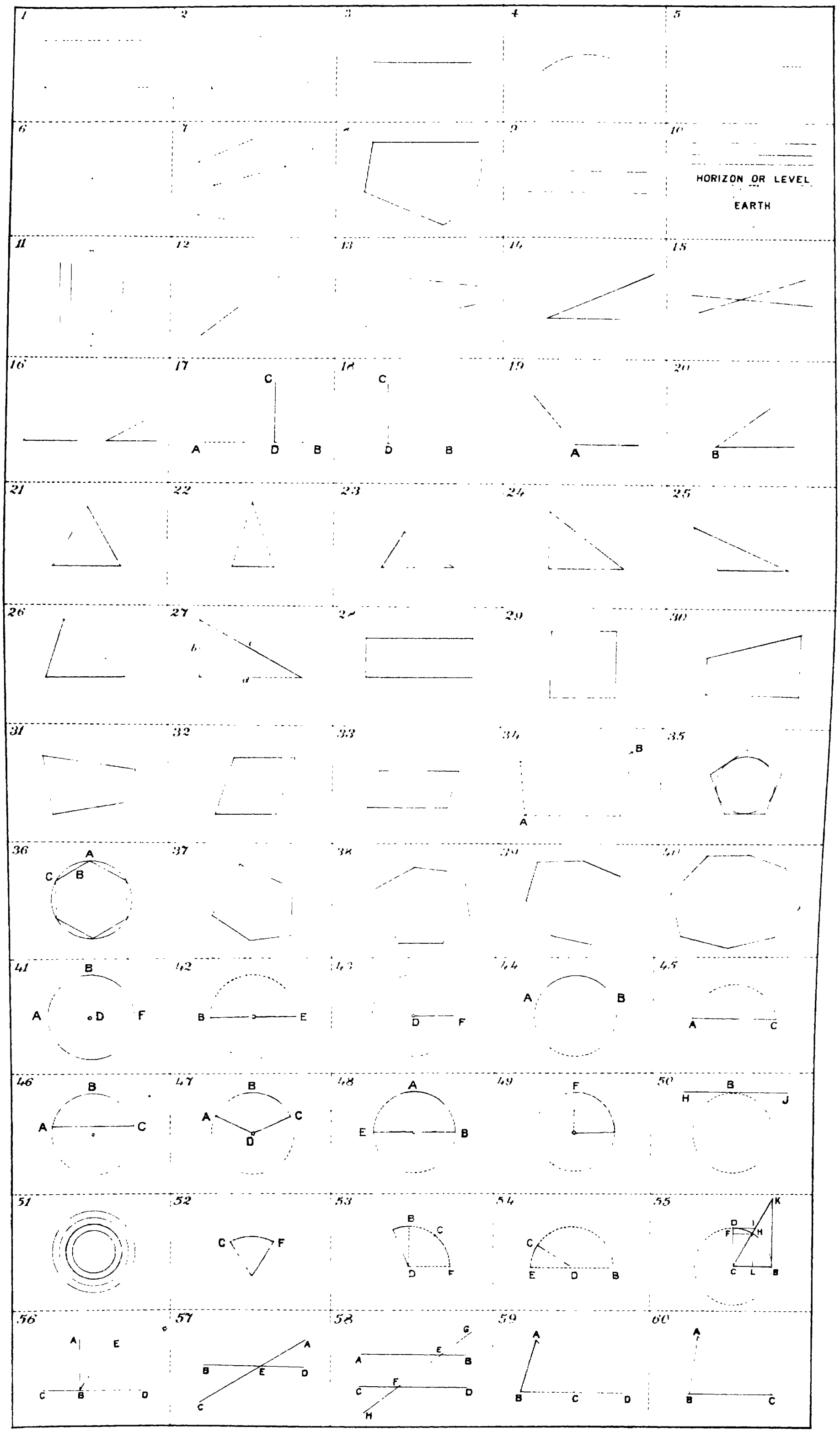
295	VII, 50.	358	XII, 12.	423	XIII, 28, 60.	488	XIII, 94; XVI, 70, 78, 79.
296	VII, 50.	359	XI, 22.	424	XV, 10, 12, 23.	488*a*	
297	VII, 50.	360	VIII, 17, 18.	425	IX, 29, 36.	489	XIII, 94; XVI, 70, 78, 79.
298	VII, 50.	361	VIII, 19.	426	III, 37.	489*a*	
299	VII, 50.	362	XI, 22.	427	III, 38.	490	XI, 23.
300	VII, 50.	363	XIII, 86, 114.	428	III, 38.	491	III, 12.
301	VII, 50.	364	X, 7.	429	XV, 33.	492	III, 13.
302	VII, 50.	365	X, 13.	430	XVI, 7—12.	493	III, 14.
303	VII, 50.	366	XV, 43.	431	XIII, 82, 83.	494	III, 15.
304	VII, 50.	367	X, 26.	432	X, 10.	495	III, 16.
305	VII, 50.	368	X, 25.	433	XIV, 21.	496	XI, 26.
306	VII, 50.	369	IX, 14, 41.	434	XIII, 77.	497	XI, 26.
307	VII, 50.	370	IX, 42.	435	XIV, 21.	498	XI, 22.
308	VII, 50.	371	IX, 43.	435*a*	XIII, 123.	499	XVI, 69.
309	VII, 50.	372	IX, 44.	436	XVI, 66.	500	VIII, 23.
310	VII, 50.	373	VIII, 21.	437	XVI, 76.	501	IX, 19.
311	VII, 50.	374	IX, 52.	438	XV, 8.	502	XIV, 23.
312	VII, 50.	375	IX, 46.	439	XVI, 66.	503	VIII, 14.
313	VII, 50.	376	IX, 47.	440	V, 21.	504	VII, 59.
314	VII, 50.	277	X, 9.	441	VI, 37.	505	XVI, 71.
315	VII, 50.	378	X, 9, 10.	442	X, 24.	506	XVI, 70, 71.
316	VII, 50.	379	X, 15.	443	IX, 8.	507	XI, 11.
317	VII, 50.	380	X, 17, 18.	444	XIV, 1, 25.	508	XI, 11.
318	VII, 50.	381	X, 15—18.	445	XVI, 39.	509	XI, 28.
319	VII, 50.	382	X, 20.	446	XVI, 39.	510	XVI, 55.
320	VII, 50.	383	X, 13.	447	XI, 19.	511	XVI, 55.
321	VII, 50.	384	XV, 43.	448	XI, 19.	511*a*	XVI, 55.
322	VII, 50.	385	VI, 47.	449	XI, 19.	512	XVI, 55.
323	VII, 50; XI, 11.	386	X, 18.	450	XIII, 26.	513	XVI, 55.
324	VII, 47, 50.	387	XIII, 116, 123, 131.	451	XII, 2.	513*a*	XVI, 55.
325	VII, 47, 50.	388	I, 110.	452	XIII, 62.	514	XVI, 55.
326	VII, 47, 50.	389	IX, 27.	453	XIII, 68, 79, 80.	515	XVI, 55.
327	VII, 47, 50.	390	X, 20.	454	XIV, 19.	516	XVI, 22, 23.
328	VII, 47, 50.	391	X, 28.	455	XV, 22.	517	XVI, 55.
329	VII, 47, 50.	392	VII, 57; VI, 50.	456	XIII, 62.	518	XVI, 55.
330	VII, 47, 50.	393	XIII, 2.	457	XIII, 69, 70, 80.	519	XVI, 55.
331	VII, 47, 50.	394	XIII, 4.	458	XV, 2, 3, 7.	520	XVI, 55.
332	VII, 47, 50.	395	X, 20.	459	XV, 22.	521	XVI, 55.
333	VII, 47, 50.	396	X, 19, 27.	460	XV, 22.	522	XVI, 55.
334	XI, 10.	397	IX, 26, 27.	461	XIII, 27.	523	XVI, 55.
335	X, 21; XIV, 2—4, 6, 7, 9, 11—18.	398	X, 32.	462	XV, 4.	523*a*	XVI, 55.
336	VII, 43; X, 4.	399	X, 32.	463	XIII, 28.	524	XVI, 55.
337	IX, 14, 32, 36.	400	XI, 15.	464	XV, 43.	525	XVI, 55.
338	IX, 17, 18, 24, 32, 36.	401	XI, 12, 14.	465	XV, 9.		
339	VIII, 22.	402	XI, 2, 3.	466	XV, 19.		
340	VIII, 16.	403	XI, 17.	467	XV, 24, 29.		
		404	X, 36.	468	XV, 23.		
		405	XI, 7, 8, 18.	469	V, 5.		
				470	XV, 34, 41.		

ILLUSTRATIONS.



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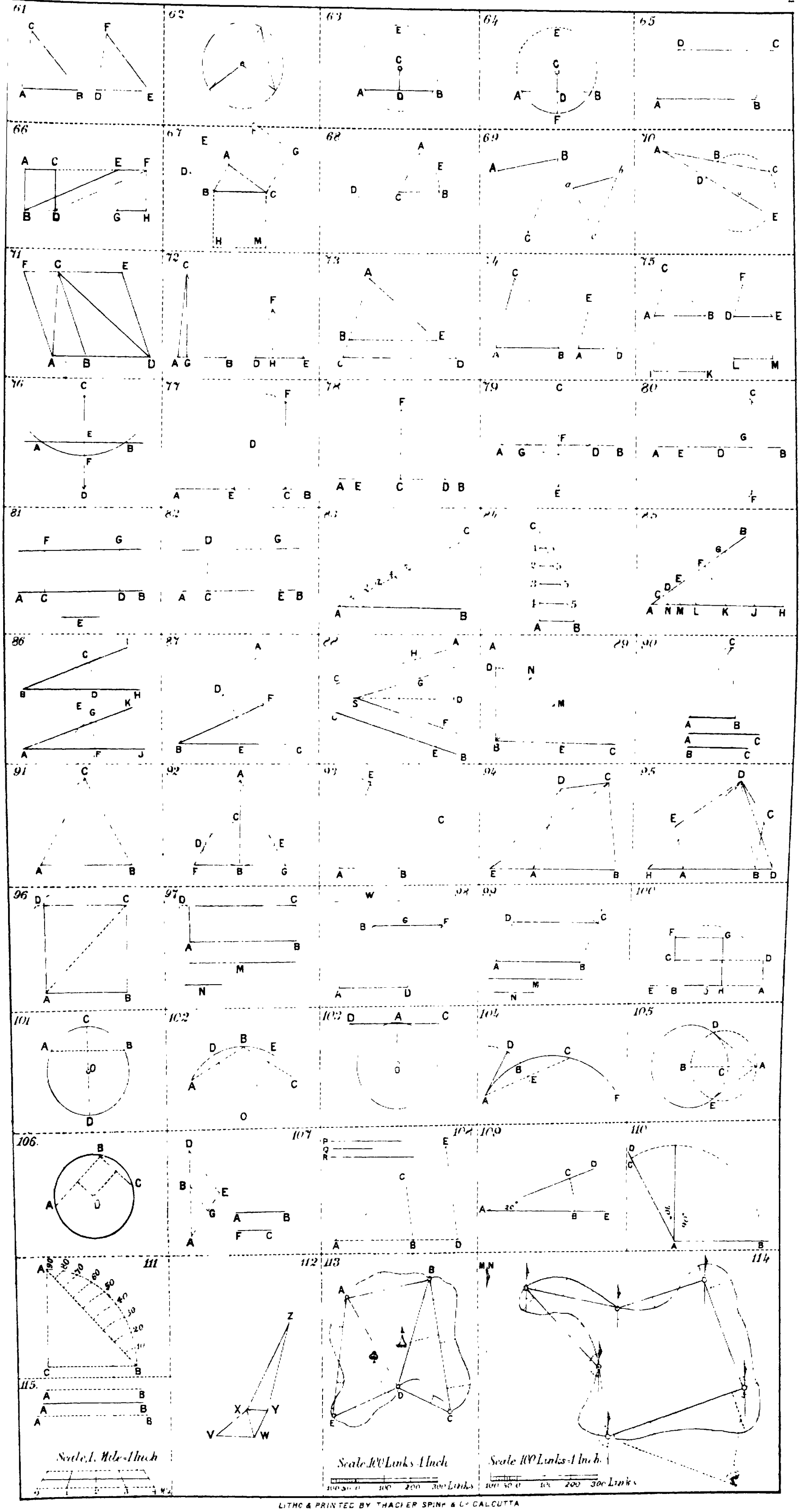
ILLUSTRATIONS.



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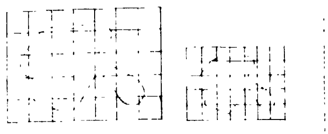

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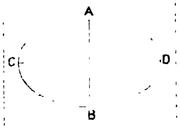

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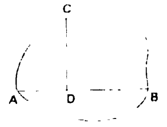

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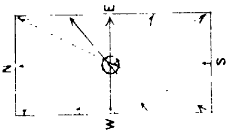

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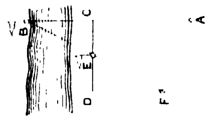

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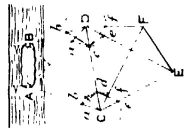

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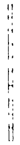

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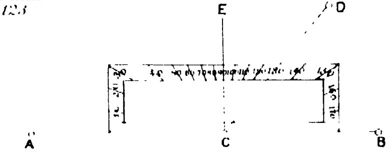

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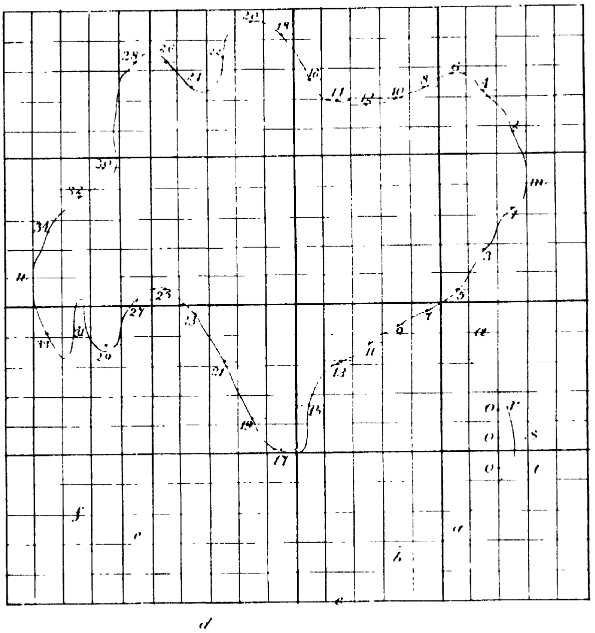

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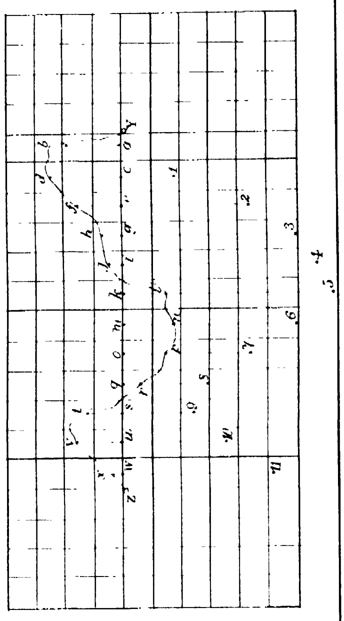

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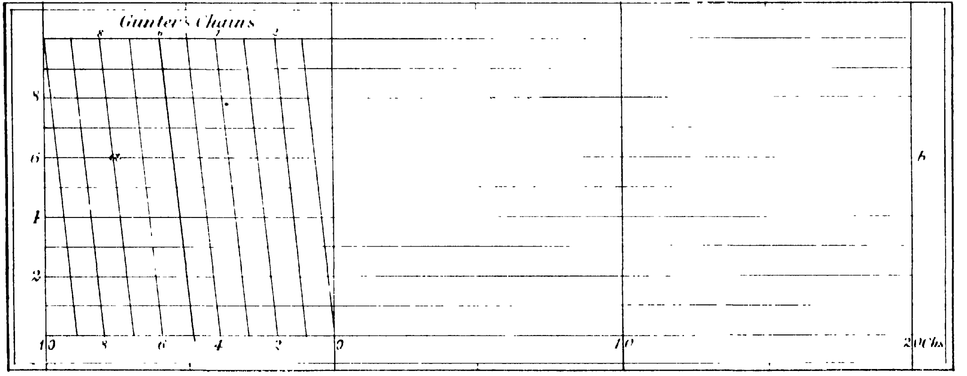
Scale, 5 Chains to the Inch or 16 Inches to the Mile



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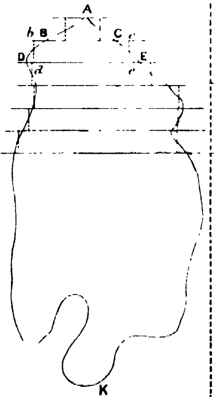

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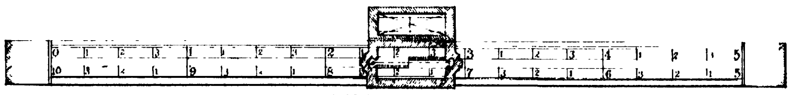

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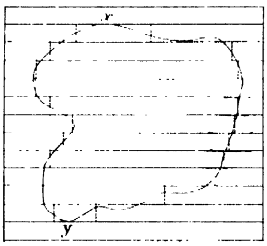

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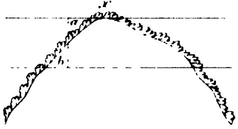

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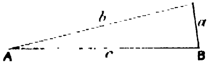

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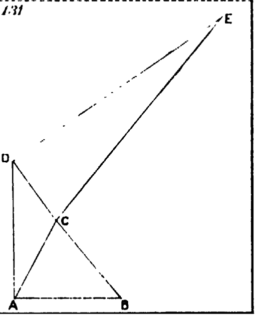

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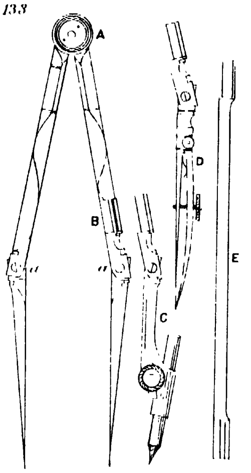

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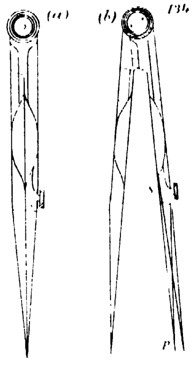

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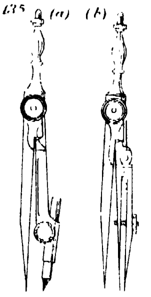

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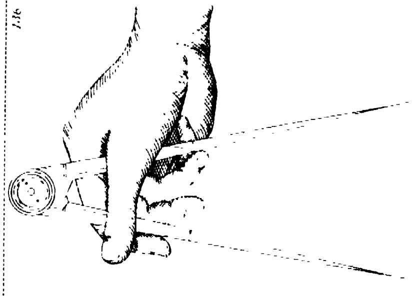

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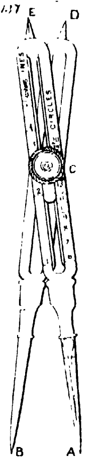

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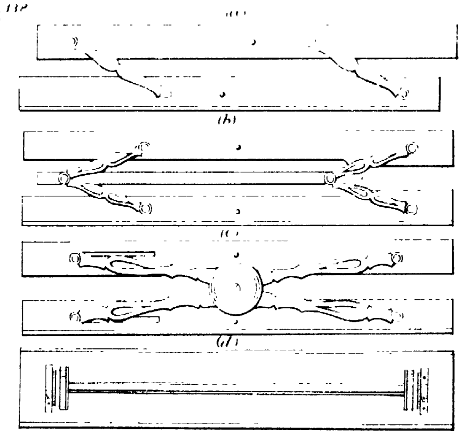

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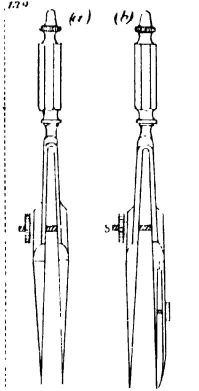

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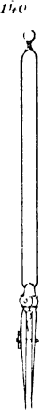

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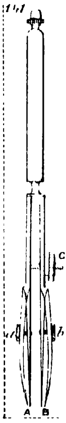

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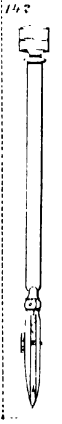

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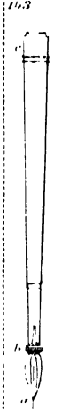

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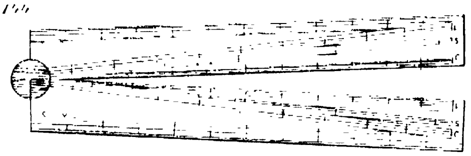

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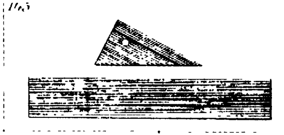

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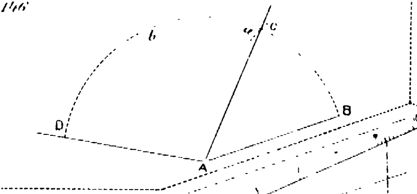

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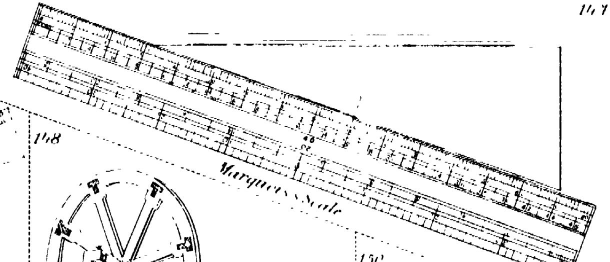

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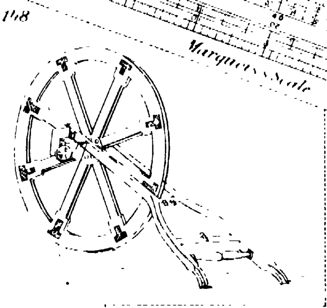

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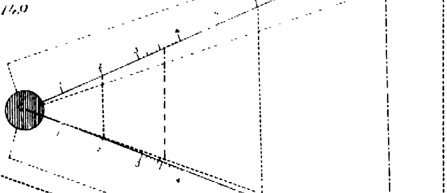

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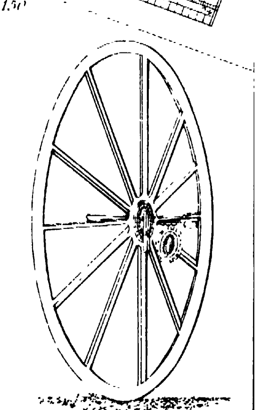

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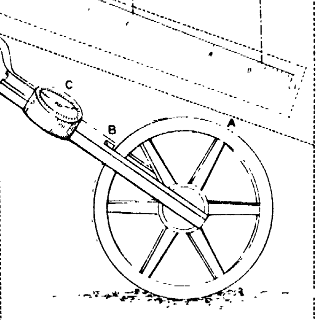

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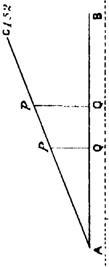

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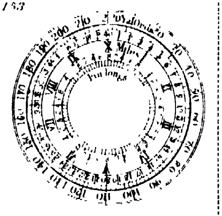

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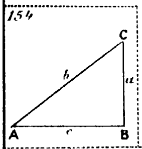

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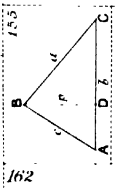

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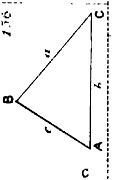

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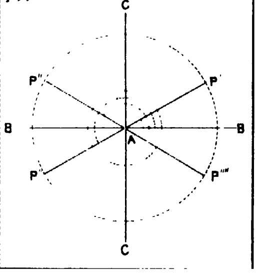

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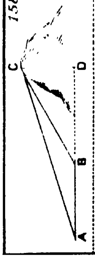

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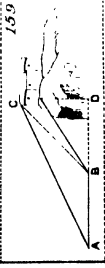

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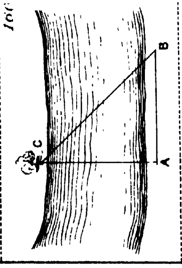

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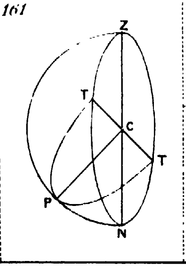

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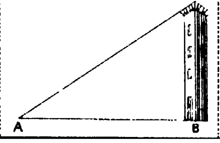
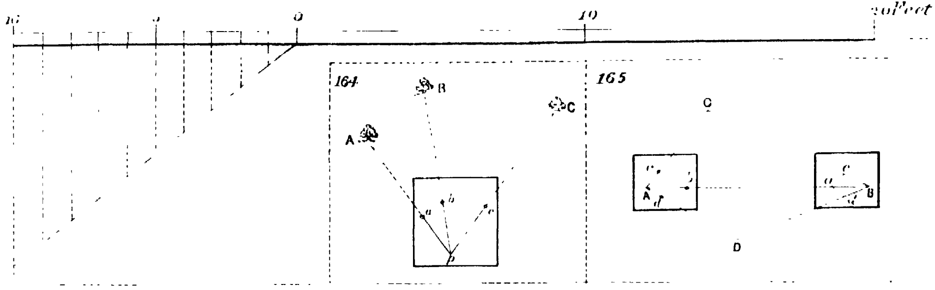


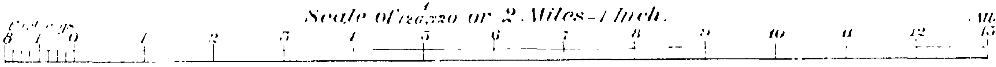
163.

Scale of $\frac{1}{60}$ or 5 feet -1 Inch.



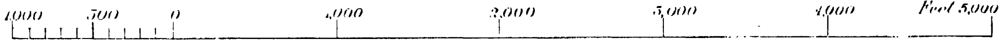
166.

Scale of $\frac{1}{126,720}$ or 2 Miles -1 Inch.



167.

Scale of $\frac{1}{10,560}$ or 6 Inches - 1 Mile.



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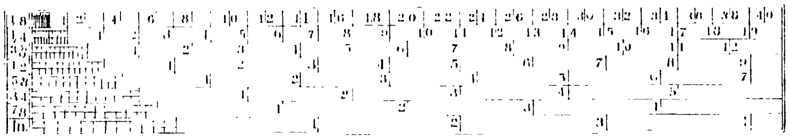
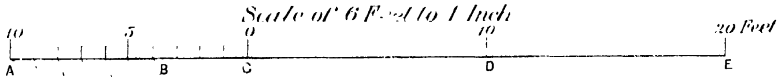


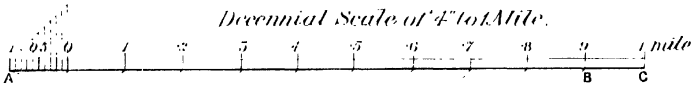
169.

Scale of 6 Feet to 1 Inch



170.

Decennial Scale of 4" to 1 Mile.



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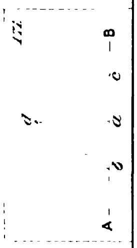
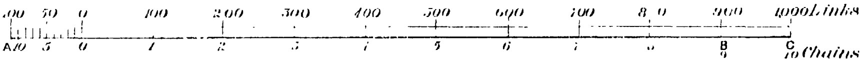


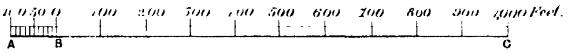
172.

Scale of 2 chains to 1 Inch, in Links (upper), and in Chains (lower).



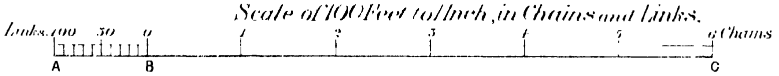
173.

Scale of 5 chains to 1 Inch, in Feet.



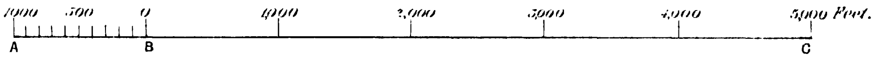
174.

Scale of 100 Feet to 1 Inch, in Chains and Links.



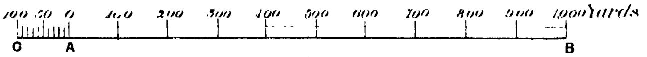
175.

Scale of 5" to 1 Mile, in Feet.



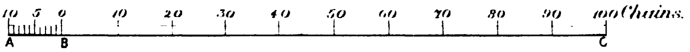
176.

Scale of 6" to 1 Mile, in Yards.



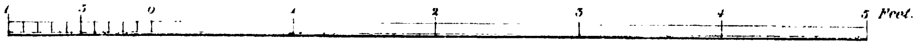
177.

Scale of 3" to 1 mile, in Chains.



178.

Scale, 1 Foot -1 Inch, in Tenths.



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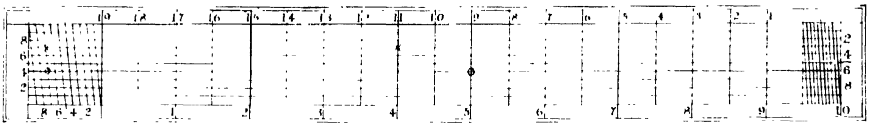
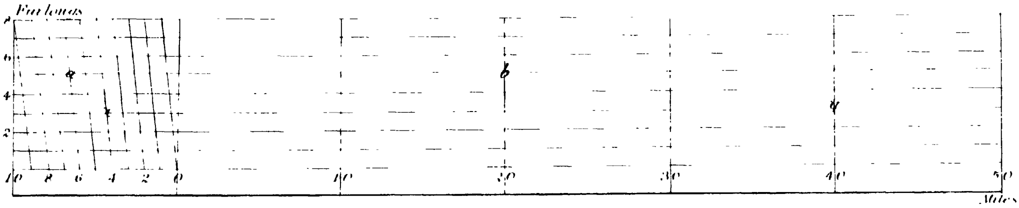
179



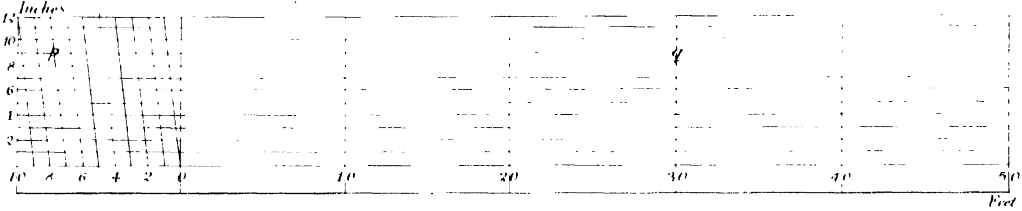
180

Scale of $\frac{1}{570240}$ or 9 Miles to the Inch (to read furlongs)



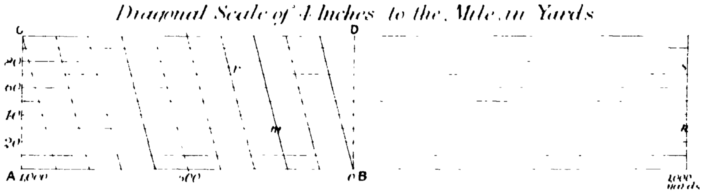
181

Scale of $\frac{1}{108}$ or 9 Feet to the Inch (to read Inches)



182

Diagonal Scale of 4 Inches to the Mile, in Yards



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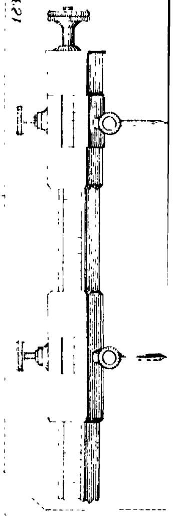

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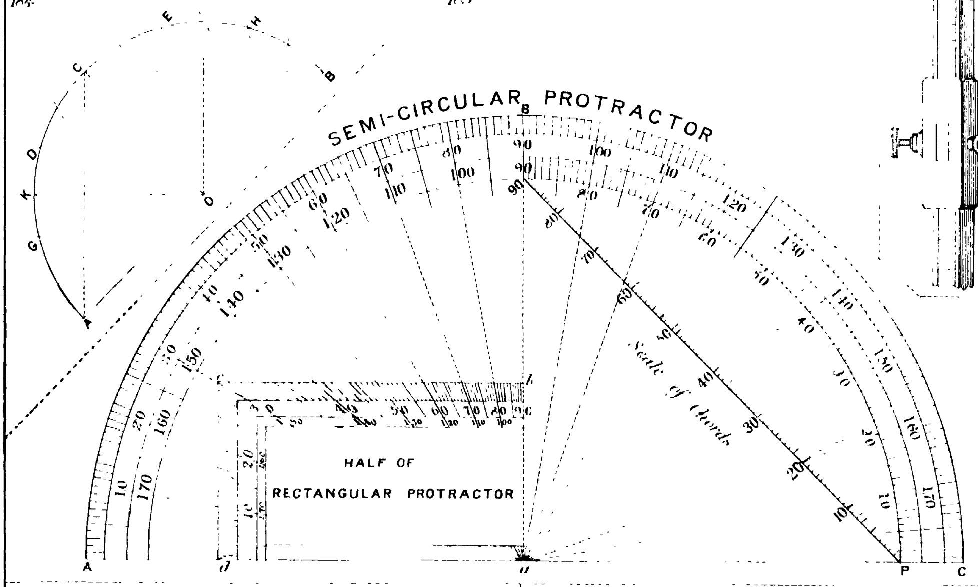

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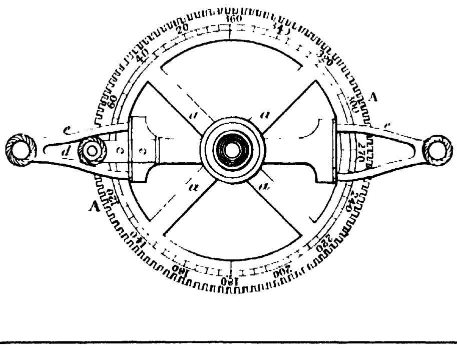

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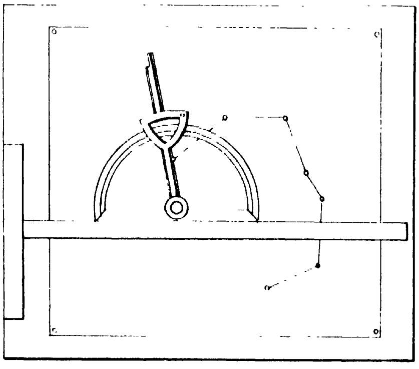

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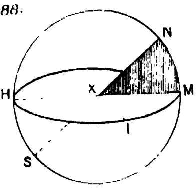
188.



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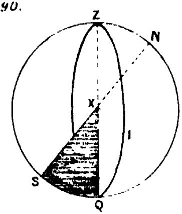

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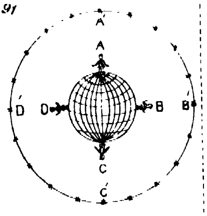
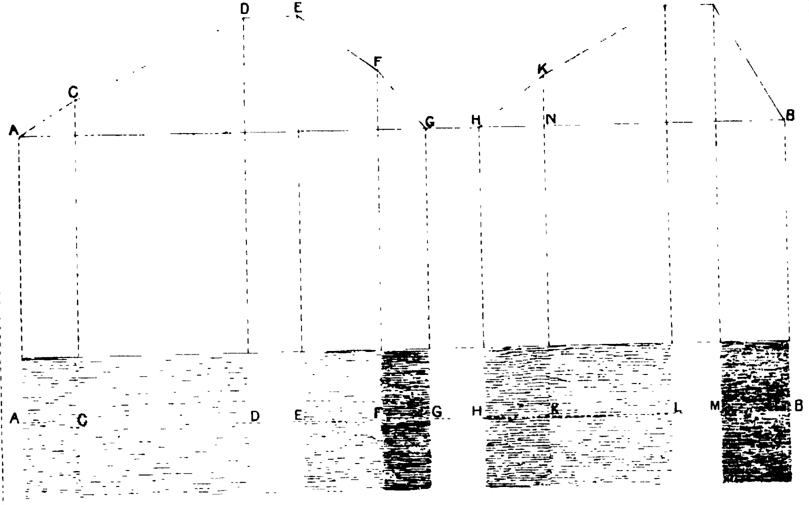


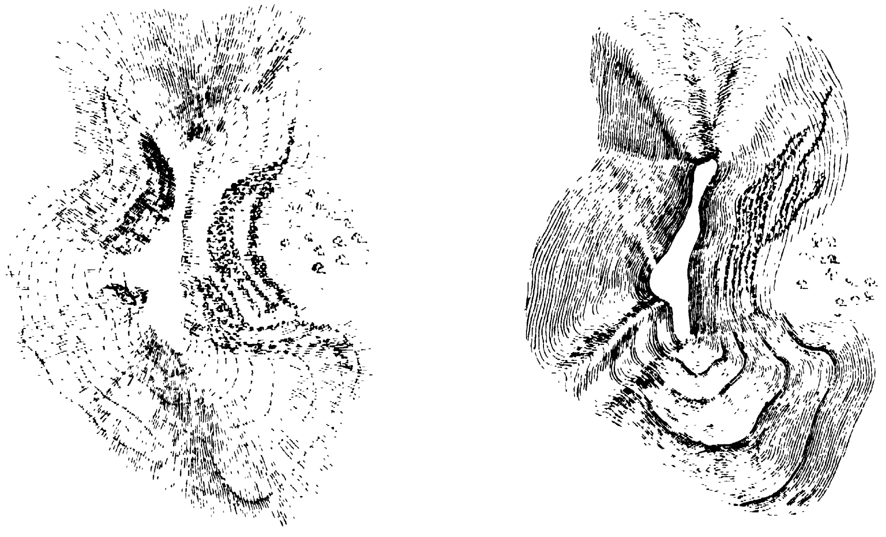
189.

Section on A.B.



Ground Plan.

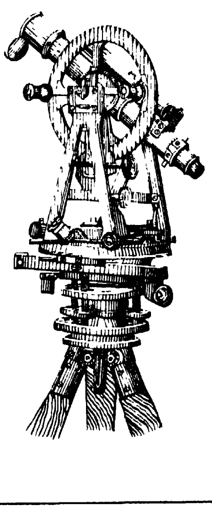
192.



Vertical Style.

Horizontal Style.

193.



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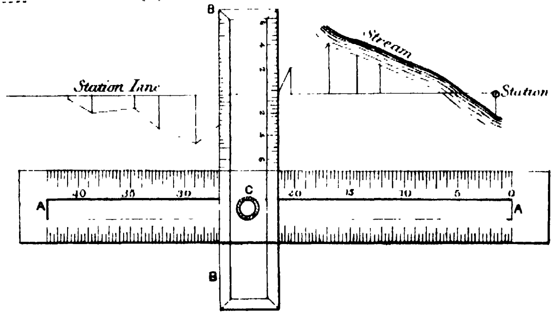
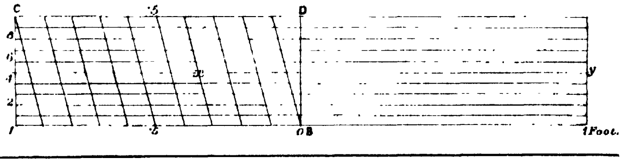


195.

Diagonal Scale of 3 Inches to 1 Foot.



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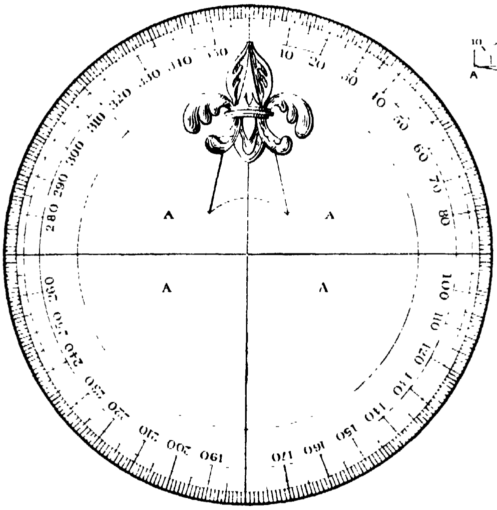
196.



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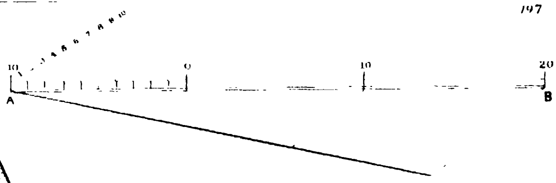

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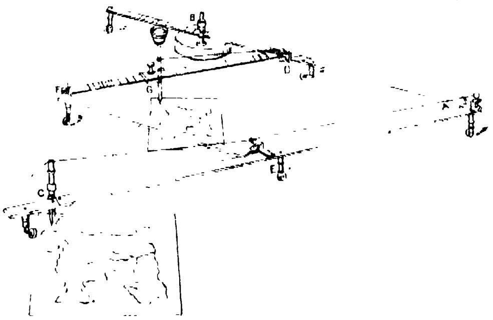

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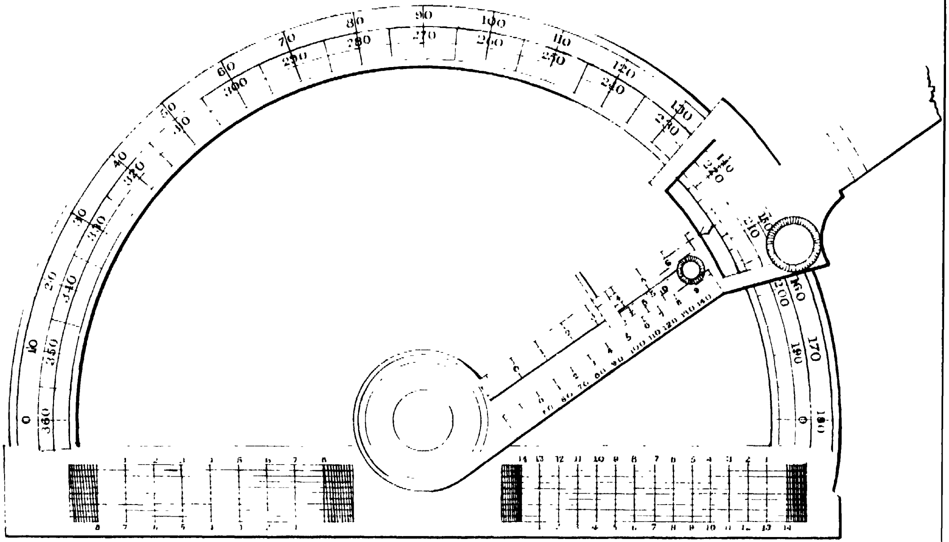
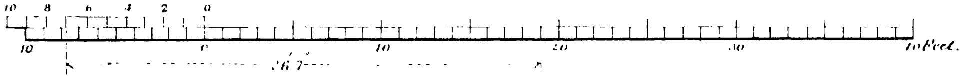


200.

Scale of 1/100



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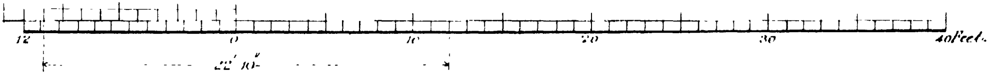
Scale of 1/100



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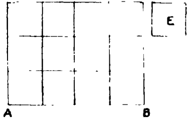

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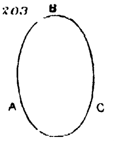

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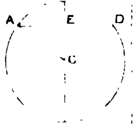

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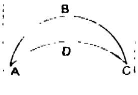

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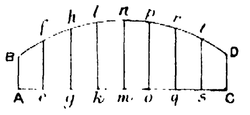

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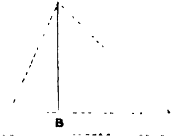

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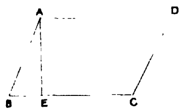

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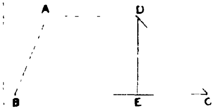

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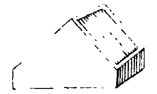

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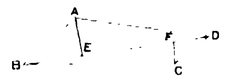

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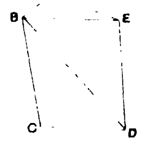

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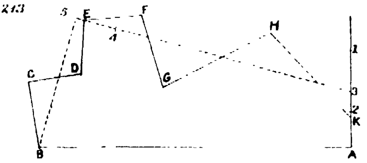

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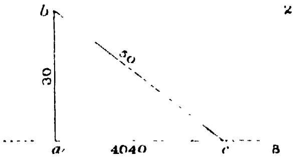

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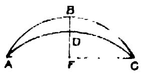

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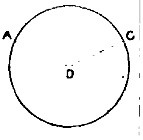

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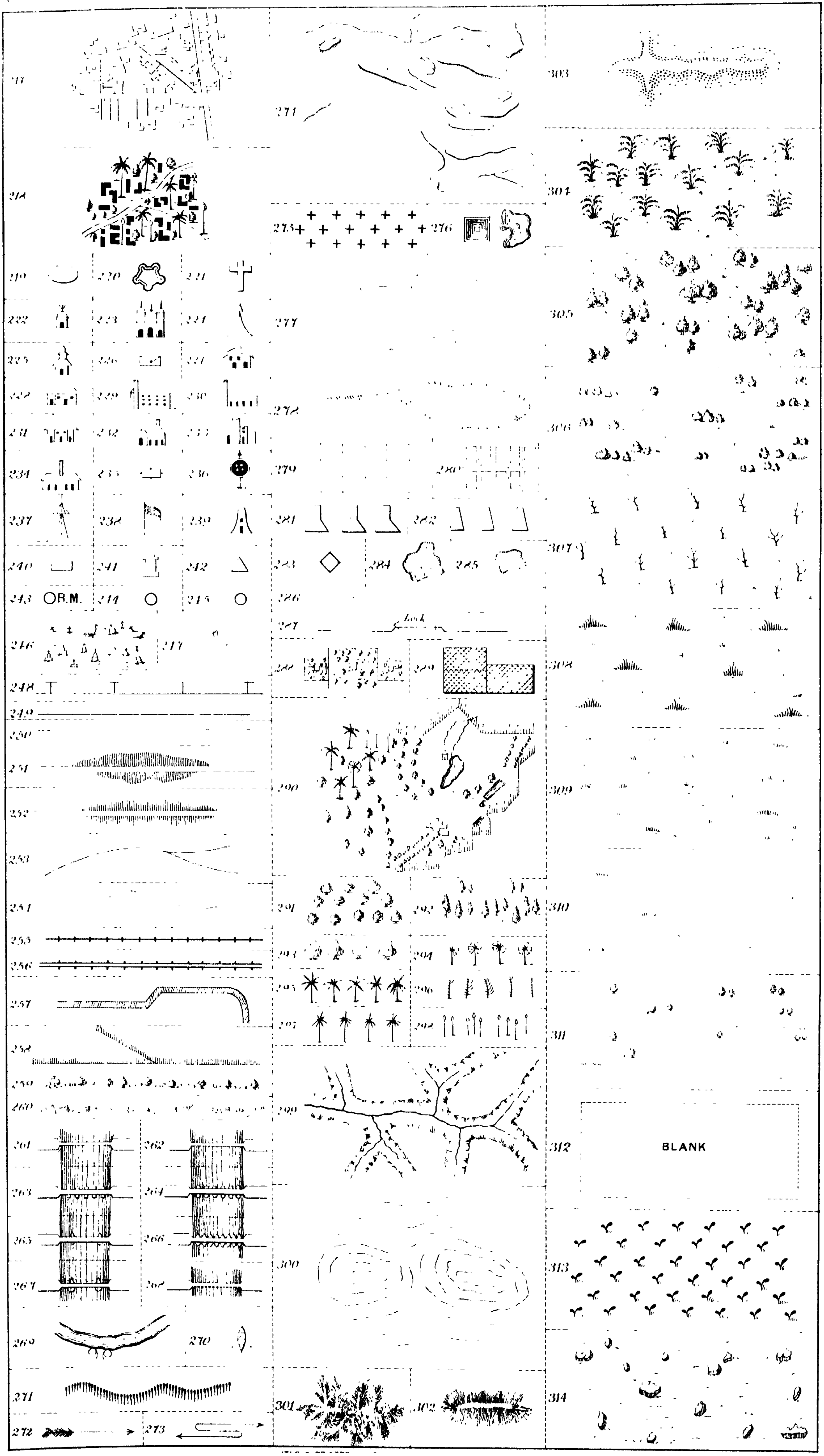
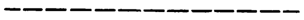
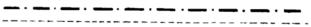
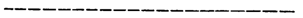
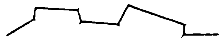
ILLUSTRATIONS.

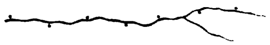


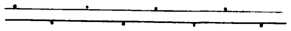
315 

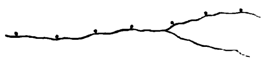
316 

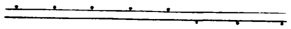
317. 

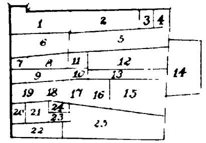
318 

319 

320 

321 

322 

323 

324 **PEGU**

325 **HAnthawaddy.**

326 **KYAUKtan.**

327 **IRRAWAddy**

328 **RANGOON**

329 SYRIAM

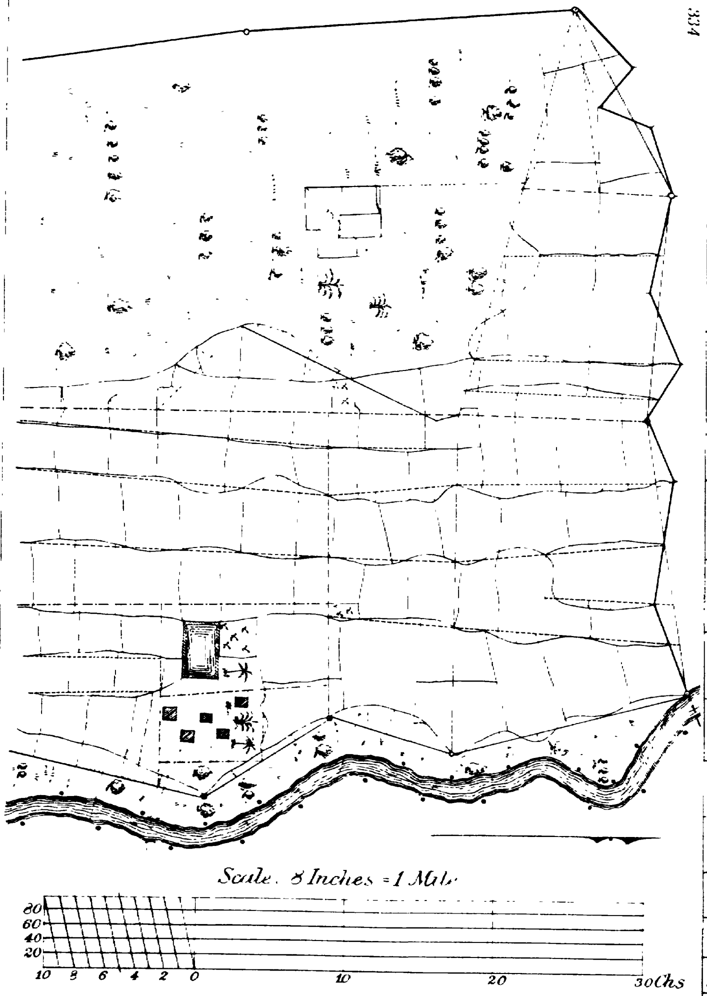
330 **DAWBAUK**

331 Prome

332. *Kamakalôk*

333 *Nwatabo*

334



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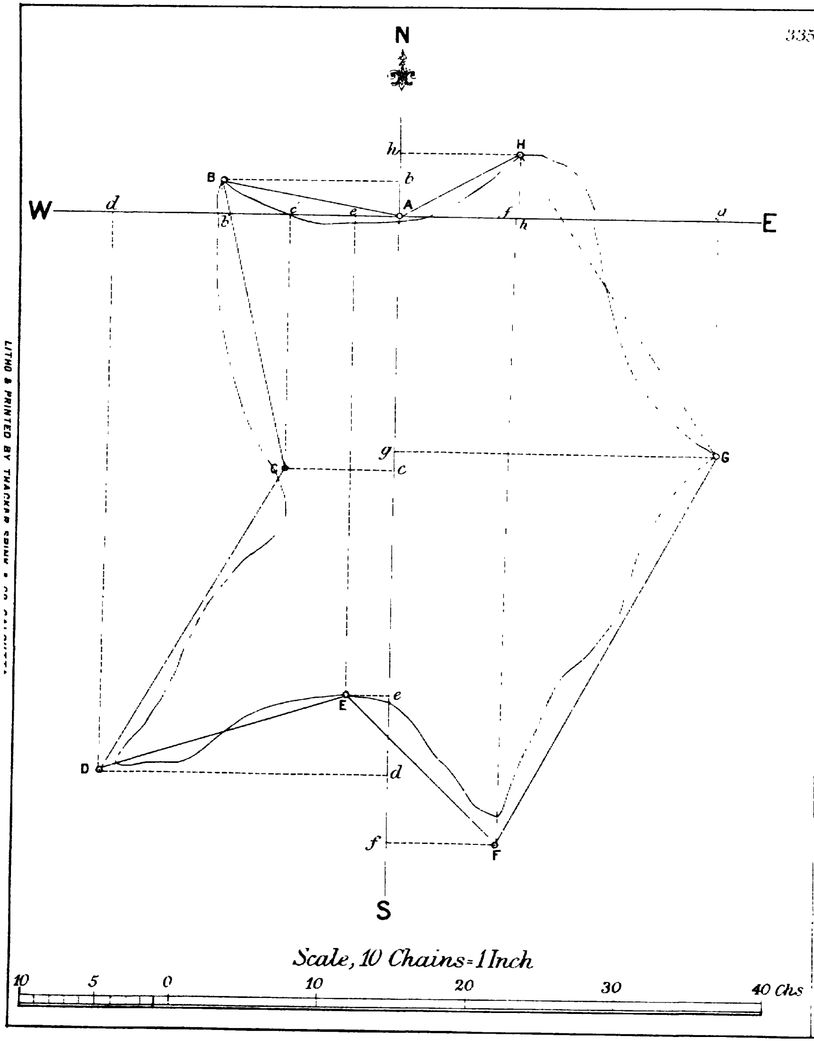

ABCDEFGHIJKLMN

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A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

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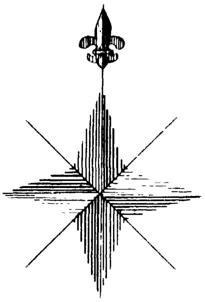
a b c d e f g h i j k l m n o p q r s t u v w x y z æ œ & 1 2 3 4 5 6 7 8 9 0.

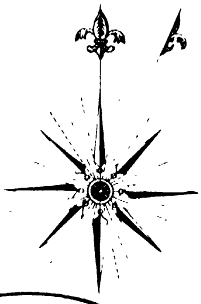
PLAN. *Section.* *Datum Line.* **HILL SHADING.**

Bad Printing spoils an otherwise good Drawing. 1. 2. 3.

This Italic Printing is the sort best suited for small drawings. 1 2 3 4 5 6 7 8 9 0.



Plan

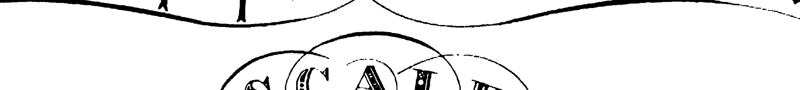


OF

MOULMEIN

Shewing the proposed Line of Water Works

SCALE



10 Chains to an Inch.

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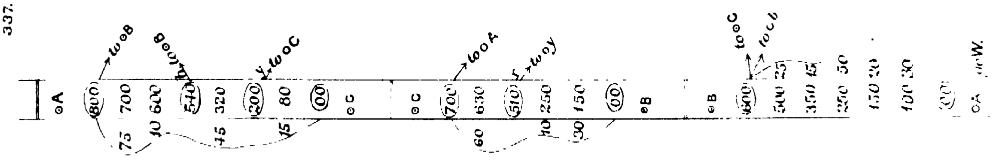
337.



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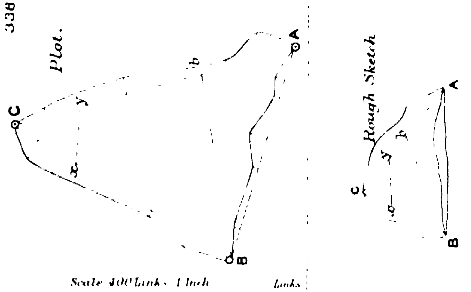

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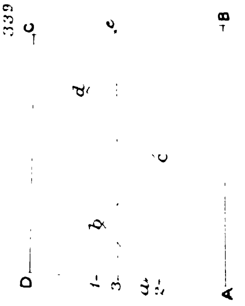

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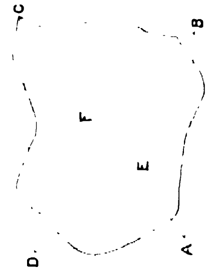

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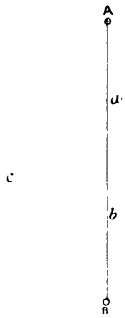

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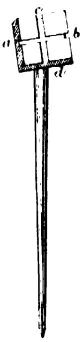

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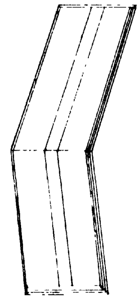

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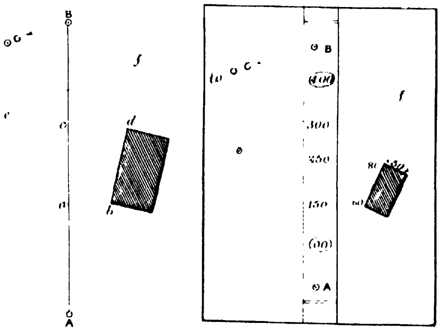

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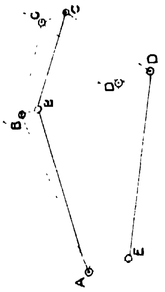

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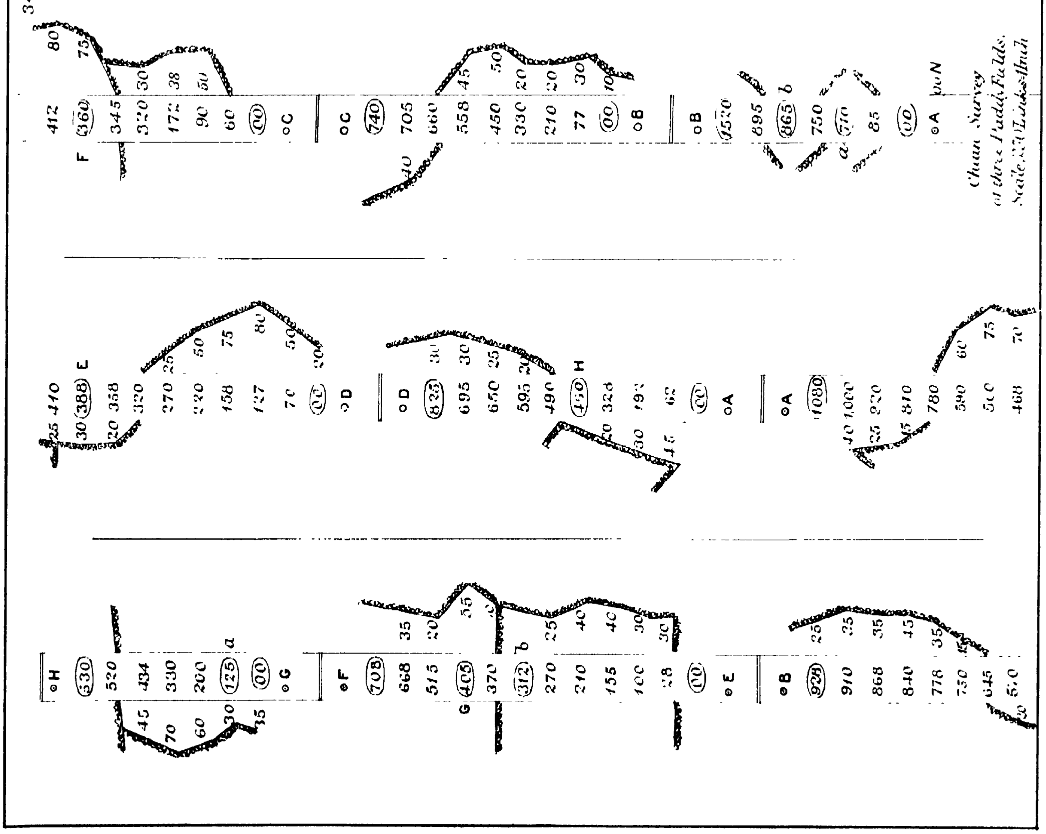

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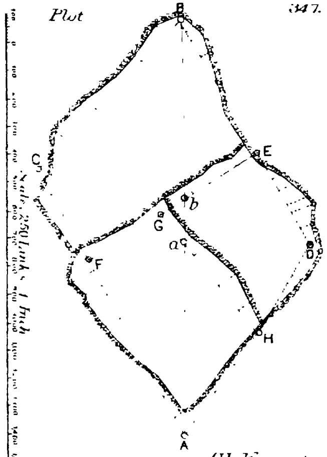
347.

Plot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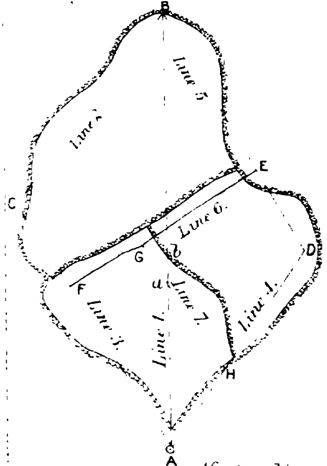
Scale, 250 Links = 1 Inch
100 0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 Links

(Half-size)

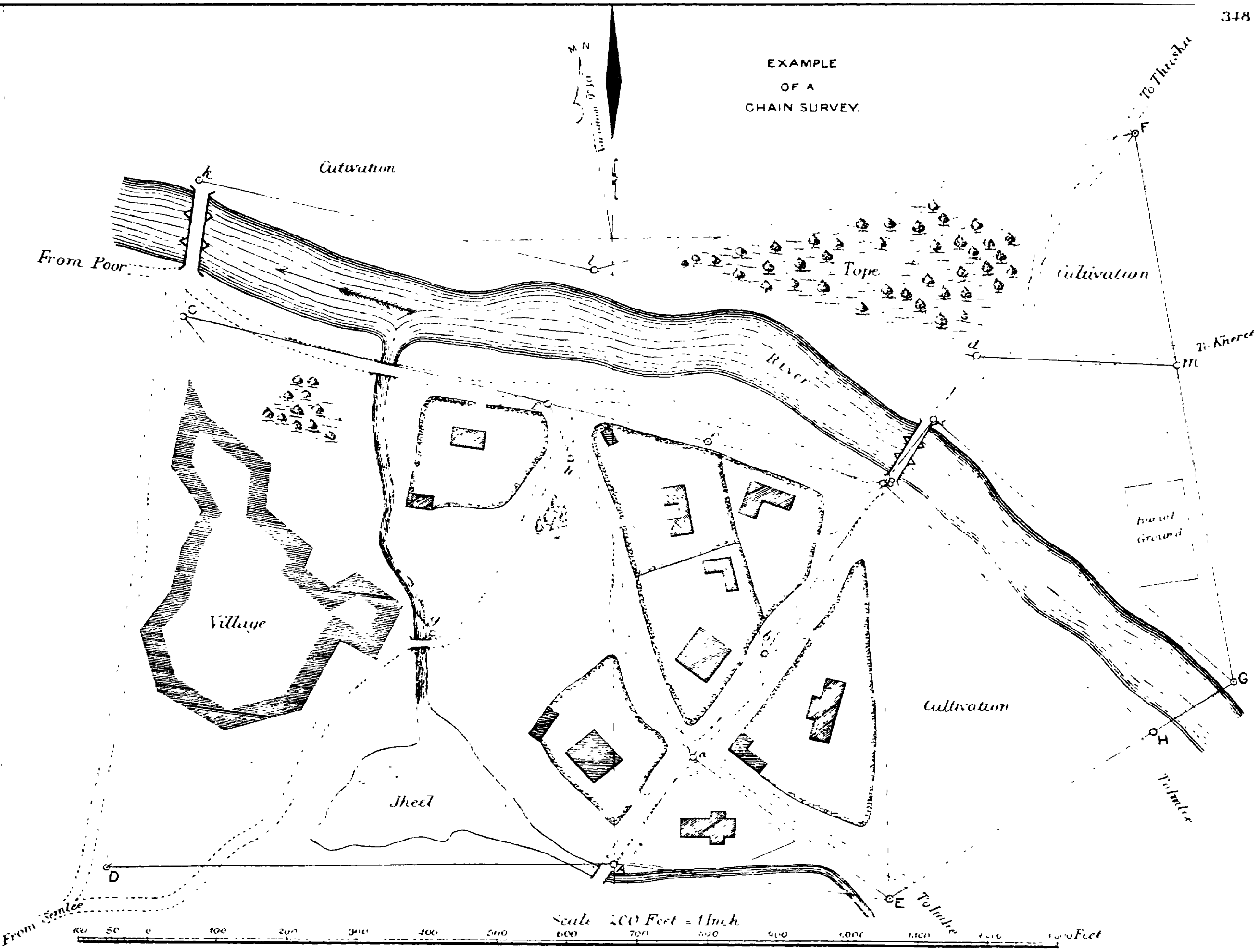
Rough Sketch in Field Book.



(Continued) 346.

348

EXAMPLE OF A CHAIN SURVEY.



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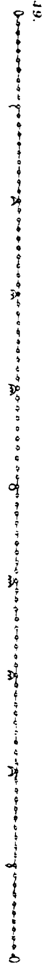

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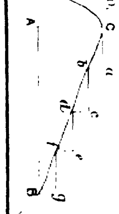
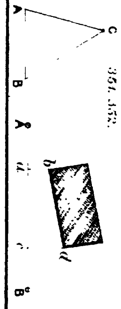


351. 352.



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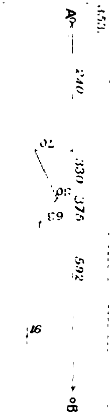
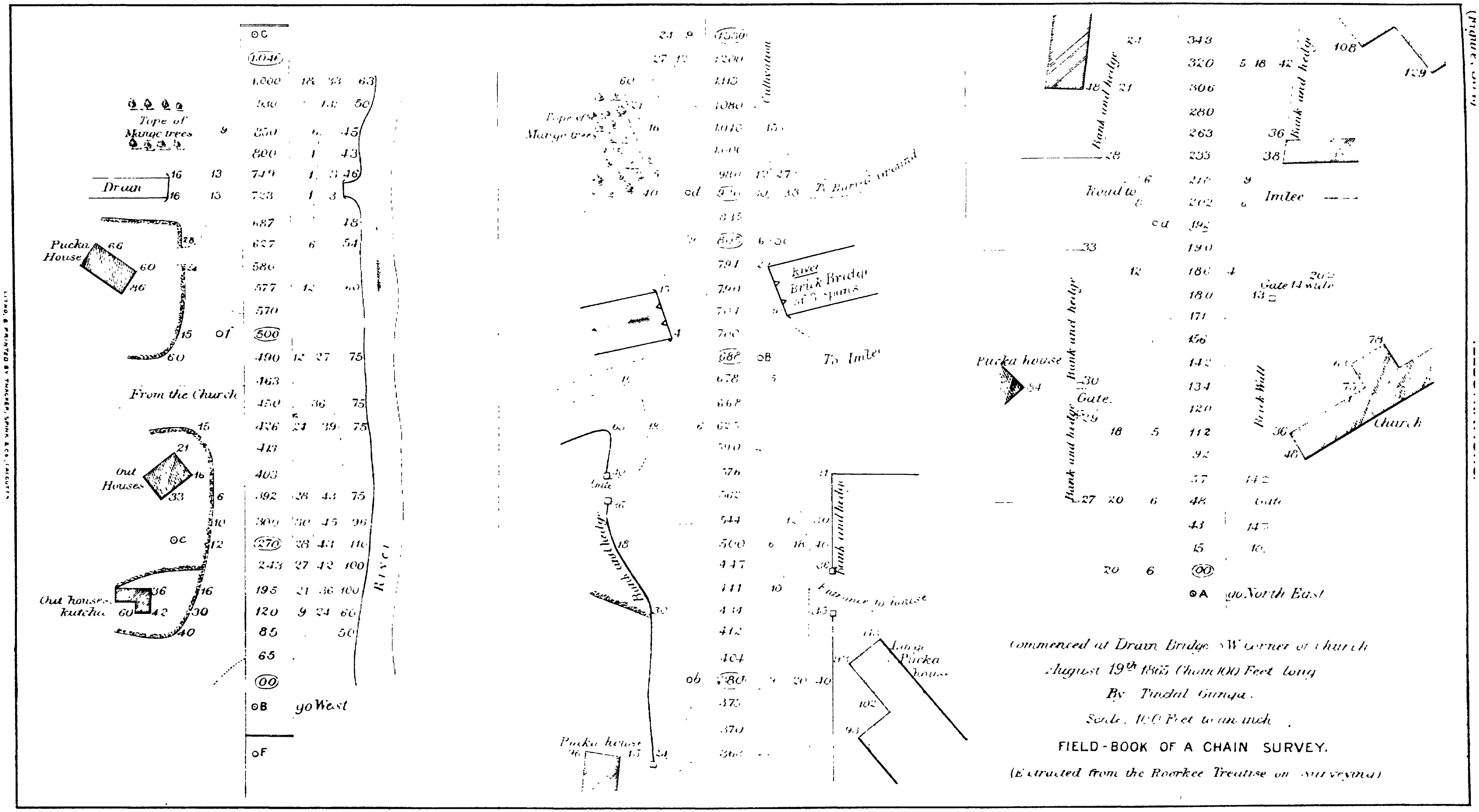


LITHD & PRINTED BY THACKER SPINK & Co, CALCUTTA.



Commenced at Drain Bridge S W corner of church
August 19th 1865 Chain 100 Feet long
By Tindal Gunga.
Scale, 100 Feet to an inch.

FIELD-BOOK OF A CHAIN SURVEY.

(Extracted from the Roorkee Treatise on Surveying)

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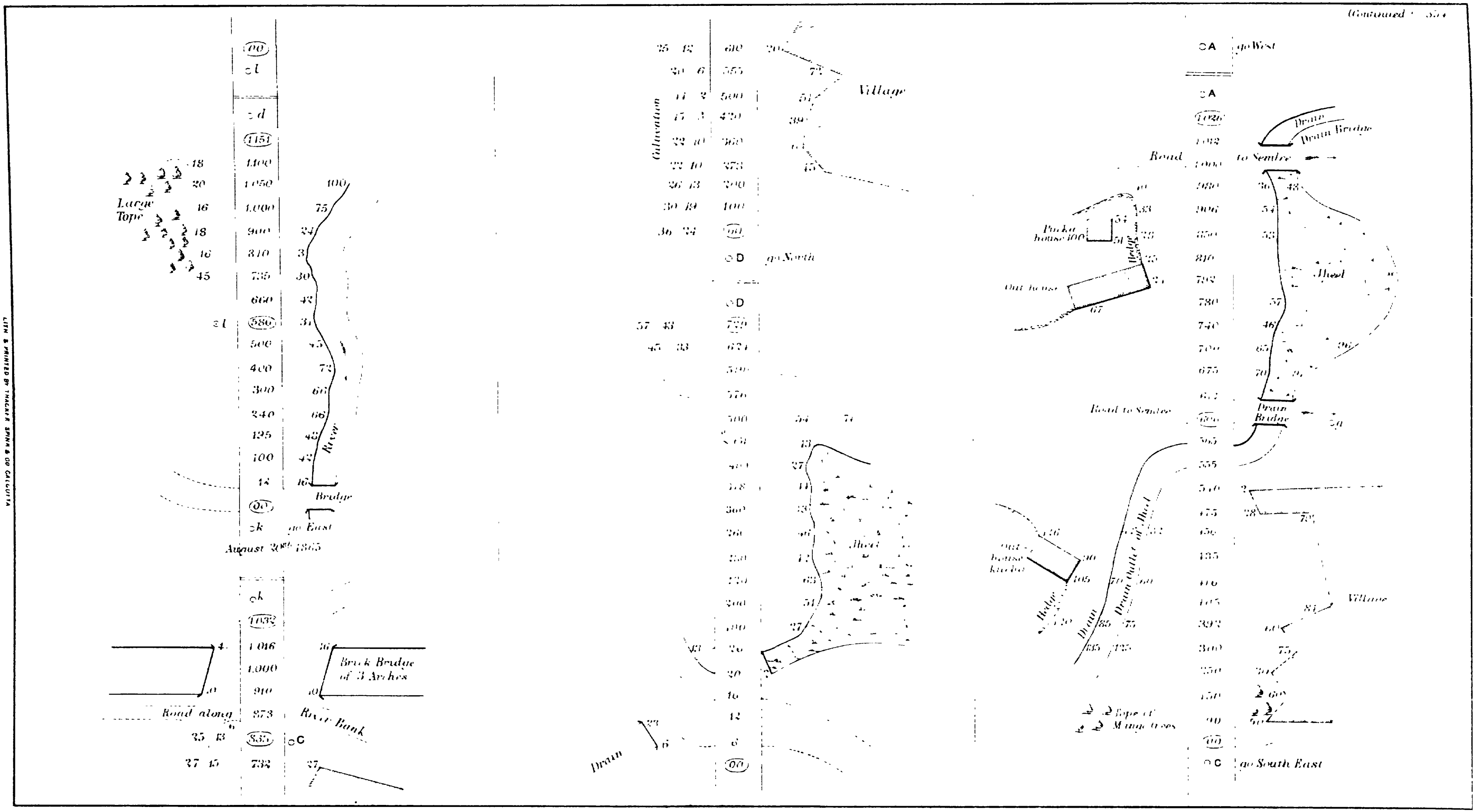
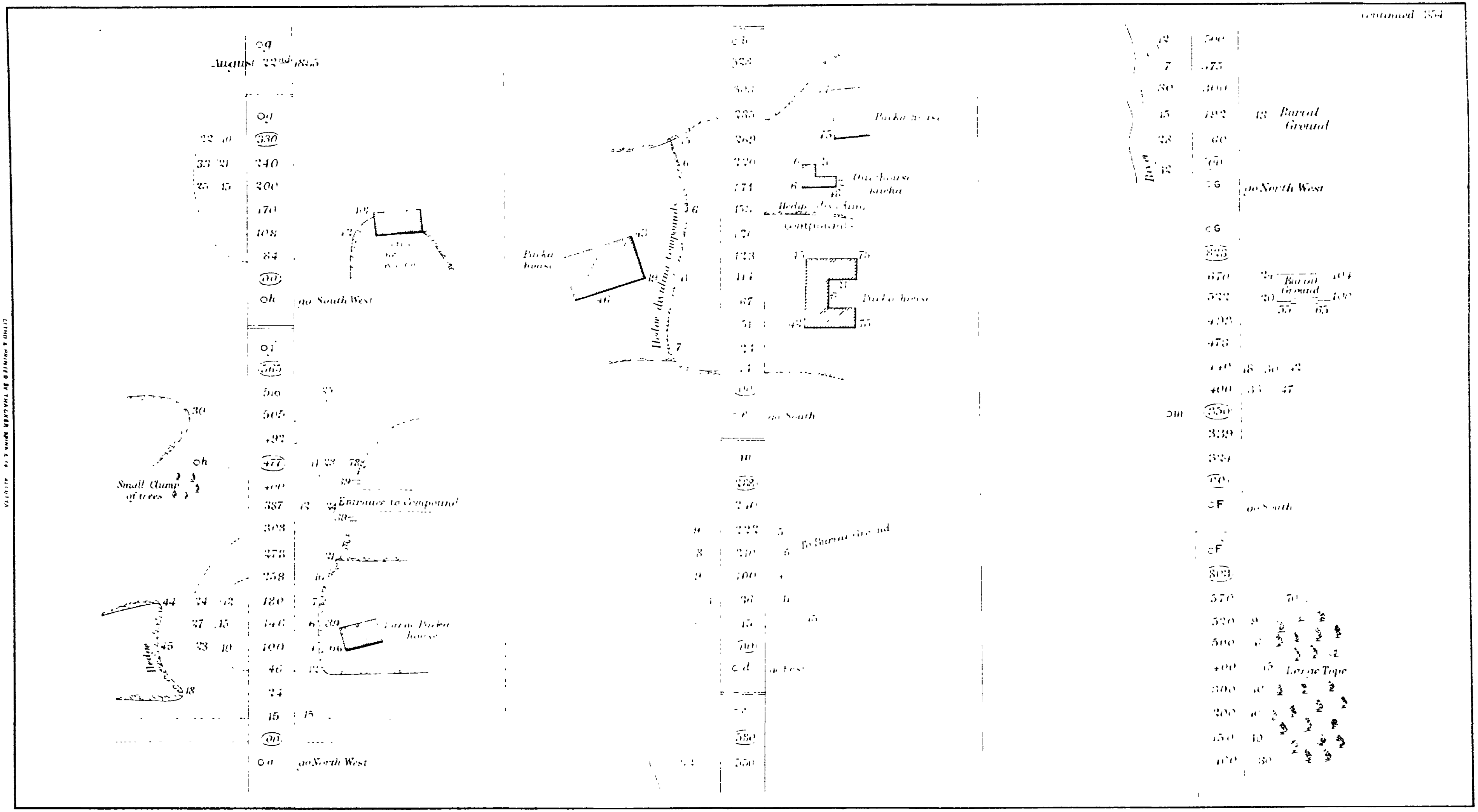
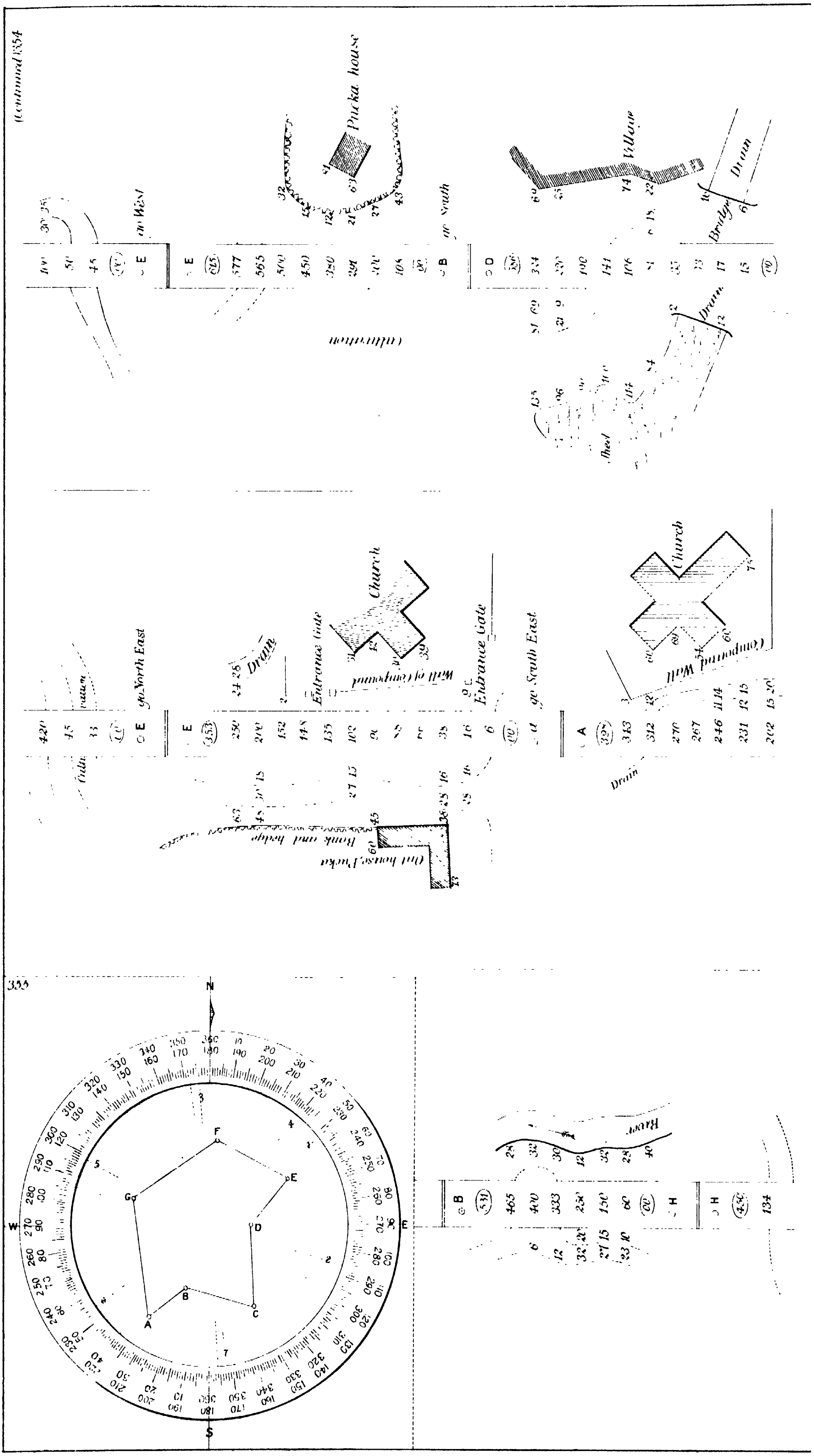


LITH & PRINTED BY THACKER SPINK & CO CALCUTTA

continued 354

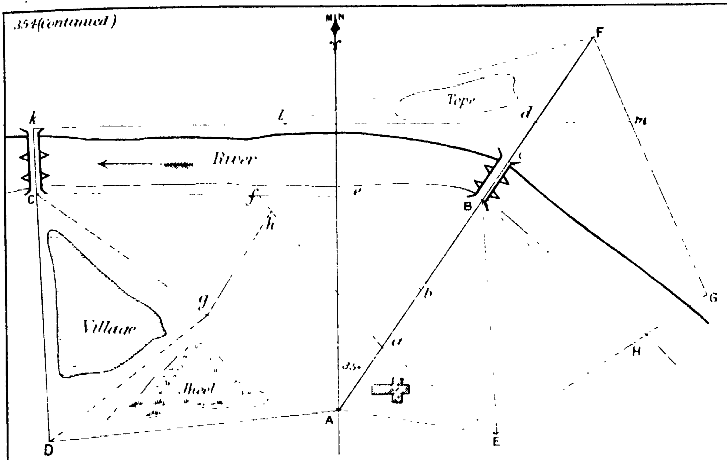


LITHO & PRINTED BY THACKER SPINK & CO CALCUTTA



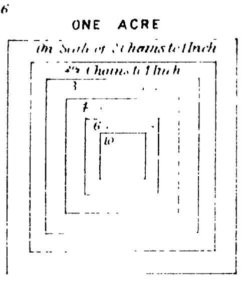
LITHO & PRINTED BY THACKER, SPINK & Co CALCUTTA

354 (continued)



Rough Sketch in Field Book
showing the positions of the measured lines and the various stations
The line ABF *bears 35° from Magnetic North Variation of compass 9°30' West of North*

356



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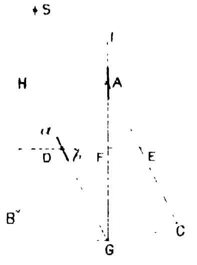

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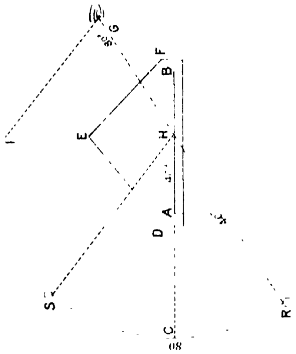

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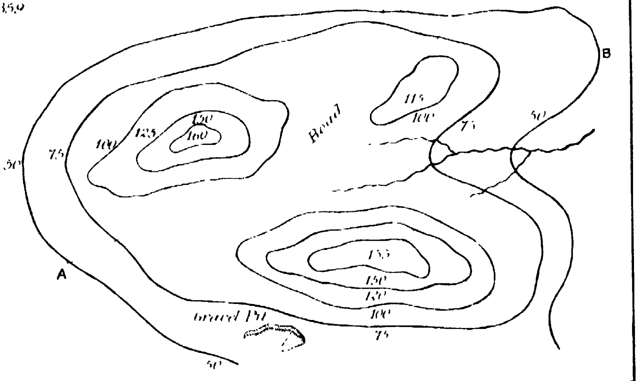

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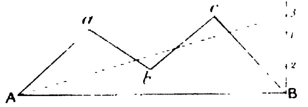

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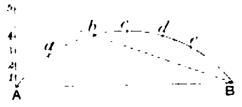

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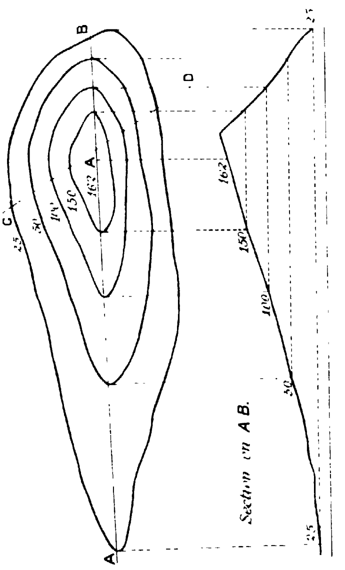

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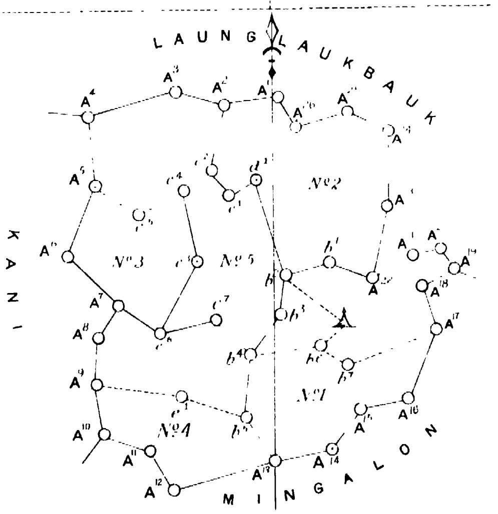

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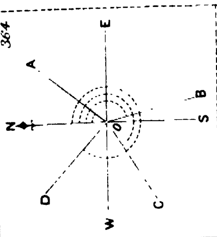

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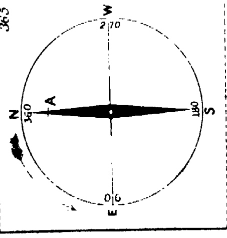

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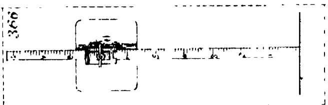

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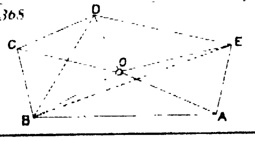

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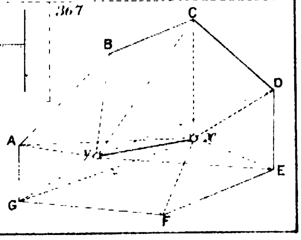

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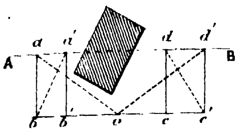
(Figures 369 to 384.)

369.



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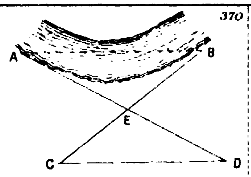

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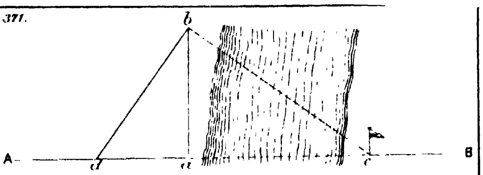

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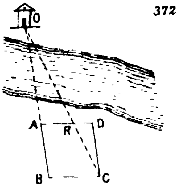

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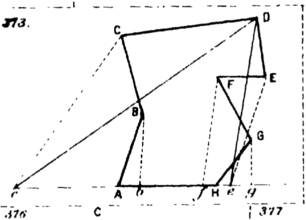

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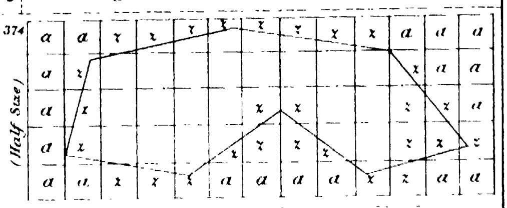

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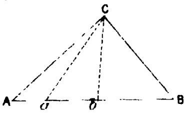

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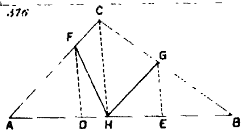

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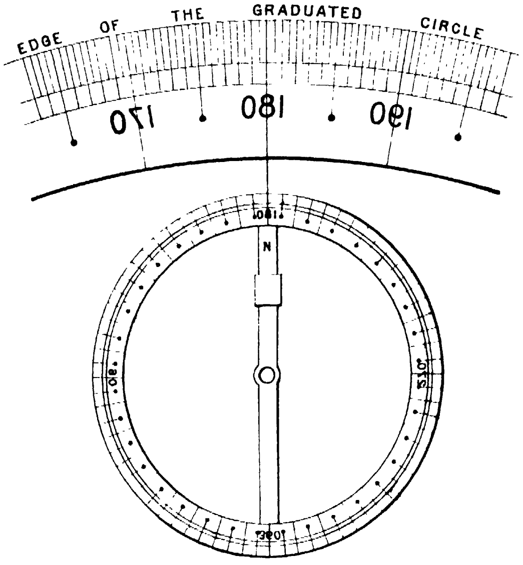
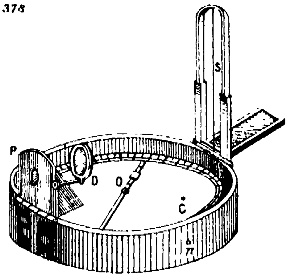


Sketch of the graduated circle of the Prismatic Compass. The figures and divisions are not all shown; but a portion of the edge is represented above it to an enlarged scale.

378



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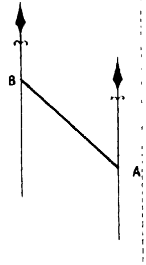
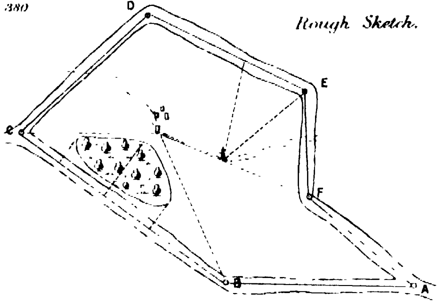


380

Rough Sketch.



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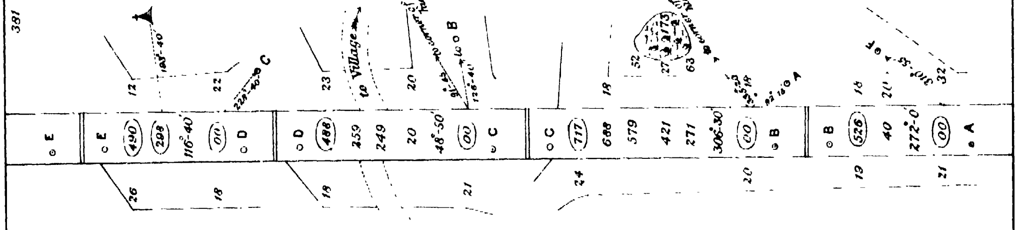

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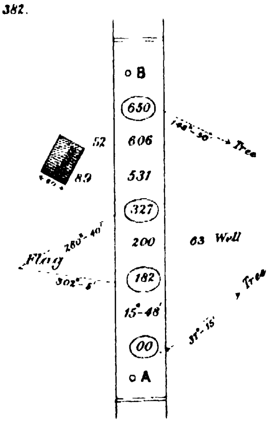

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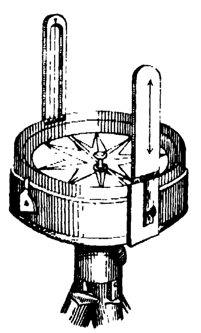

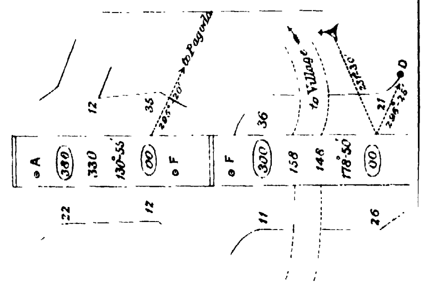

384.

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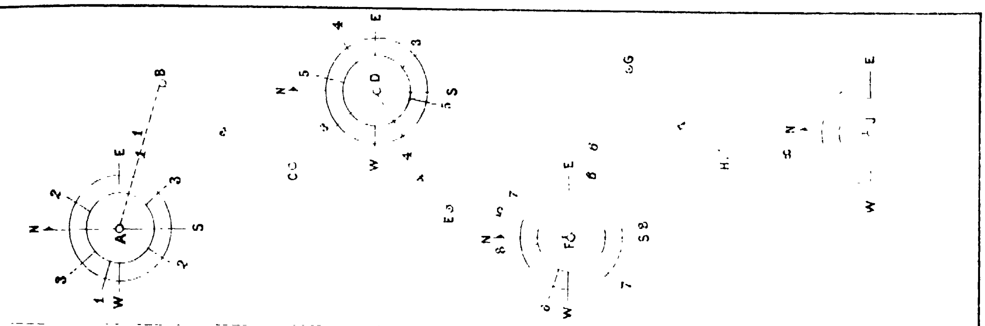
Figures

385



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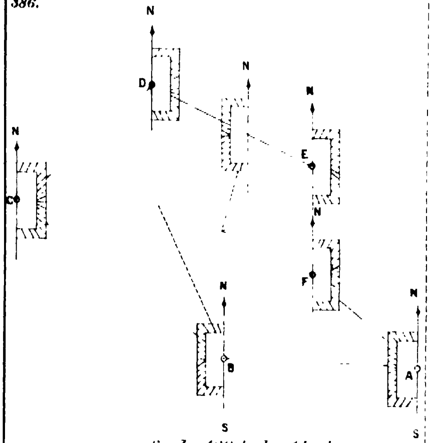
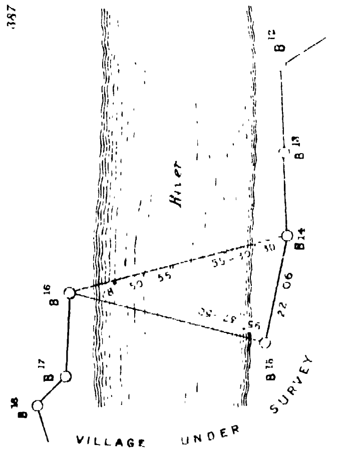


Scale, 400 links 1 Inch

387



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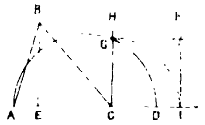

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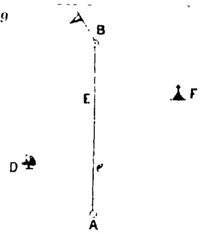

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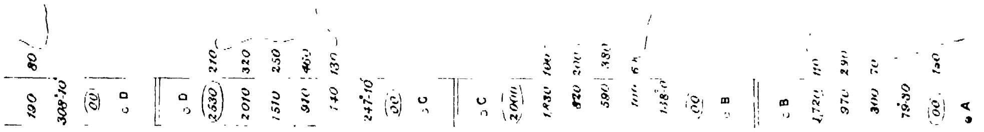

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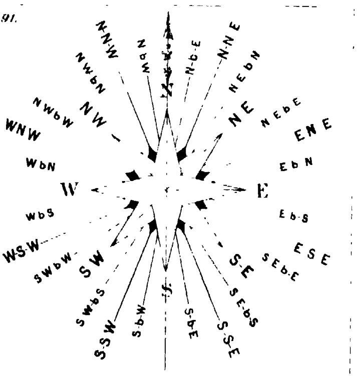

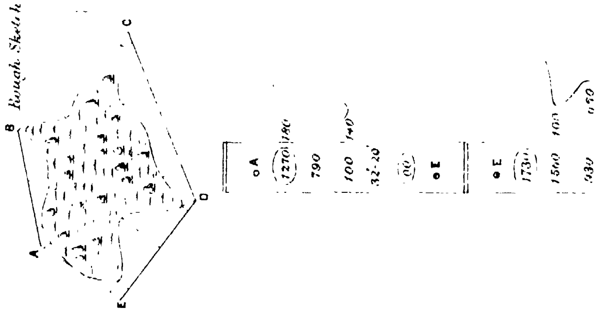

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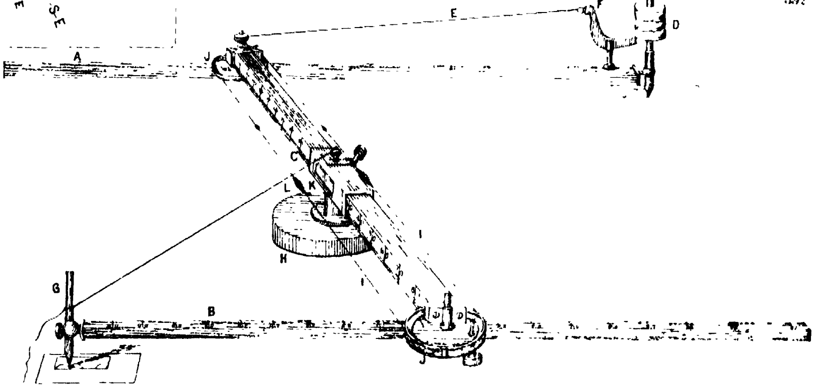

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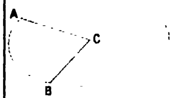

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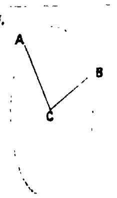

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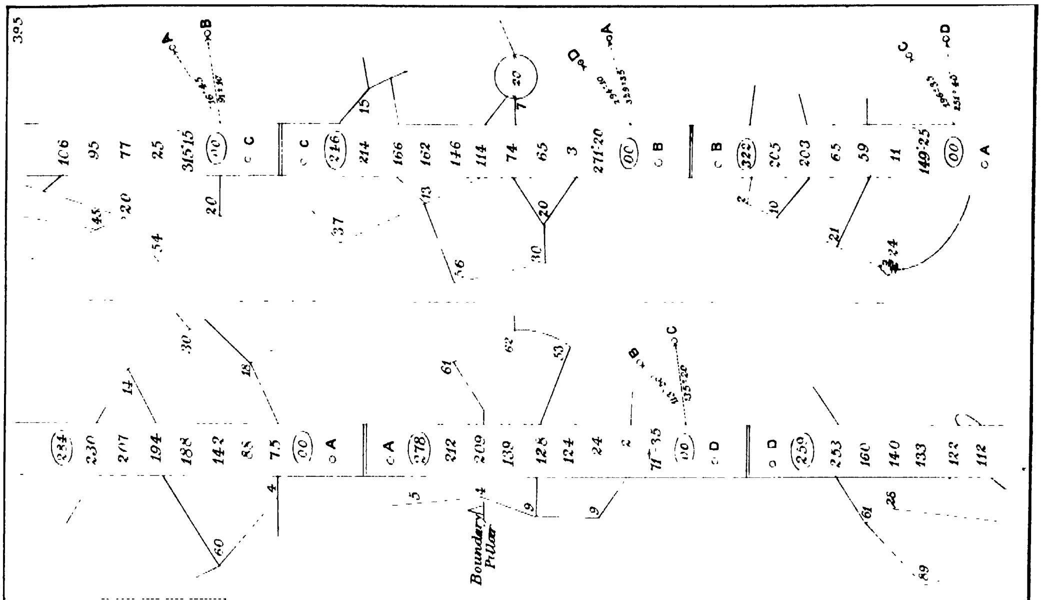

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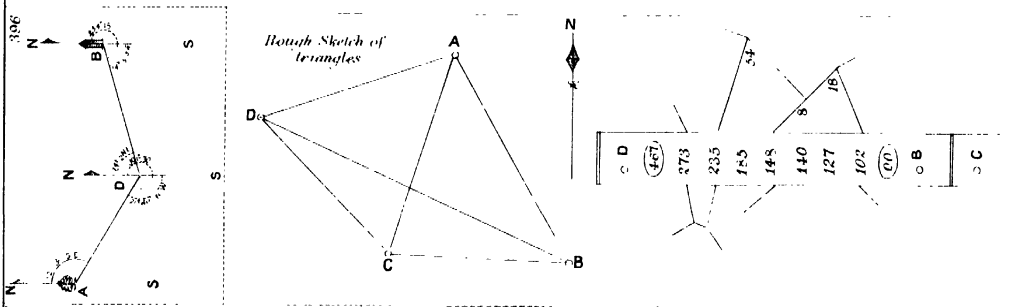

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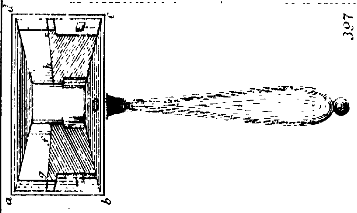

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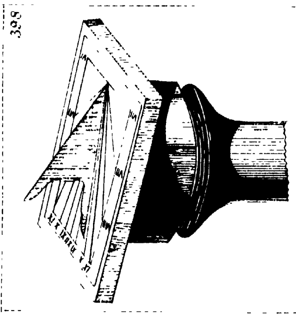

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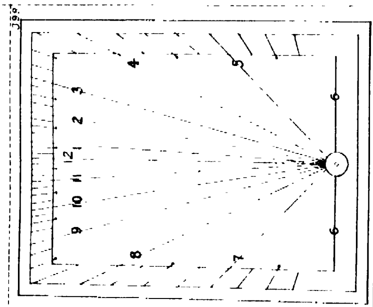

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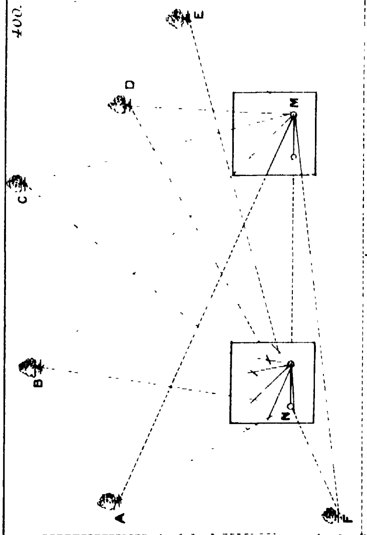

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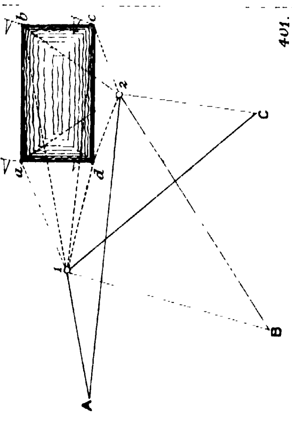

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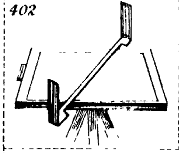

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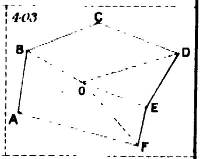

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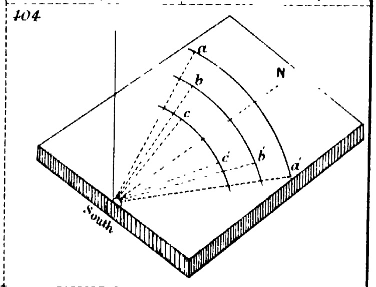

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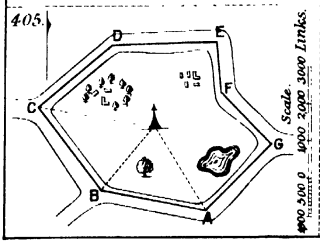

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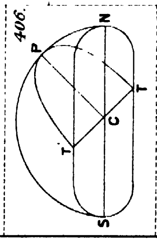

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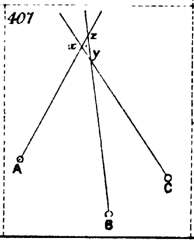

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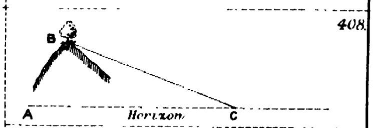

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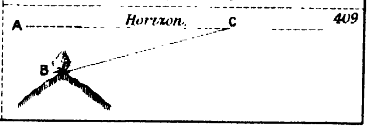

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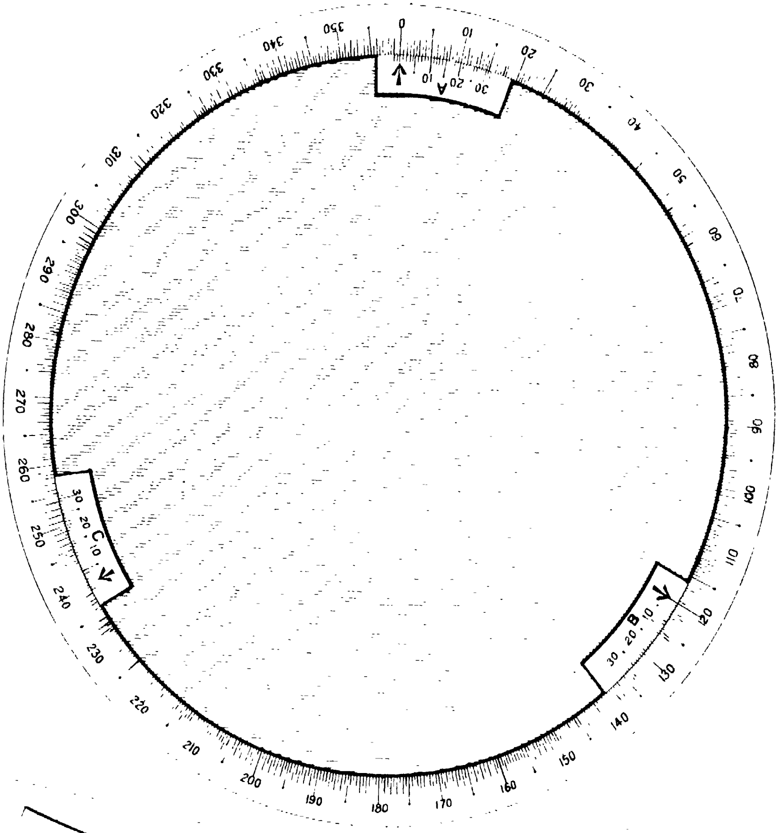
410



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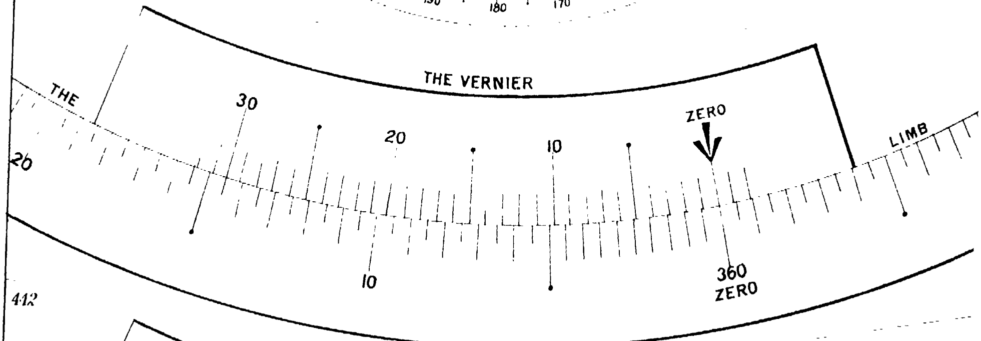

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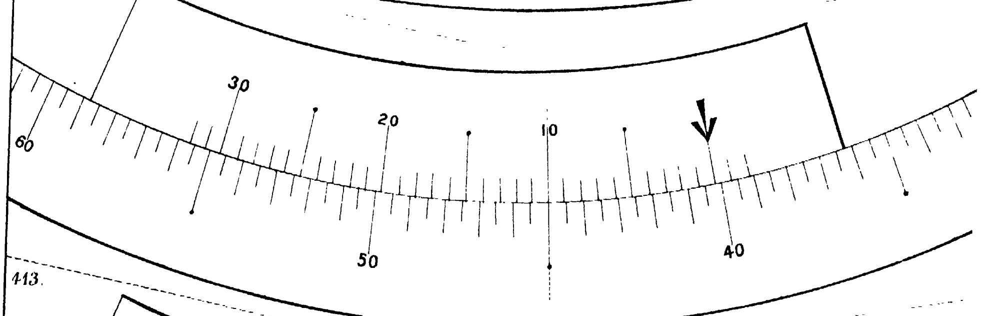

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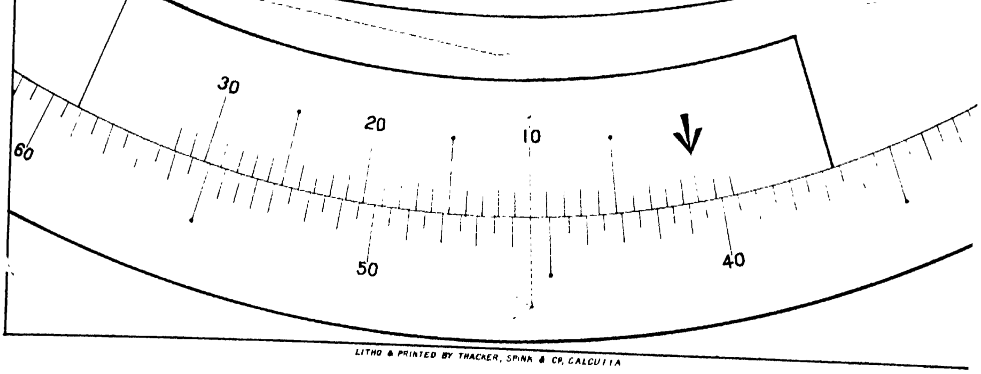

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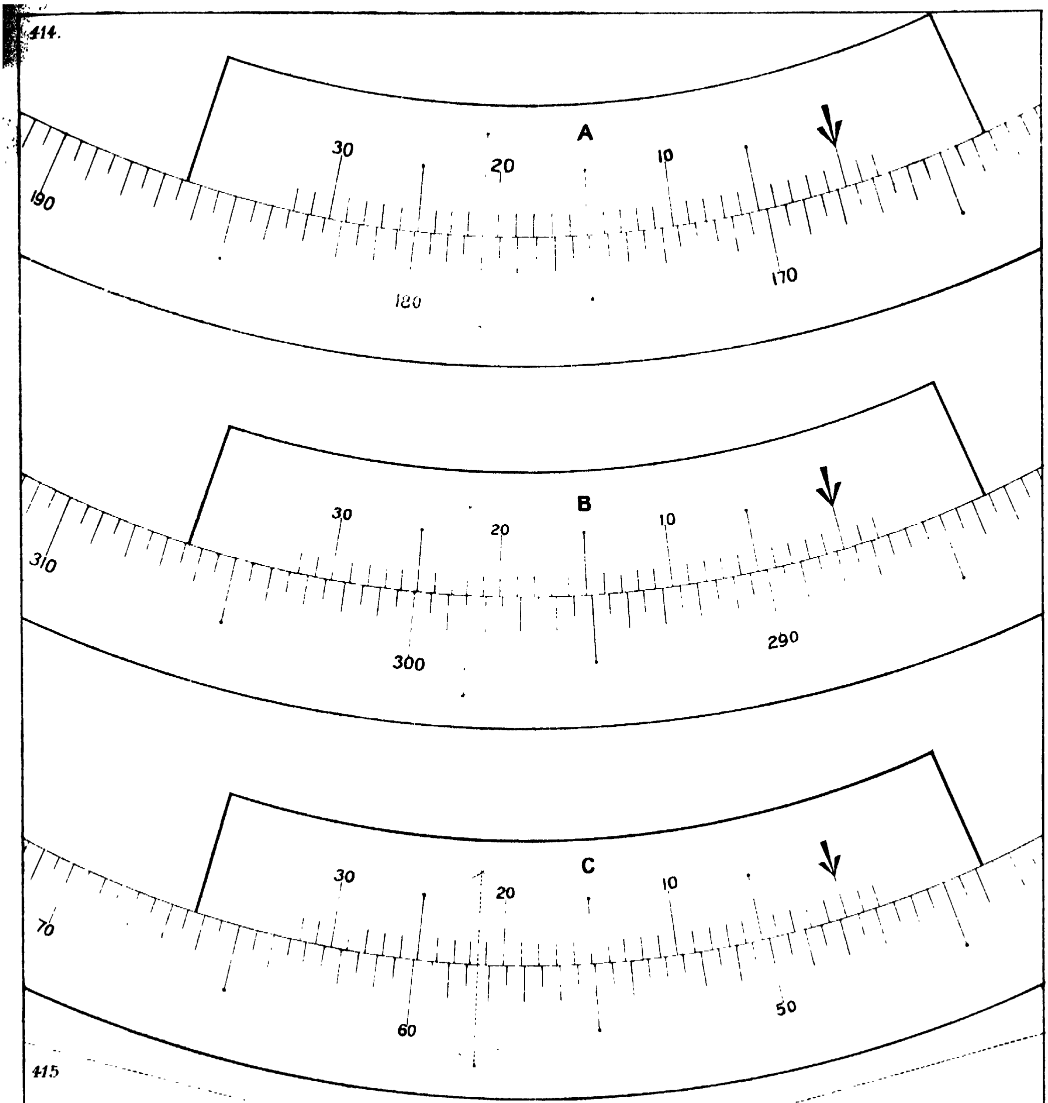
414.



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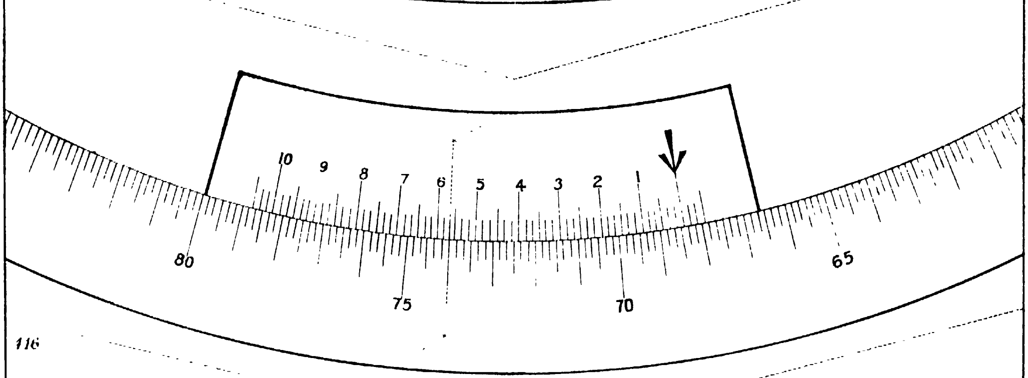

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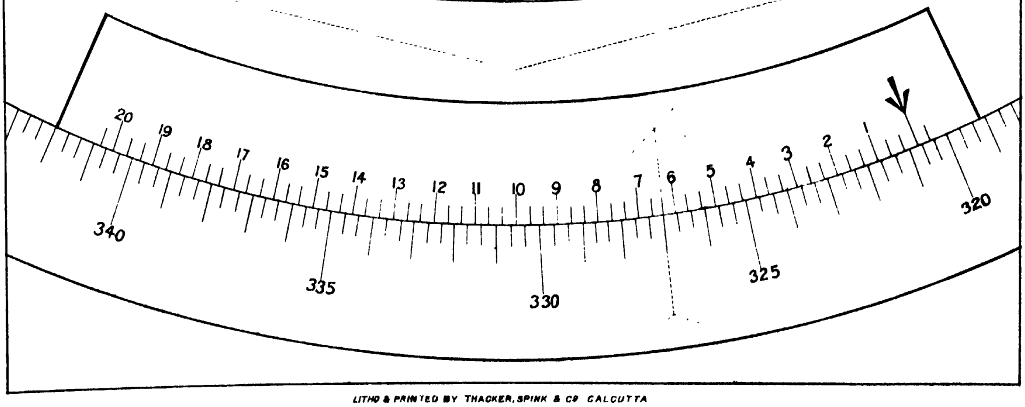
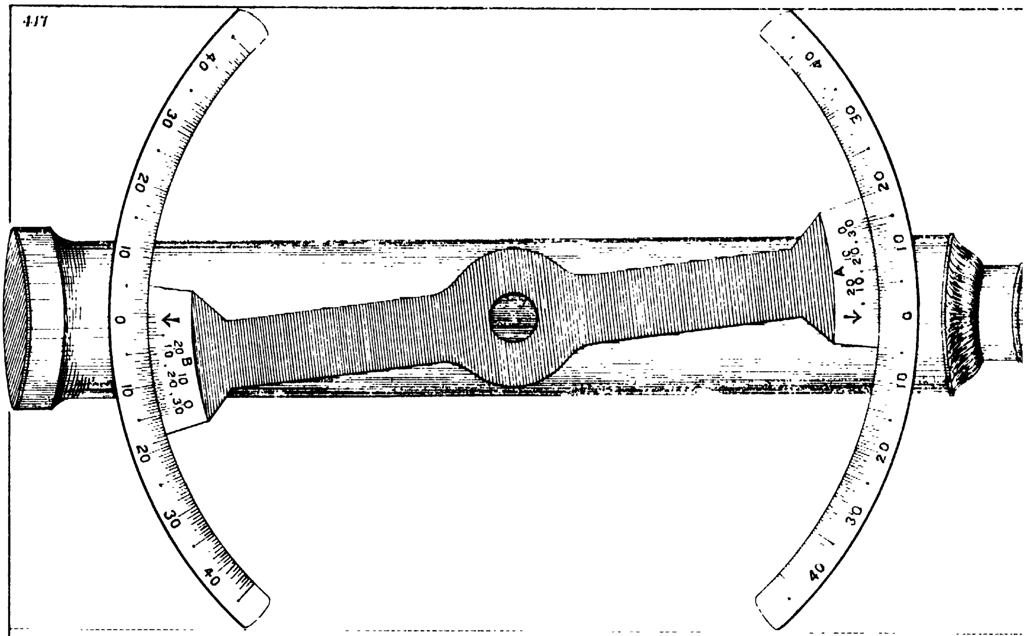


LITHO & PRINTED BY THACKER, SPINK & Co CALCUTTA

417



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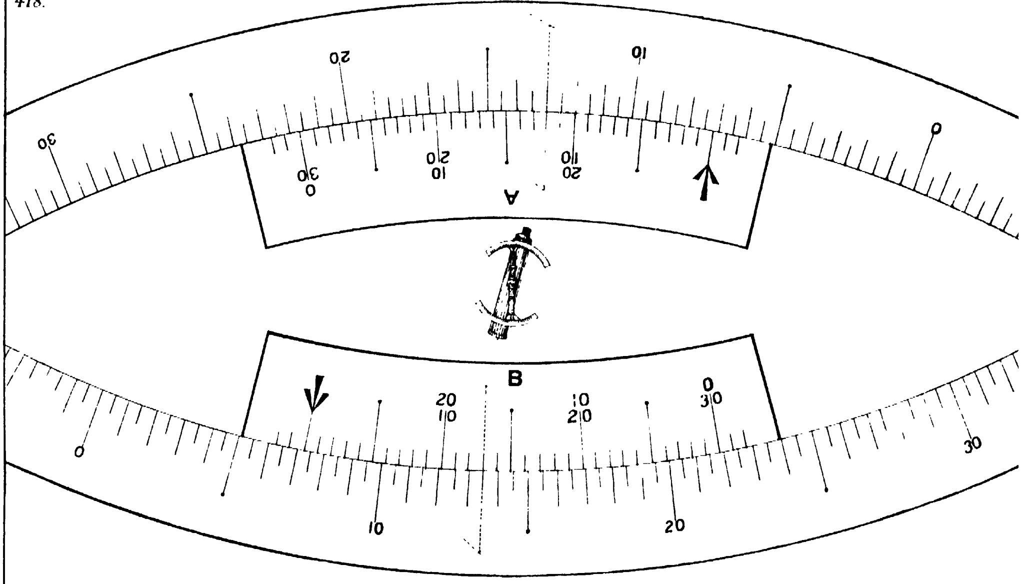

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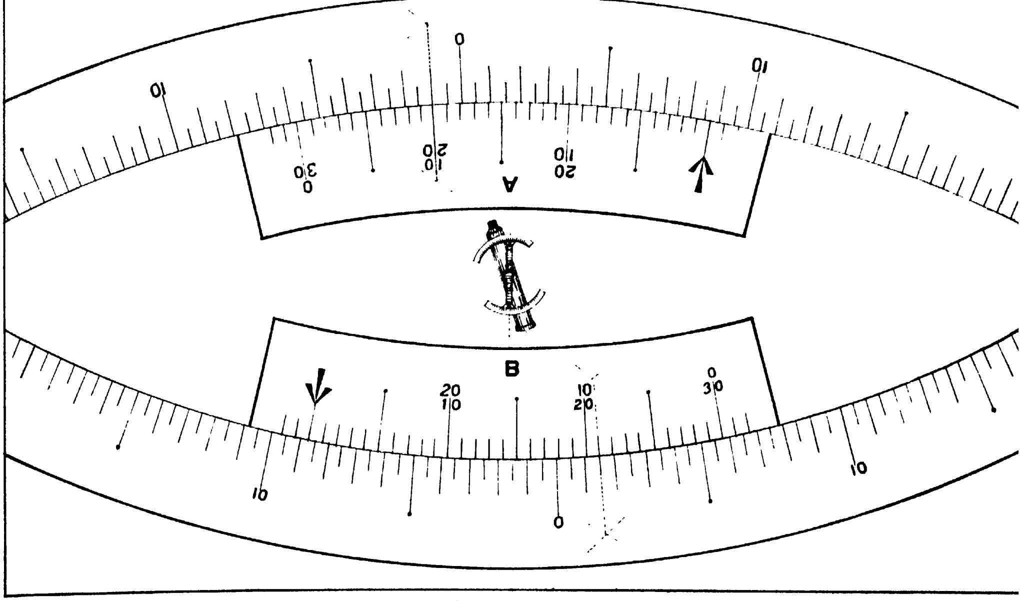
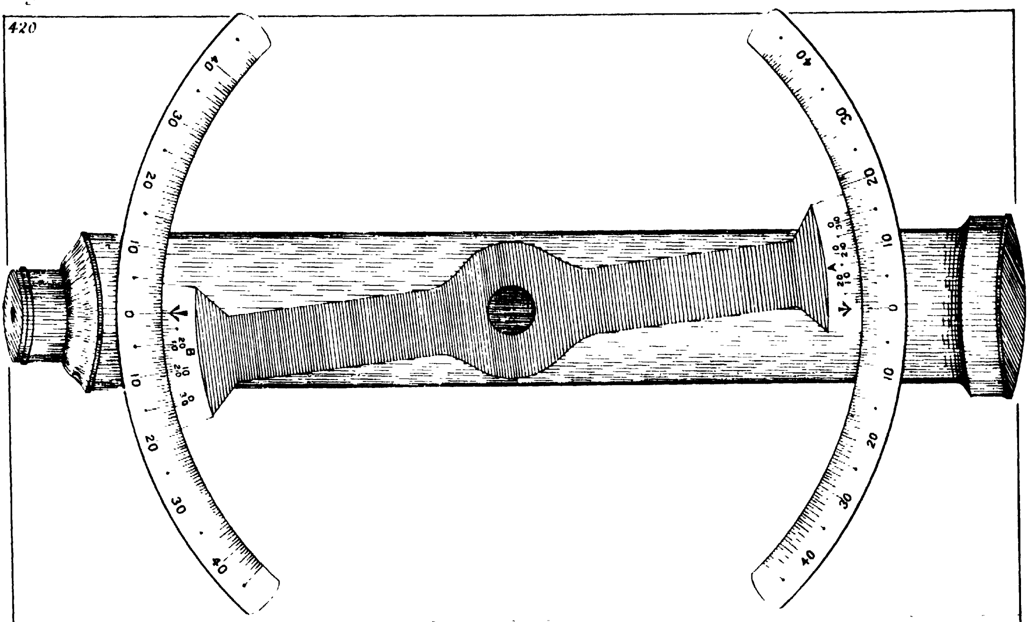


LITHO. & PRINTED BY THACKER, SPINK & Cº CALCUTTA

(Figures 420-422.)

420



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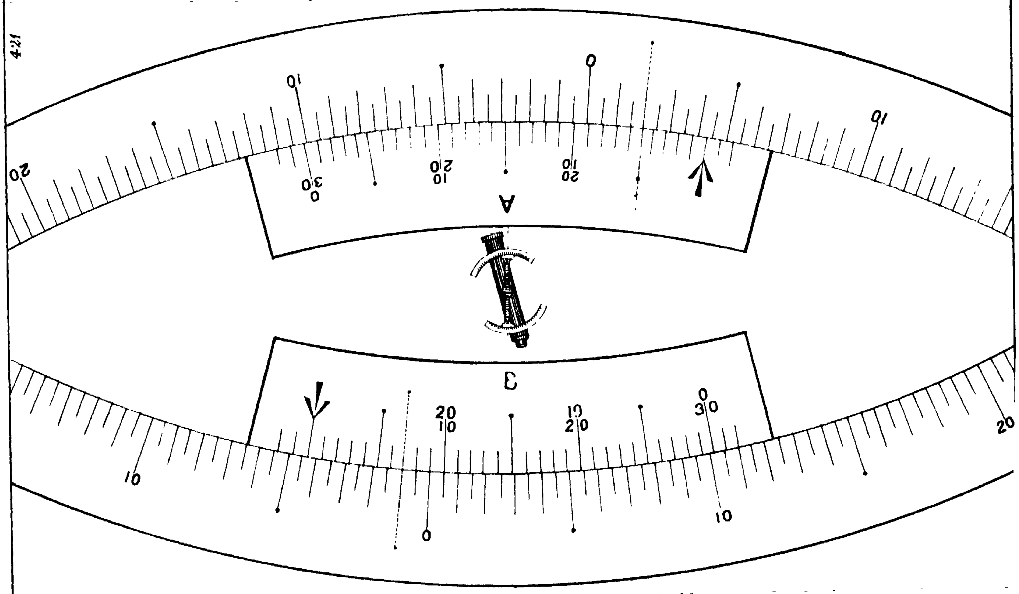

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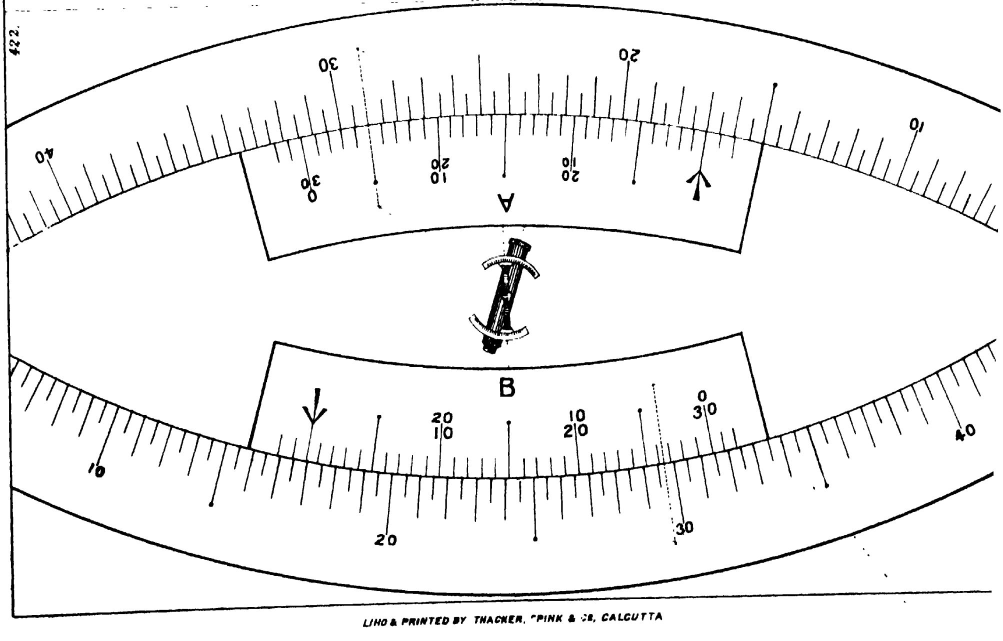
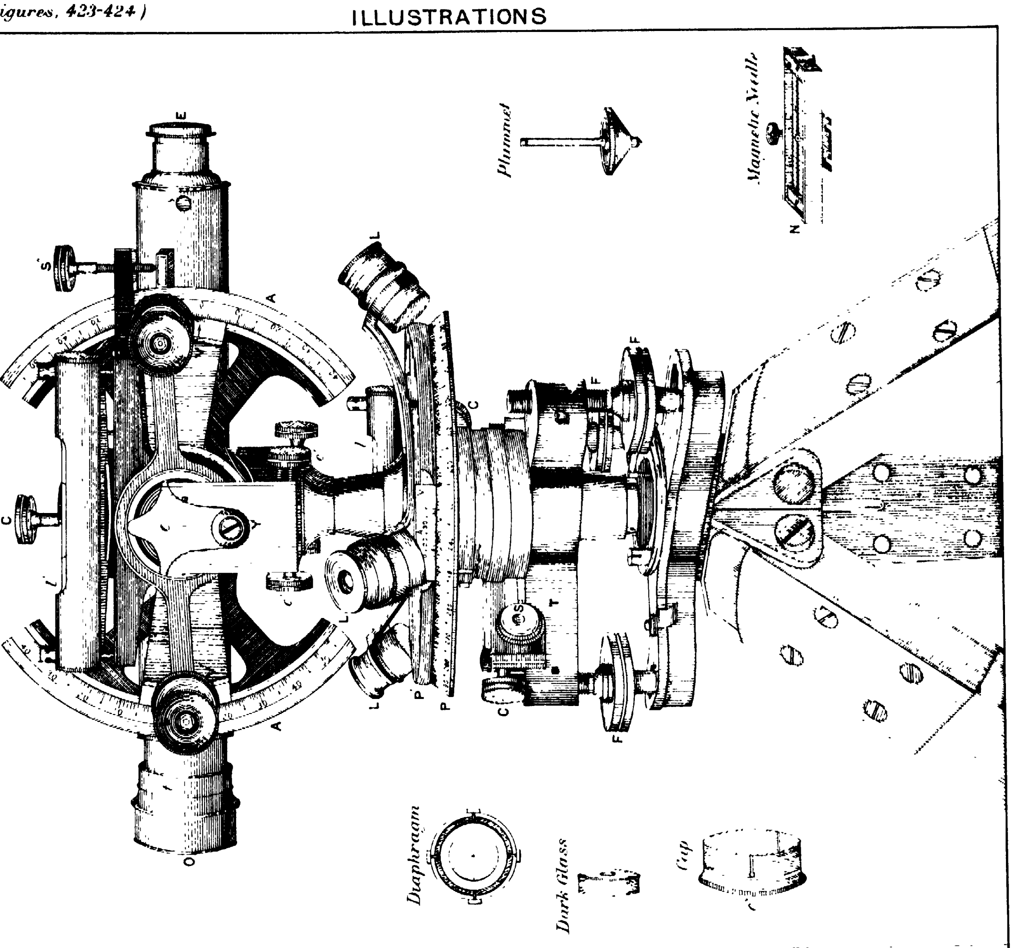


LITHO & PRINTED BY THACKER, SPINK & CO, CALCUTTA

LITHO & PRINTED BY THACKER, SPINK & CO. CALCUTTA

ILLUSTRATIONS

423



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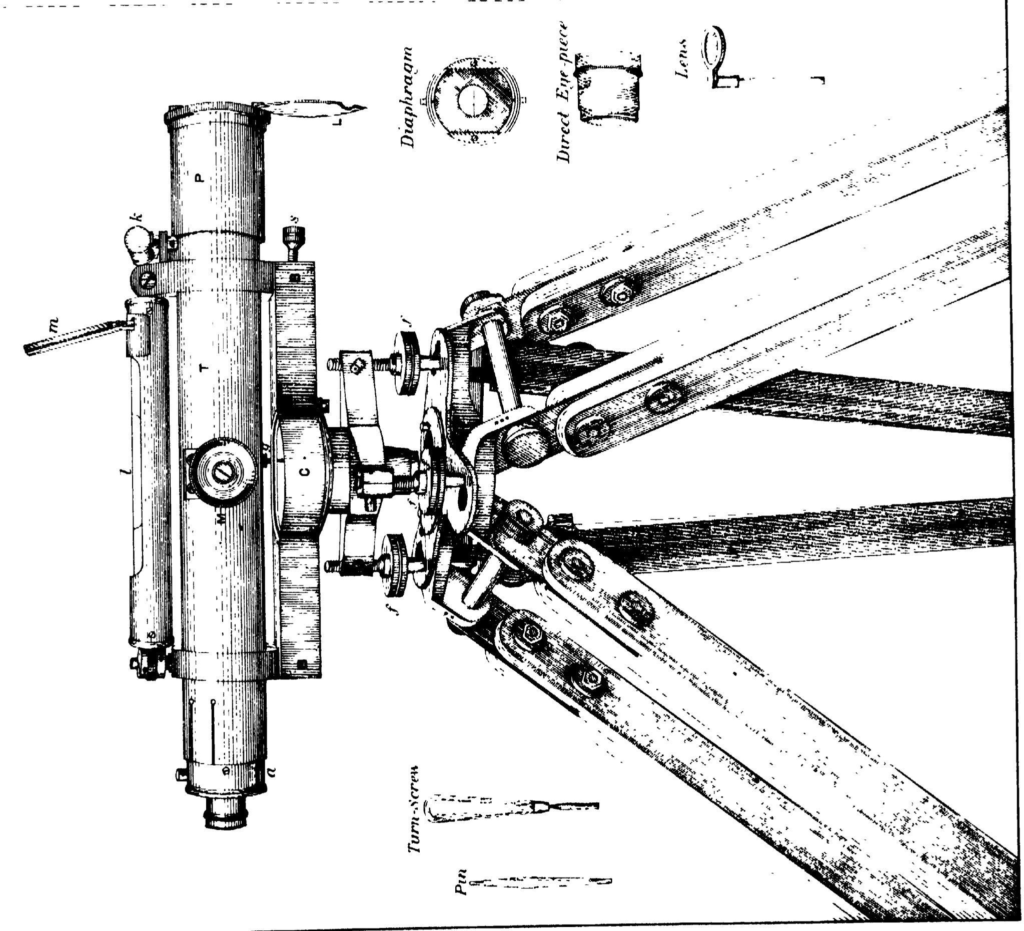
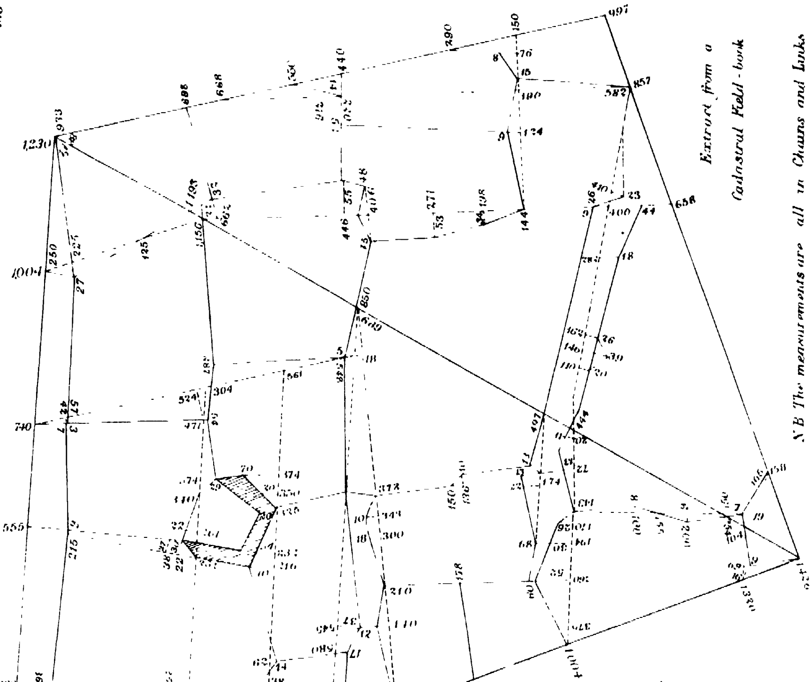


Litho & Printed by Thacker, Spink & Co, Calcutta.

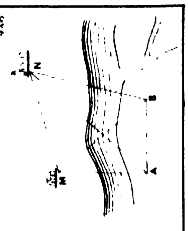
425



Extract from a Cadastral Field-book

N.B. The measurements are all in Chains and Links

426



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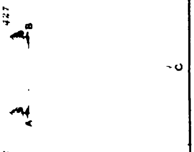

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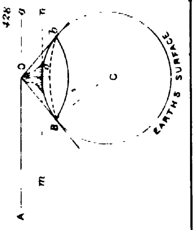

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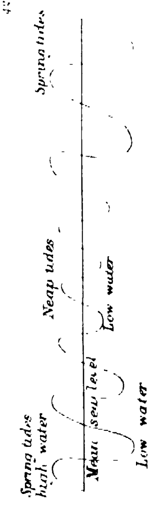

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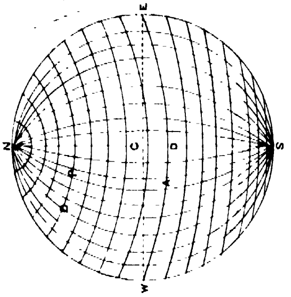

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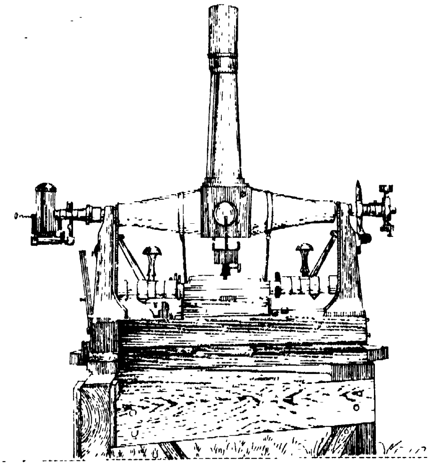

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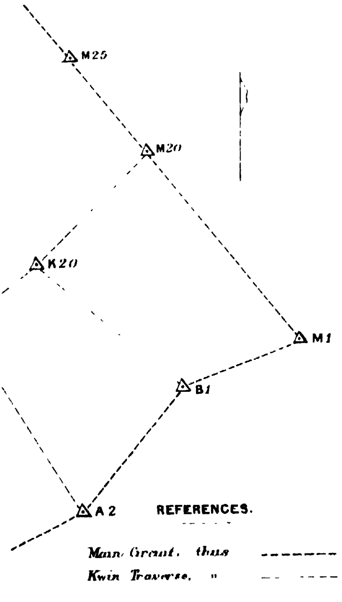

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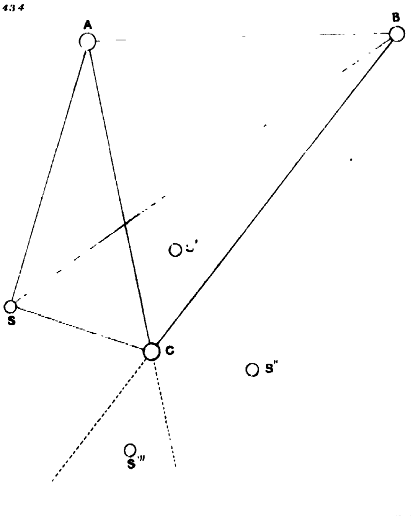
Main Triant. thus ------

Kwin Traverse, " -- ---

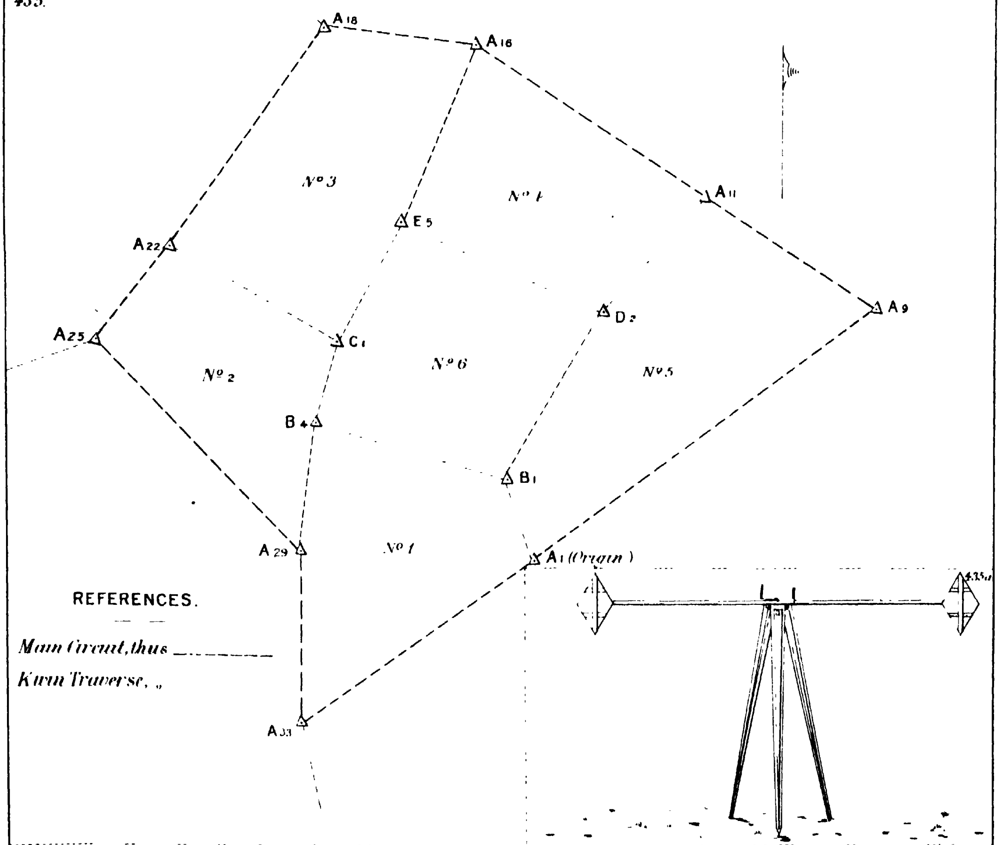
434



LITHO & PRINTED BY THACKER SPINK & CO CALCUTTA

ILLUSTRATIONS.

435.



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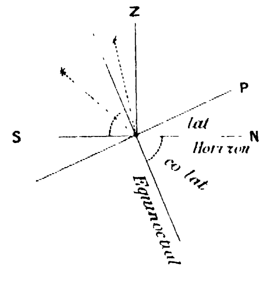
Main Circuit, thus ————

Kum Traverse, „

436



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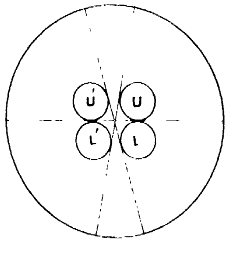

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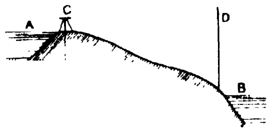

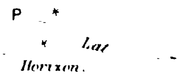

439

440

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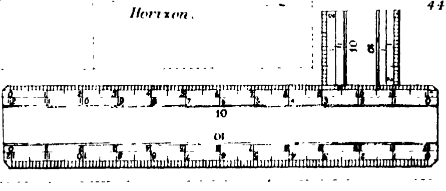

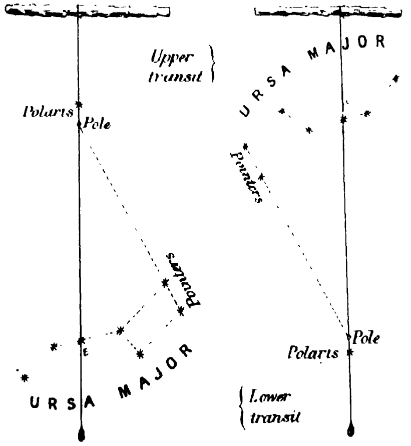

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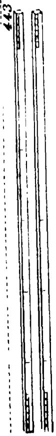

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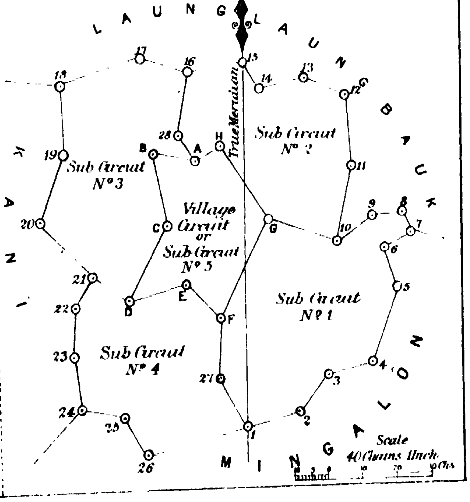

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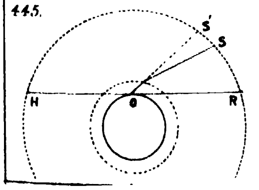

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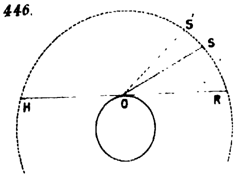

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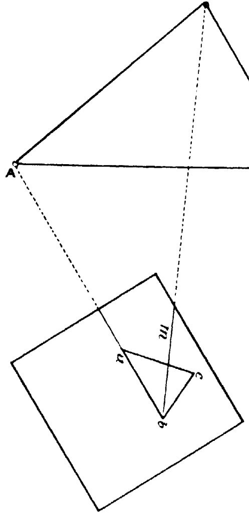

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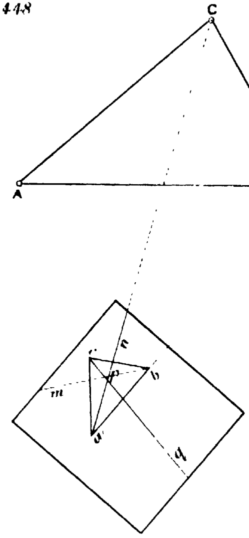

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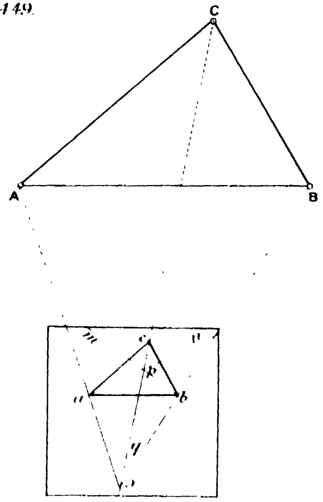

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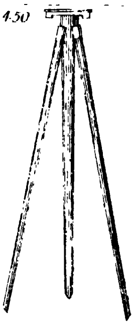

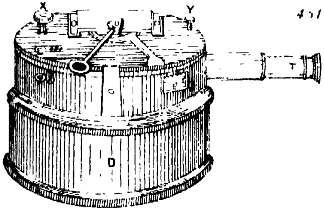

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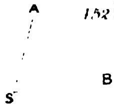

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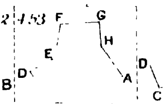

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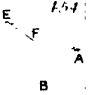

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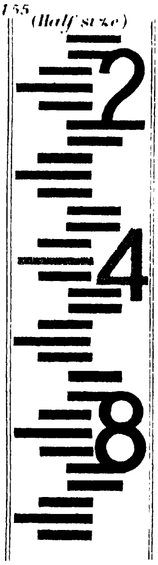

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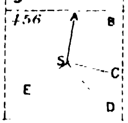

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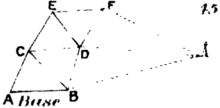

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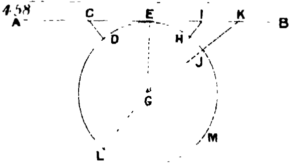
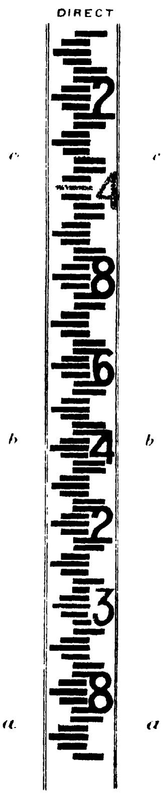
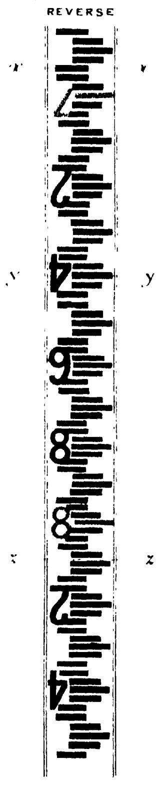


459 (Quarter size) DIRECT



460 (Quarter size) REVERSE



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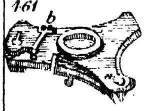

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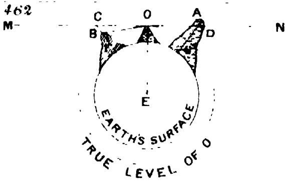

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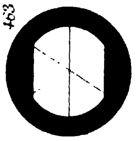

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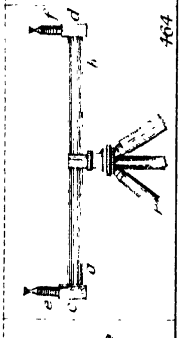

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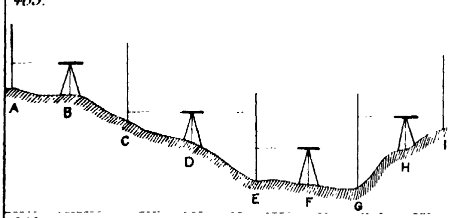

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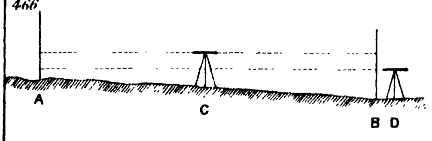

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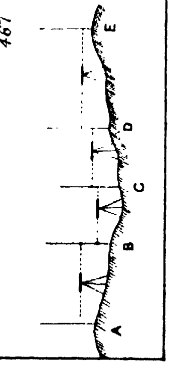

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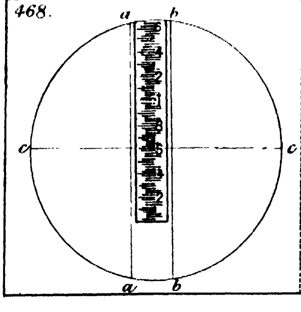

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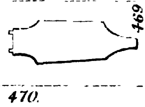

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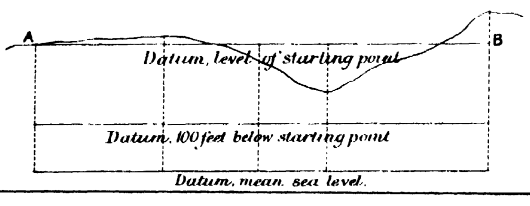
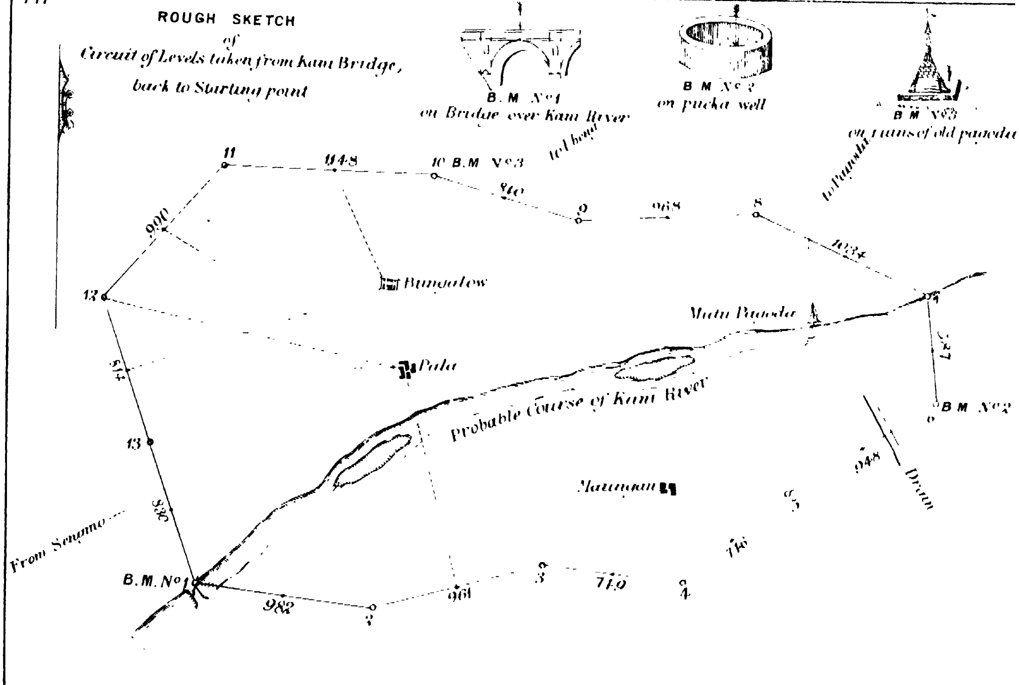

A
B
Datum, level of starting point
Datum, 100 feet below starting point
Datum, mean sea level.

LITHO & PRINTED BY THACKER SPINK & CO.

ILLUSTRATIONS.

471



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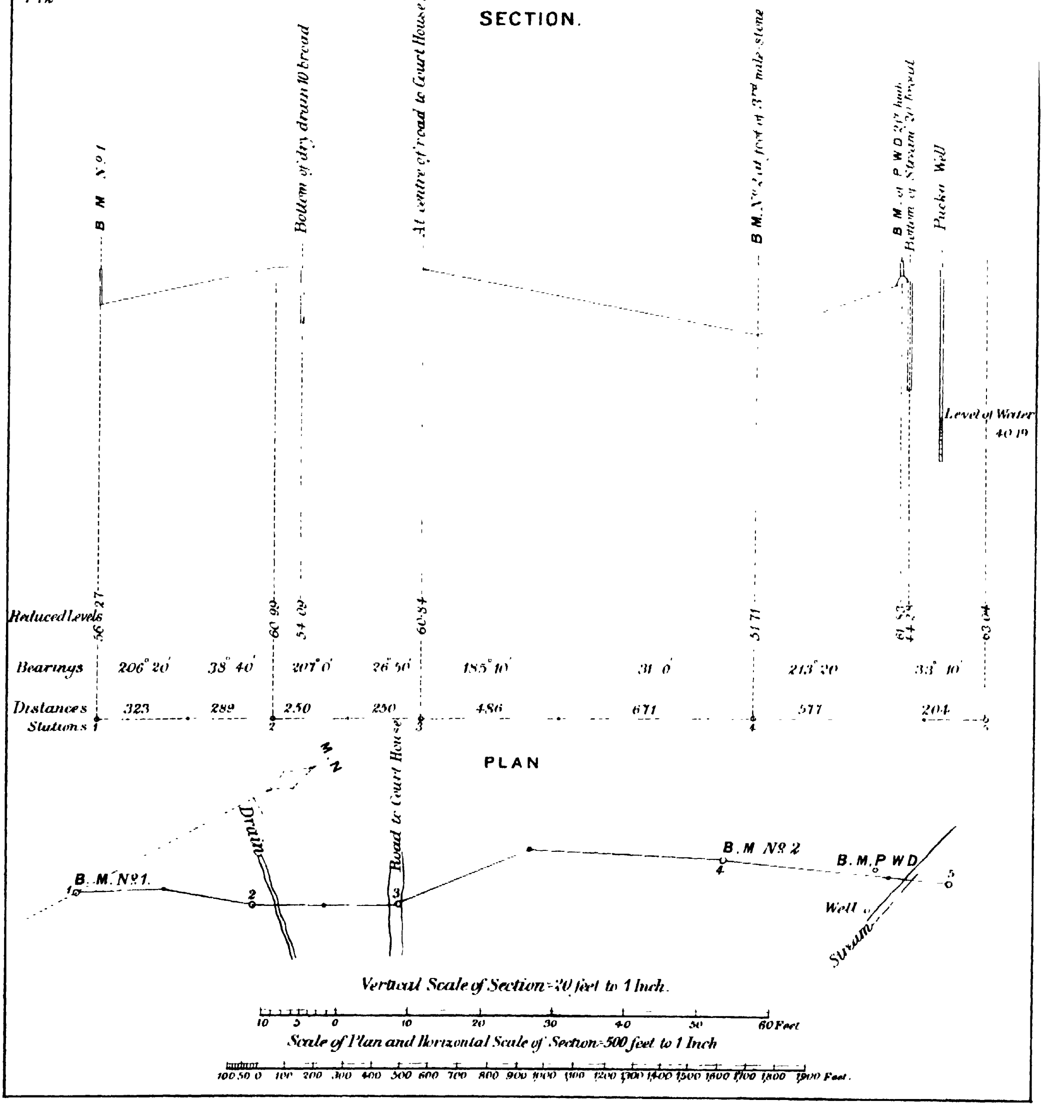
SECTION.
B. M. No. 1
Bottom of dry drain 10 broad
At centre of road to Court House.
B. M. No. 2 at foot of 3rd mile stone
B. M. of P. W. D. 2/7 inch
Bottom of Stream 20 broad
Pucka Well
Level of Water 40.19
Reduced Levels
56.27
60.99
54.02
60.84
51.71
61.53
44.24
63.04
Bearings
206° 20′
38° 40′
207° 0′
26° 56′
185° 10′
31° 0′
213° 20′
33° 10′
Distances
323
289
250
250
486
671
577
204
Stations
1
2
3
4
5
PLAN
M. N
Drain
Road to Court House
B. M. No. 1.
1
2
3
B. M. No. 2
4
B. M. P. W. D.
5
Well
Stream
Vertical Scale of Section = 20 feet to 1 Inch.
10 5 0 10 20 30 40 50 60 Feet
Scale of Plan and Horizontal Scale of Section = 500 feet to 1 Inch
100 50 0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 Feet.

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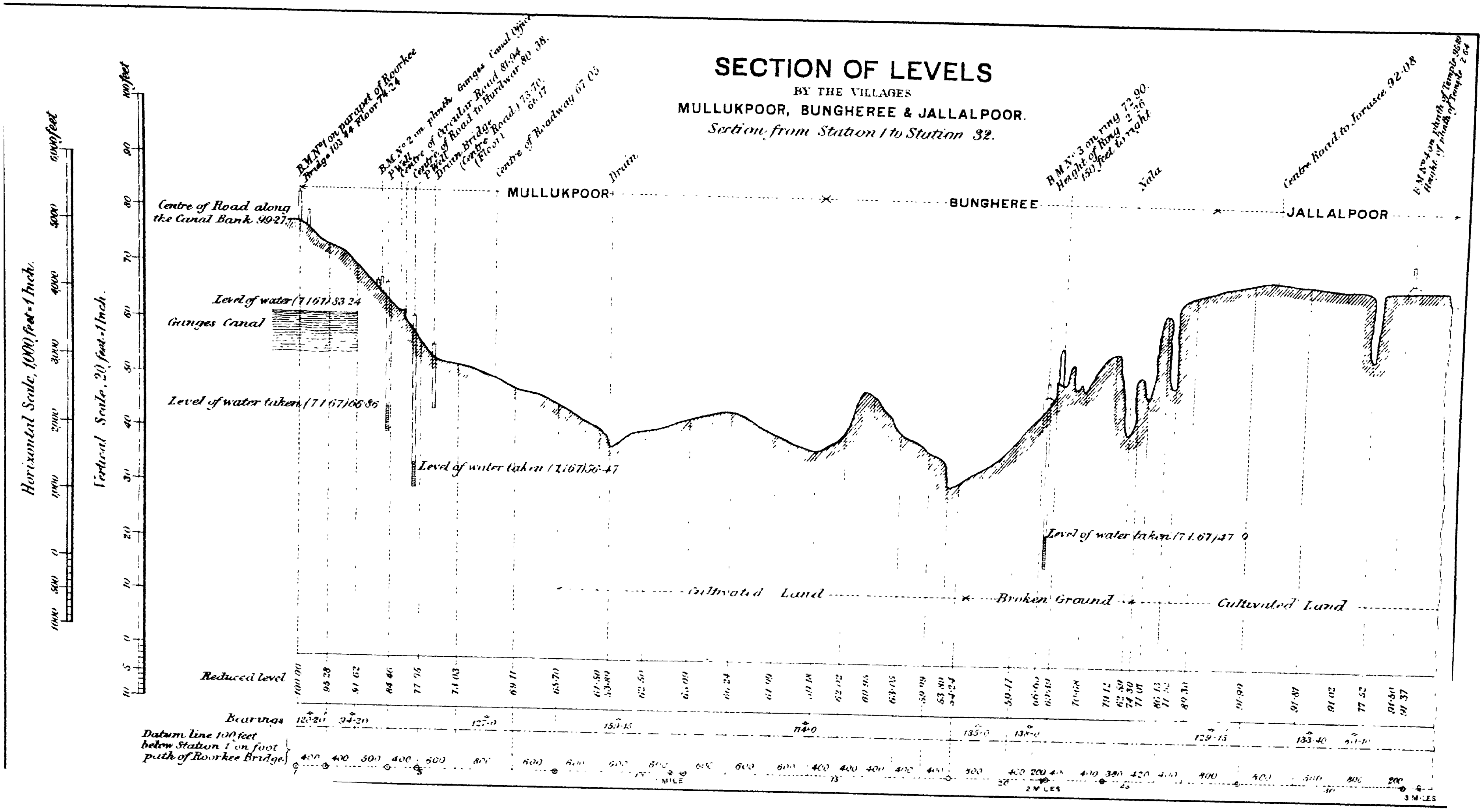
SECTION OF LEVELS

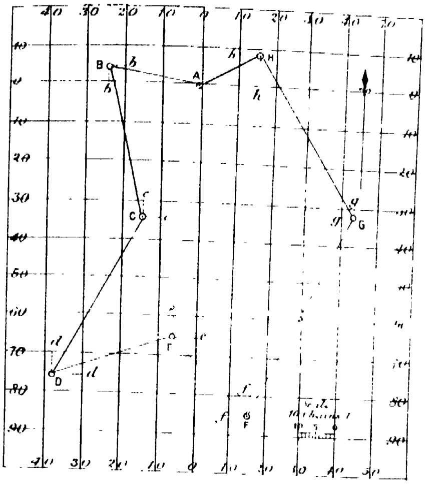
BY THE VILLAGES

MULLUKPOOR, BUNGHEREE & JALLALPOOR.

Section from Station 1 to Station 32.

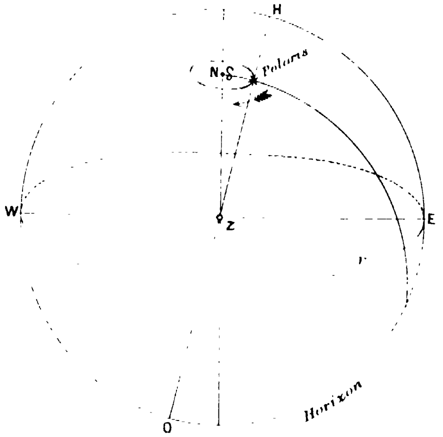


LITHO BY THACKER, SPINK & CO, CALCUTTA



(Quarter size)

475



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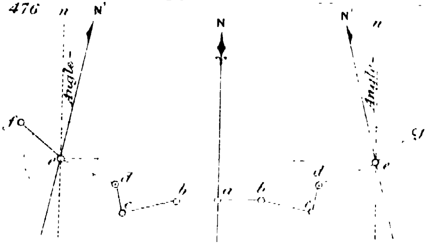

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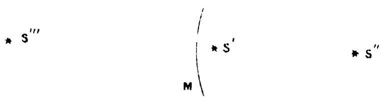

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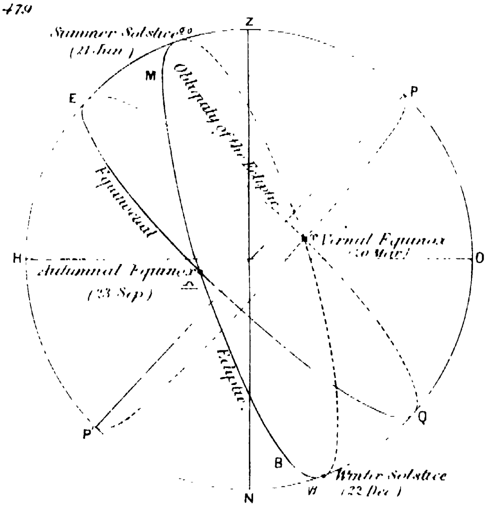

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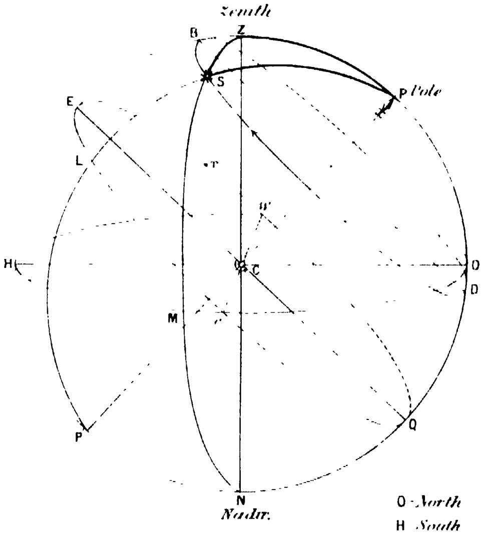

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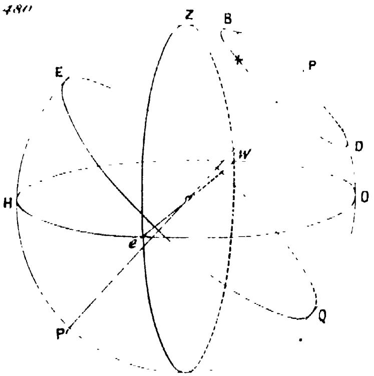

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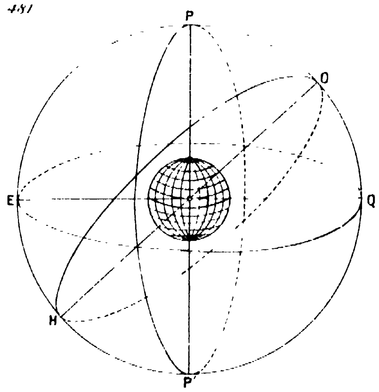

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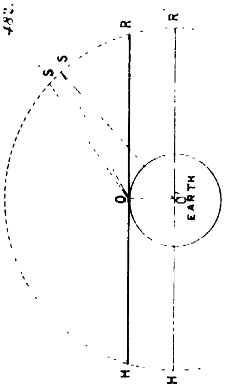

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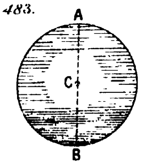

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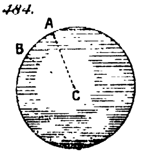

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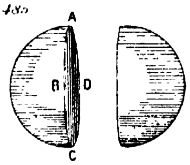

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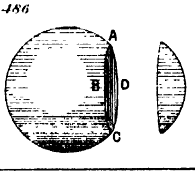

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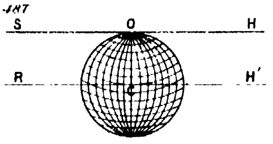

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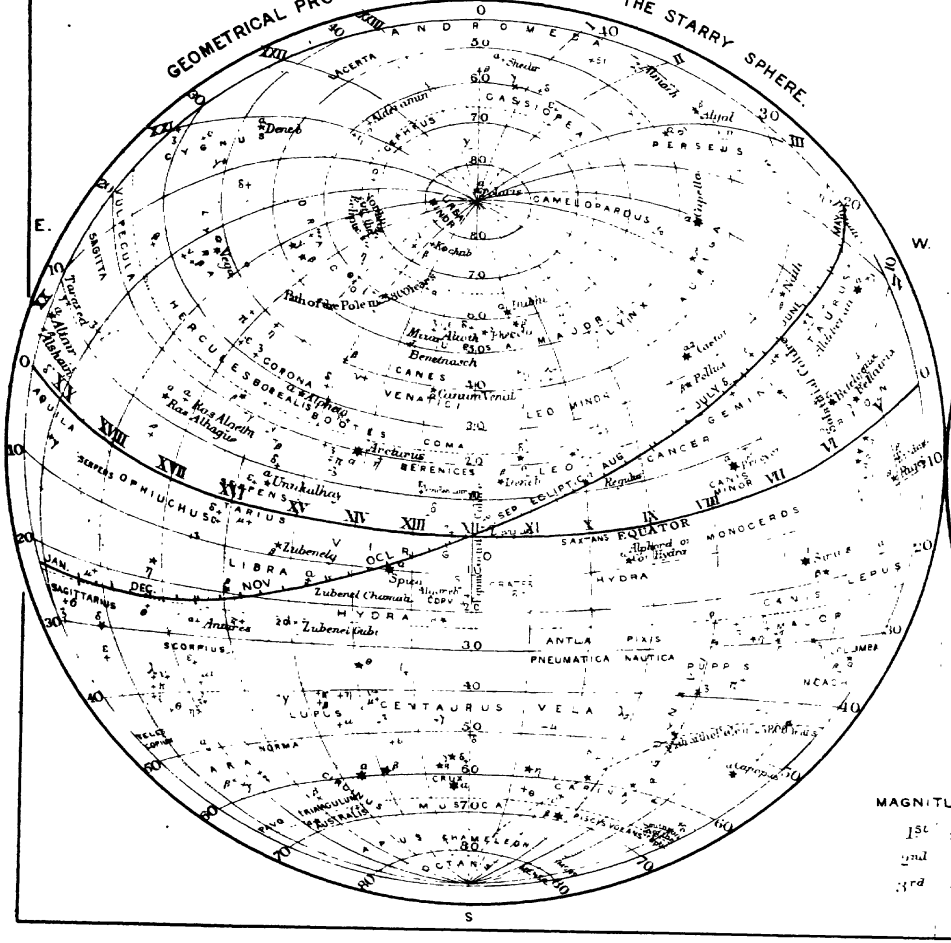
GEOMETRICAL PROJECTION OF TWO-THIRDS OF THE STARRY SPHERE.



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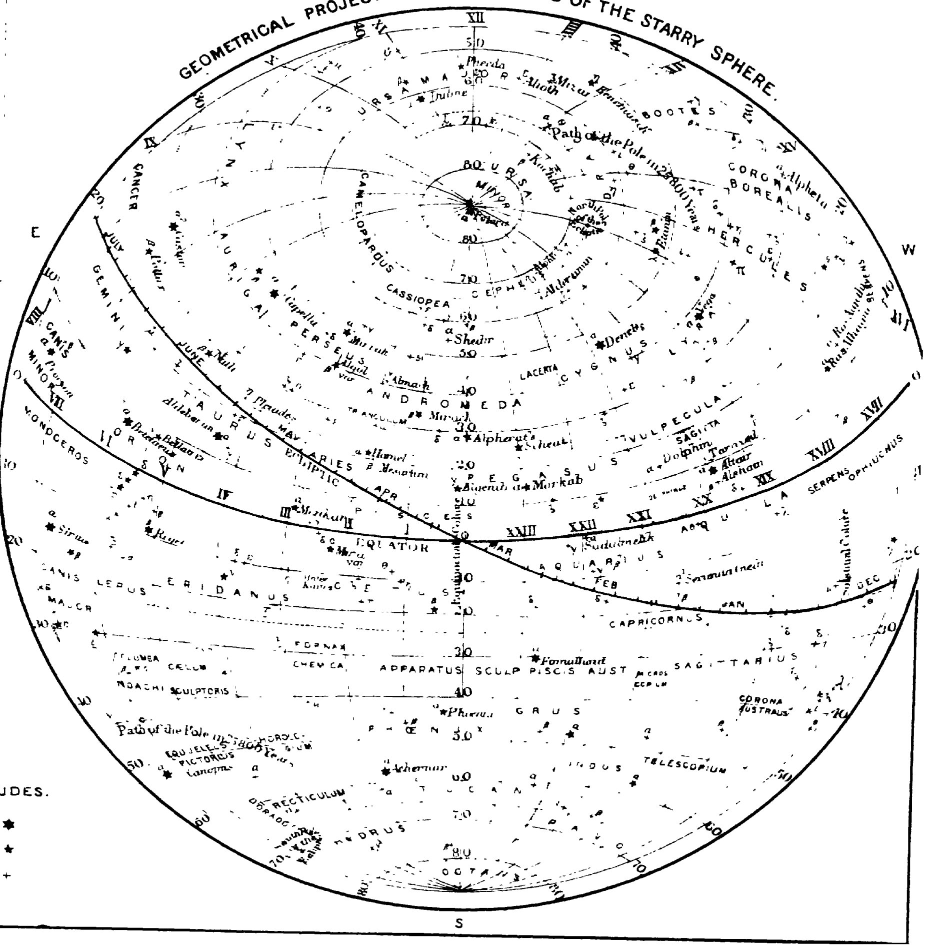

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Nº 4

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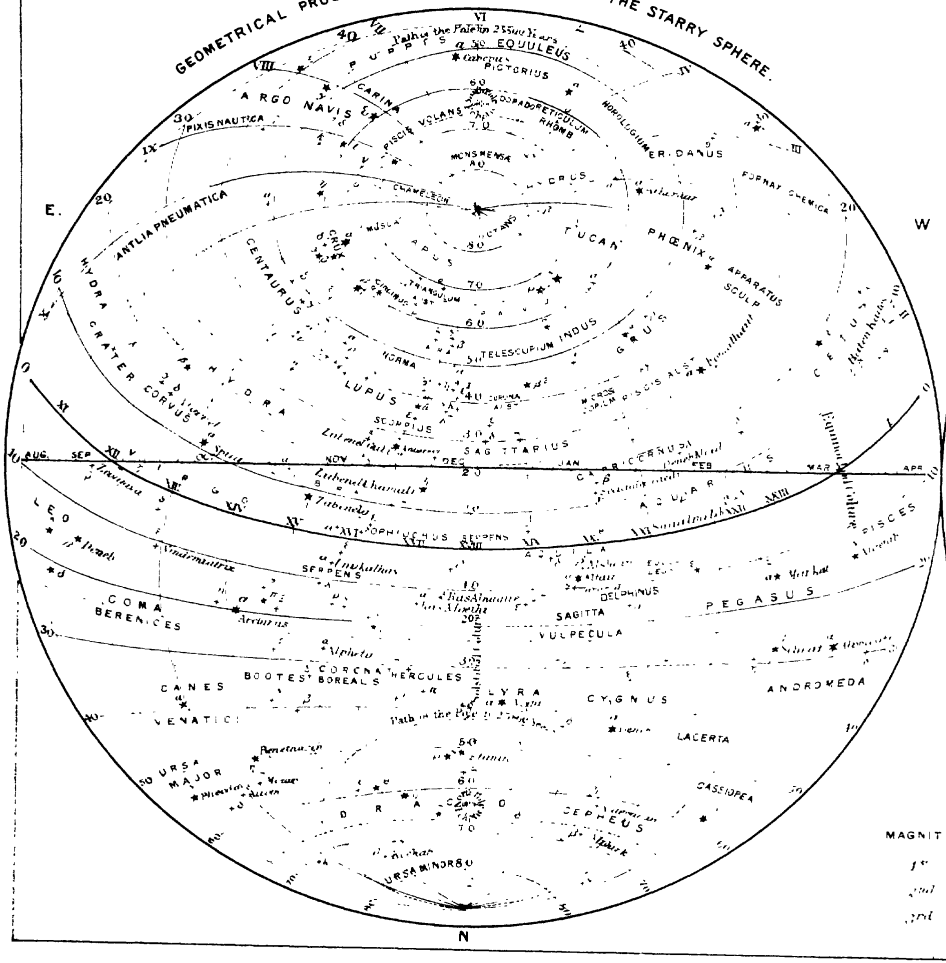

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Nº 3.

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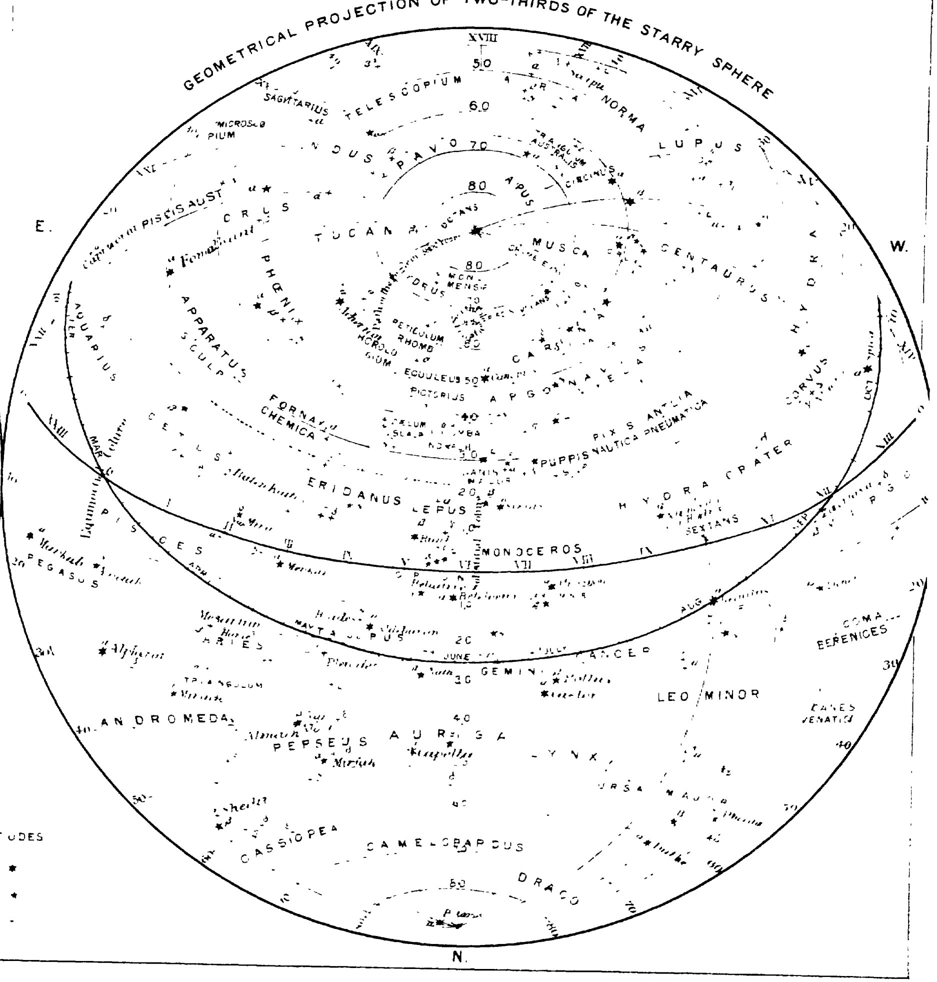

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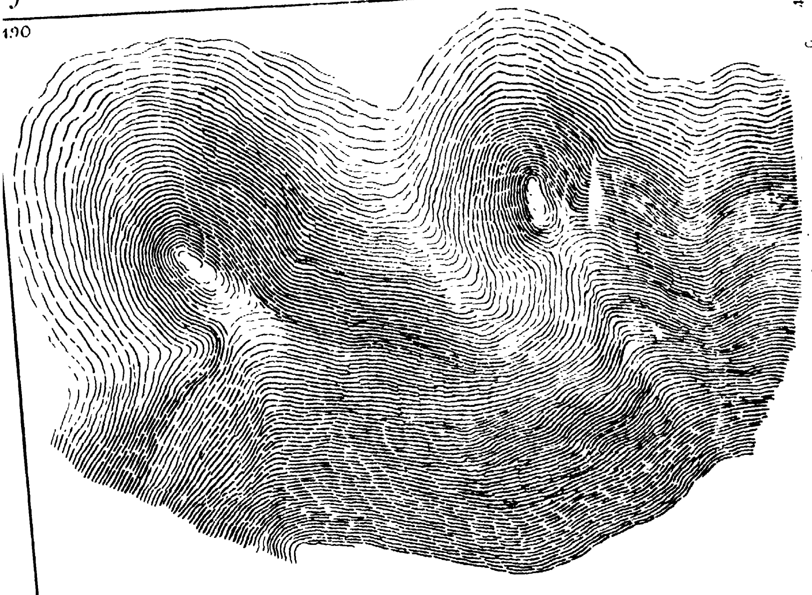

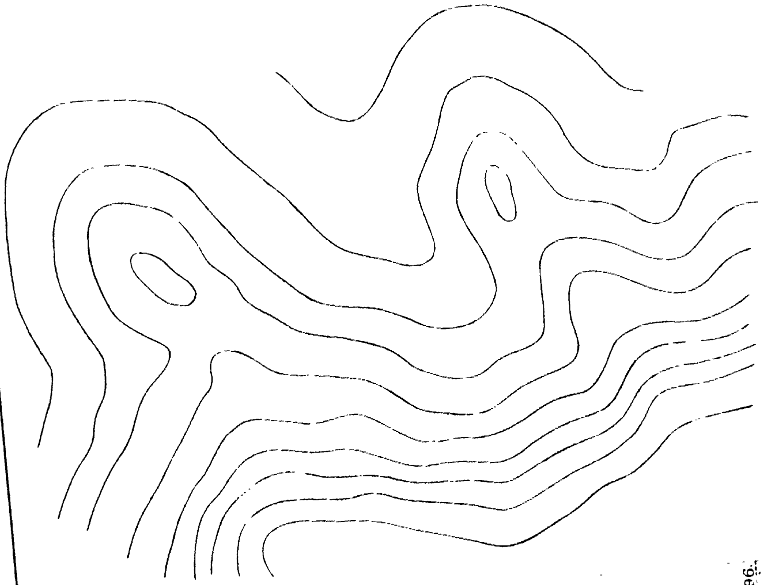

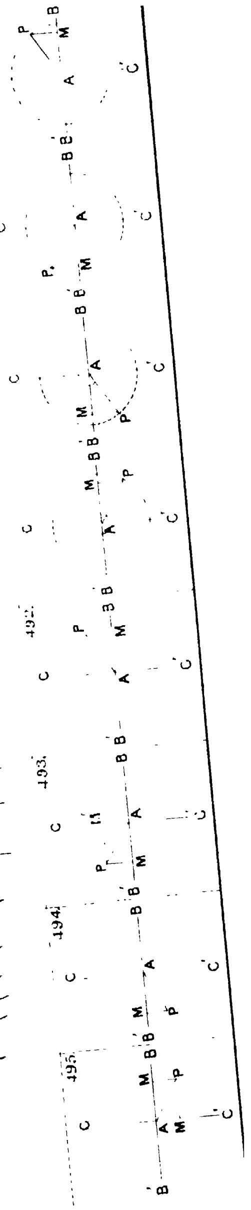

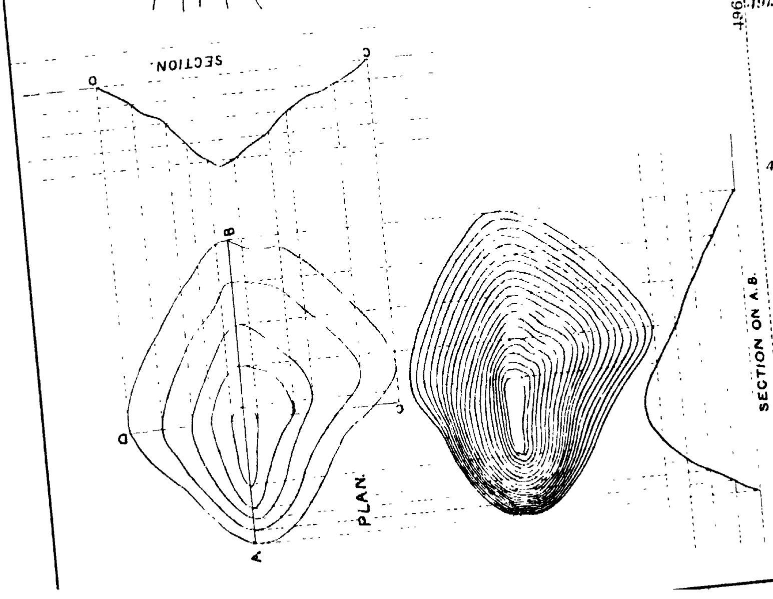

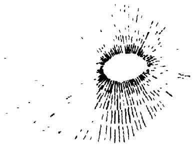

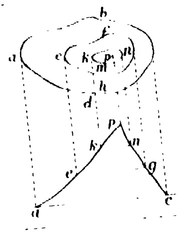

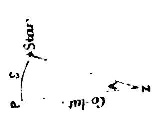

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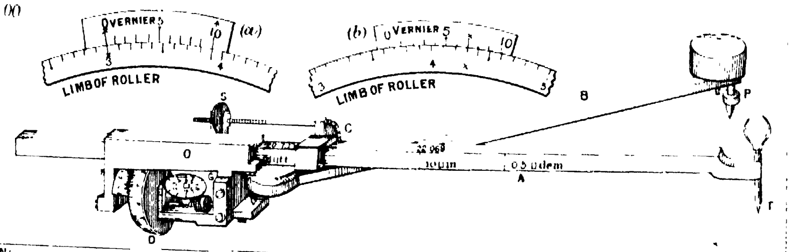

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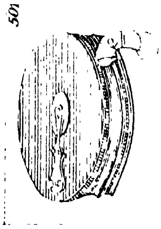

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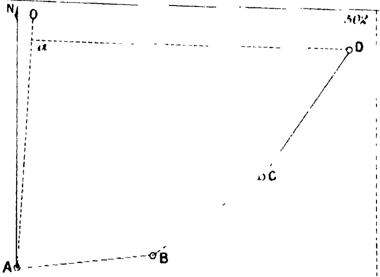

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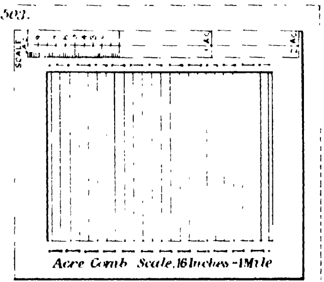

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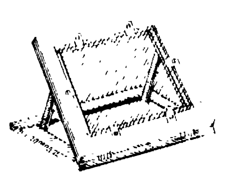

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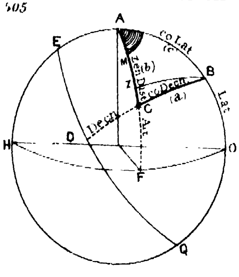

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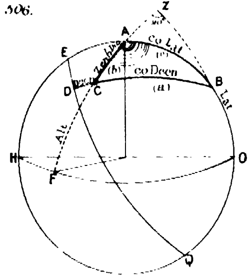

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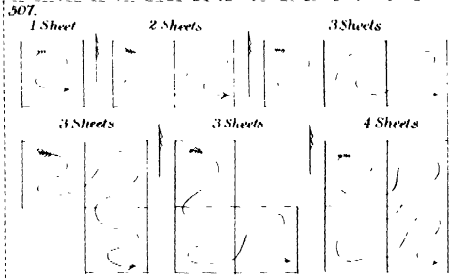

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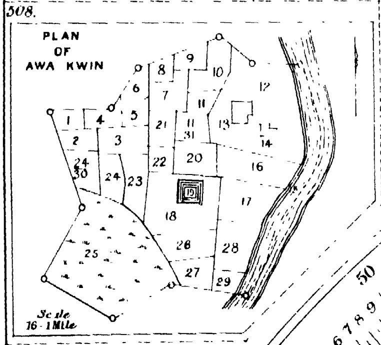

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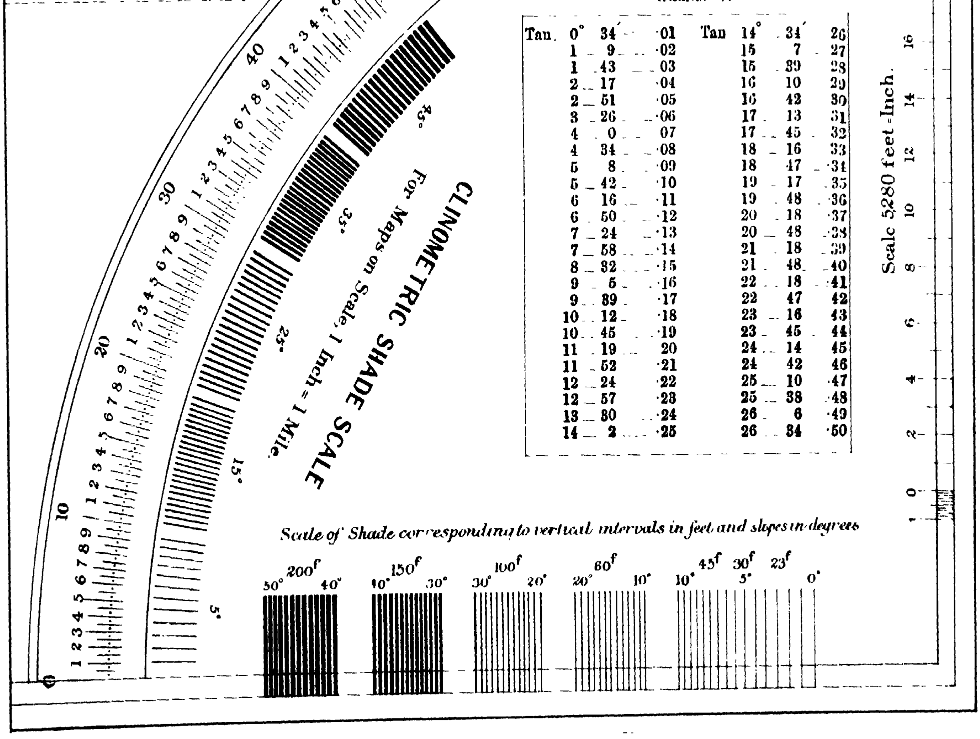

TABLE OF TANGENTS.

Radius = 1.

Tan.	0°	34′	·01	Tan	14°	34′	26
	1	9	·02		15	7	27
	1	43	03		15	39	28
	2	17	·04		16	10	29
	2	51	·05		16	42	30
	3	26	·06		17	13	31
	4	0	07		17	45	32
	4	34	·08		18	16	33
	5	8	·09		18	47	·34
	5	42	·10		19	17	35
	6	16	·11		19	48	·36
	6	50	·12		20	18	·37
	7	24	·13		20	48	·38
	7	58	·14		21	18	39
	8	32	·15		21	48	40
	9	5	·16		22	18	·41
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	11	19	20		24	14	45
	11	52	·21		24	42	46
	12	24	·22		25	10	·47
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	14	2	·25		26	34	·50

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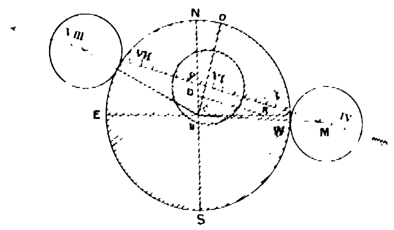

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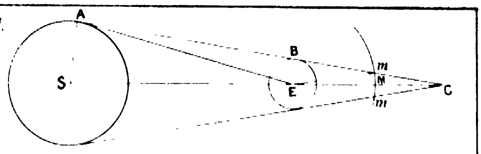

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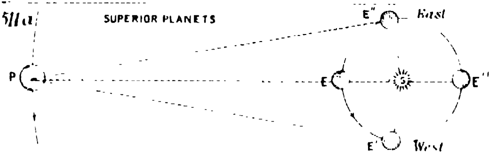

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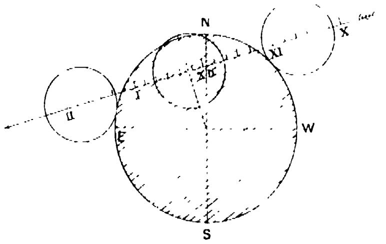

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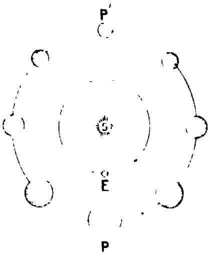

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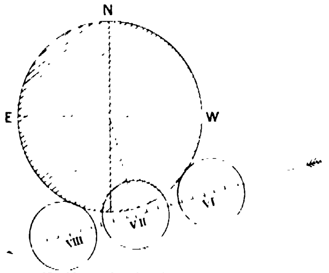

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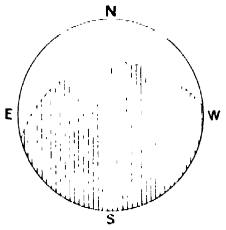

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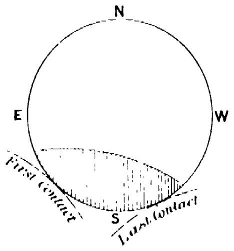

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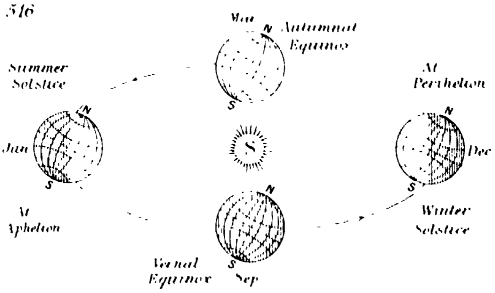

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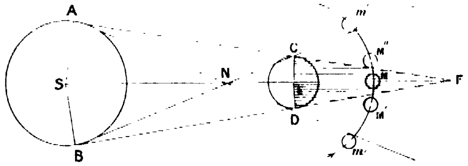

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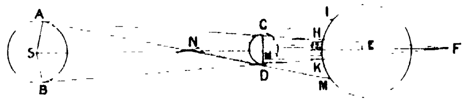

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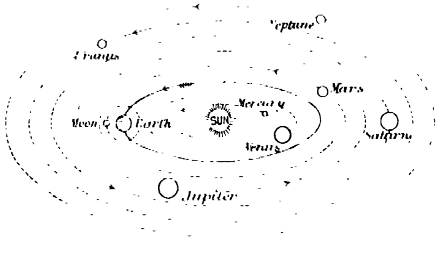

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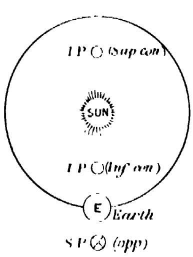

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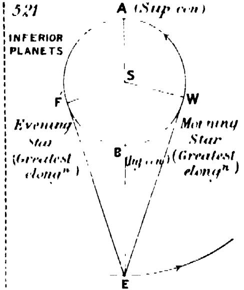
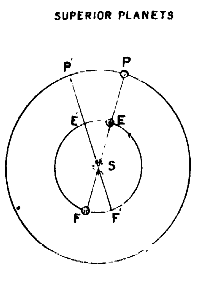
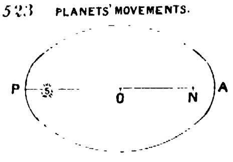

A (Sup con)
INFERIOR PLANETS
S
F
W
Evening Star (Greatest elong^n)
Morning Star (Greatest elong^n)
B
(Inf con)
E

522



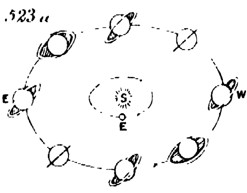
F in opposition with Earth at E
P „ conjunction „ „ „ F
Planet most brilliant at P

523



A in Aphelion P in Perihelion

523a



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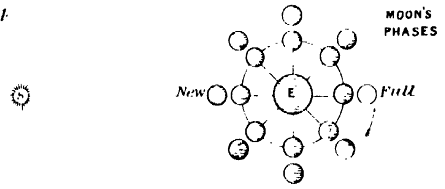

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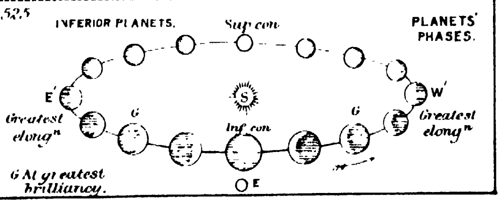

G At greatest brilliancy.

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,, VII.—Map drawing.
,, VIII.—Calculation of Areas.
Chapter IX.—Chain Surveying.
X.—Compass Surveying.
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(*a*) " ....What I have seen appears to me to be well adapted to attain the ends specified in his preface, namely, that it should serve as a text-book for the Survey classes recently established in Burma, and as a guide to Officers in the Commission, Planters, and others, to whom a knowledge of surveying might often prove very useful.......................... Mr. Clancey has evidently spared no pains. The rules he gives are clearly expressed and they are illustrated by numerous examples."

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"........I have looked carefully through the work and commend it as being admirably arranged; it is in several parts—each part complete in itself—so as to lead the student step by step from elementary principles to the fullest information required. Each part is accompanied by a series of illustrative diagrams, very neatly executed, and with miscellaneous examples for solution. The work has besides the advantage of containing in one volume, in small compass, an amount of information which is generally found in two or three and more treatises. It will be found as useful to professional men as to students, and I readily recommend it as a most serviceable, intelligent, and handy treatise."

N. Belletty, Esq., Surveyor, Survey of India, Calcutta.

"........Under the orders of the Government of India a Committee composed of the Surveyor-General, the Consulting Engineer to the Government of India, and others, was formed, who reported most favourably on your book. Thirty copies of it have accordingly been purchased by the Government of India this day for distribution to the Heads of Departments."

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"........It has been prepared with great care and with great accuracy. The illustrations are copious, the descriptions are clear and full, and the general arrangement is extremely methodical. I am sure it will prove a very useful work, not alone to those who are about to take up the study of surveying, but also to those who are already initiated, but may require a reliable book of reference."

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W. J. Addis, Esq., C.E., Secretary and Engineer to the Moulmein Municipality.

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John Coles, Esq., F.R.A.S., etc., Instructor in Surveying and Practical Astronomy, Royal Geographical Society, London.

" I have carefully looked through both editions of your *Aid to Land-Surveying* and I have no hesitation in saying that I consider them to be among the most useful and practical works dealing with the subject of surveying, that have come under my notice. They possess the very great advantage of being written in so plain a style, that any student of ordinary ability ought to find no difficulty in making himself acquainted with the theory of land-surveying; practice, of course, must be learned in the field. A word of praise is also due to the well-selected tables and the explanatory diagrams. In conclusion permit me to congratulate you on the production of such a useful work."

(*Of the Press.*)

The *Indo-European Correspondence*, Calcutta.

"........Far different was the impression we received from the perusal of the *Aid to Land-Surveying* by J. C. Clancey, Superintendent of Land Records, Burma. This book, sold at Newman and Company's, combines every quality that can recommend it both to students, Surveyors, and Engineers. From the first beginnings in elementary mensuration to all the secrets of practical surveying the student is literally taken by the hand with such lucid method as to dispense with the assistance of a master. In fact, the only fault that could be found with the author is that he supposes too little mathematical knowledge in the learner, a fault which, we suppose, many will be only too glad to forgive. Although the price of the book may be above the reach of ordinary students, it will prove very useful to the teacher and to those who take up the study of surveying specially. The problems show at once the practical tact of the author. As to the typography of the book, suffice it to say that Mr. C. Goubert, our able manager, has exerted his whole skill to make it a worthy specimen of this press."

The *Rangoon Times*, Rangoon.

"........We have taken the opinion of a high Government official in the Public Works Department on the book, and this gentleman has declared it to be one of the very best treatises on the subject of surveying in existence. The general get-up of the volume is excellent, and we can confidently recommend it to Professional Surveyors and Engineers throughout the province as well as elsewhere; likewise to students engaged in the study of the subject treated of and who may be qualifying for the above-named professions."

The *Rangoon Gazette*, Rangoon.

"Mr. J. C. Clancey, Superintendent of Land Records, Burma, late Surveyor, Great Trigonometrical Survey of India, has recently published a large edition of his work on lands surveying. The author opens his work with chapters on Geometry, Mensuration, and Trigonometry, and deals with each subject so far as it is likely to be required in surveying. He then describes scales and how to use them in drawing plans and maps, and then, in Chapter IX, commences the subject of chain surveying and the instruments employed in it, which he deals with very thoroughly. Surveying with the compass and theodolite are likewise dealt with fully, and the book concludes with a chapter on the use of Astronomy in surveying. The work is throughout carefully prepared and would be of great service to young Surveyors and might well find a place on the office table of the more experienced man. We can recommend the work of Mr. Clancey to all interested or engaged in surveying work."

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EXTRACTS FROM SOME OF THE MORE IMPORTANT OPINIONS.

(Of Revenue Officers and others.)

The Hon'ble Sir CHARLES MPHERSTON AITCHISON, K.C.S.I., I.C.S., LL.D., Member of the Viceroy's Council, Calcutta, late Chief Commissioner of Burma.

(*a*) "...........I wish you every success in your work and have no objection to your dedicating it to me........"

(*b*) "...........I am very glad to see your work has received such favourable notice from officers in all grades in Burma who are well qualified to pronounce an opinion on its merits, and I am very glad if anything I have done has encouraged you to produce so useful a work. With every wish for your prosperity and advancement."

Colonel W. MUNRO, S.C., Deputy Commissioner, Bassein.

"It has been registered under section 18 of Act XXV of 1867."

Extract from the Minutes of a Meeting of the Mathematical Sub-Committee held at the Office of the Director of Public Instruction, Burma, on Wednesday, the 14th June 1882.

"........The Sub-Committee next considered the question of the merits of Mr. Clancey's work, which has long been in circulation. Resolved that, in view of the high opinion of the book recorded by Survey and Engineer Officers, it should be admitted on the authorized list."

P. HORDERN, Esq., B.A., Director of Public Instruction, Burma.

"...........The Text-book Committee have resolved to admit the work on the list of authorized text-books."

D. M. SMEATON, Esq., C.S.I., M.A., C.S., Secretary to the Chief Commissioner in the Land Revenue and Agricultural Department, Burma.

"...... .....Aids to the acquirement of an exact knowledge of survey methods are placed in the hands of all Burman and Karen Surveyors. Two Vernacular manuals of surveying have been adopted for the Survey schools and are distributed to the recruits. One of these is the work of Mr. J. C. Clancey, Superintendent of the Bassein Supplementary Survey, and is especially adapted for the use of Burman and Karen Surveyors. It has copious illustrations, both of the theory and practice of surveying in all its branches. Copies of this manual have been distributed to all Departments of the Administration in which a knowledge of surveying is required."

Colonel B. A. N. PARROTT, S.C., Settlement Officer, Kyauktan, and Deputy Commissioner, Hanthawaddy.

(*a*) "...... ..I think it is an excellent work and will be very useful........"

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"Your work on land-surveying will supply a much required want. The old class of thugyis, with but a rough idea of surveying, is pretty numerous still in Burma, and a book such as yours, which can aid them or others in acquiring a more extensive knowledge of surveying, is of great value.

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Julian Rogers, Esq., Secretary to the Surveyors' Institution, London.

"........It is a most useful book."

W. T. **Hall**, Esq., B.A., LL.B., C.S., President of the Rangoon Municipality.

(*a*) "........It should be of the greatest use in the province now that the teaching of surveying has been so much extended."

(*b*) "It appears to have been very well printed. I hope the edition will be rapidly sold out."

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T. C. **Wilson**, Esq., C.S., Director of Land Records and Agriculture, Burma.

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Ernest Clarke, Esq., Secretary, Royal Agricultural Society, London.

"........It is a very valuable work."

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John Coles, Esq., F.R.A.S., etc., Instructor in Surveying and Practical Astronomy, Royal Geographical Society, London.

(See opinion against the large "Aid to Land-Surveying.")

Sir **Norman Lockyer**, K.C.B., F.R.S., F.R.A.S., Professor of Astronomical Physics, Royal College of Science, South Kensington, London.

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(See opinion against the large "Aid to Land-Surveying.")

(*Of Professional Officers.*)

General Sir **James** E. T. **Walker**, K.C.S.I., R.E., Surveyor-General of India.

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Robert Gordon, Esq., C.E., Superintending Engineer, P.W.D., Henzada.

"........It is a very useful and valuable book, and I hope will meet the success that it deserves."

The Hon'ble Major H. B. **Talbot**, R.E., Deputy Superintendent, Survey of India, in charge, Prome Survey.

"........The book is just what is required by young students commencing the study of surveying by themselves, and should be of great value to all schools, particularly in Burma.

D. J. **Clancey**, Esq., C.E., Executive Engineer, P.W.D., Rajpootana.

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(*Of the Press.*)

The *Rangoon Times*, Rangoon

(*a*) "... ...The work has been prepared by Mr. J. C. Clancey, formerly of the Great Trigonometrical Survey of India and at present Superintendent of Supplementary Survey and Registration, Burma, and is dedicated, by permission, to Sir Charles Umpherston Aitchison, late Chief Commissioner of this province. The aim and scope of the book can perhaps be best learnt from the preface. Here we are told that its object is to .... This work is by far the best of the kind ever published in Burma, or in the Burmese language, and cannot fail to prove other than a great boon to teachers of surveying as well as scholars. Of Mr. Clancey's fitness for his task the following extract from the last Report on Public Instruction in this province is sufficient testimony ....The want of a suitable text-book in the Burmese language has long been felt to be a great one by all who were called on to teach surveying, and such a work in now supplied. The illustrations accompanying the work are admirable, and are as full and complete as could be desired. The translator's work has been well done, and we can on all grounds thoroughly recommend the work to all teachers in this province."

(*b*) "Mr. J. C. Clancey has forwarded to us a copy of the new and revised edition of his *Aid to Land-Surveying*, the authorized text-book for Burma, patronized by the Governments of India and Burma and most favourably criticised by such well-known officers as Messrs. Hordern, Fryer, Hodgkinson, Macrae, Birks, Mathews, Colonels Duff, Plant, Majors Parrott, Adamson, and others. The book was out of print, but in such constant demand by students, surveyors, engineers, and all classes of Government officers, that we are glad to see Mr. Clancey has brought out a new and enlarged edition which will doubtless be popular in the province for many long years to come. We venture to say that Mr. Clancey is the only man in the province who is capable of preparing such a *vade mecum* for surveyors."

The *Rangoon Gazette*, Rangoon.

(*a*) ".. ......This work is in Burmese and English, and is intended as a text-book for the Government Survey examinations. It will be found invaluable to all students of surveying in Burma."

(*b*) "The first edition of Mr. J. C. Clancey's useful book, *The Aid to Land-Surveying* having been exhausted, a new one has just been published. It has been thoroughly revised and improved; and will doubtless meet with the same cordial reception accorded to the first one, more especially as it is the authorized text-book for the Thugyiship, Myoôkship, and Anglo-Vernacular Middle School examinations. The work is in both English and Burmese, the vernacular portion of the present edition having been carefully revised by Mr. A. W. Lonsdale, Editor of the Vernacular Text-Books for Burma. It is satisfactory to find that the book has stood the test of experience, and we wish the author every success with the new edition."

(*c*) "The third edition of Mr. J. C. Clancey's *Aid to Land-Surveying* has now been published. The first edition was published 13 years ago and the second edition in 1890. The present edition, which is in English and Burmese, has been carefully revised and brought up to date. It is publised under Government auspices and is the authorized text-book for the various examinations of the province of which surveying forms a part."

The *Moulmein Advertiser*, Moulmein.

(*a*) "........The work is by Mr. J. C. Clancey, Superintendent of Supplementary Survey and Registration, Burma, late Surveyor, Great Trigonometrical Survey of India. The object of this treatise, as set forth in the preface, is to............... We have carefully looked through this book and the companion volume of illustrations, and being somewhat familiar with the subject treated of, we are in a position to say that it is well calculated to suit the requirements of the province, and we therefore recommend it as being not only a simple and intelligent treatise on surveying, but the very *help* required by those for whom the author has provided. The illustrations are all neat and well drawn and will be of incalculable assistance to the student. The exercises are all simplified by clear and precise explanations, and this has been done so fully for the beginner that it is a wonder to us how the author could have had the patience that is exhibited on every page of his book. It is needless for us to enter into the question of Mr. Clancey's capabilities; these are already well known to, and appreciated by, the local Government, and it only remains for us to congratulate Mr. Clancey on the very excellent work he has written and the service he has rendered to education in the province."

(*b*) "........We have received from Mr. J. C. Clancey, a copy of his second edition of the smaller *Aid to Land Surveying*. The book has been considerably revised and appears in better print and covering. That Mr. Clancey has devoted much time and labour on the preparation of the work cannot be doubted. The parts are well arranged, the illustrations very good, and the student is led, step by step, from elements to the fullest information required. To schools the book will be found of great use and value."

(*c*) "The new edition of Mr. J. C. Clancey's work on land-surveying, which is the third one of its kind, we doubt not will be received with the satisfaction it deserves. It is printed in English and Burmese and is accompanied by 236 illustrations on the subject. Its Burmese translation has been exceedingly well effected by the well-known Burmese scholars, Rev. Father D'Cruz and Mr. Lonsdale. It is quite sure to meet the object of its issue. Students will find it to contain most useful information upon the different subjects treated of, from Practical Geometry to Questions on Surveying. The author seems to have spared no pains in rendering clear all that relates to chain-surveying, calculation of areas, etc. The present edition has been carefully revised and is an authorized text-book."

The *Akyab Educational Gazette*, Akyab.

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The *Mandalay Herald*, Mandalay.

"We are in receipt of a copy of the *Aid to Land-Surveying*, in English and Burmese, by J. C. Clancey, F.R.G.S., Assistant Director of Land Records and Agriculture, Burma. The production of such a work will come as a boon to students, surveyors, thugyis, and others directly interested in land-measurements, while it will be invaluable to those desirous of studying the subject in its every detail. The present, or third, edition of Mr. Clancey's book has fully been brought up to date; a copious index has been inserted, which should materially add to its unquestionable value. The volume contains 236 illustrations, which are turned out in an unusually clear and finished manner, while the mass of useful information with which it is replete stamps the work as being an indispensable addition to the published authorities in land-surveying. In a word, Mr. Clancey's work cannot be too highly recommended, and no student desirous of perfecting himself in the study of the subject should be without a copy. It is printed in the excellent style peculiar to the Burma Government Press, and the price is Rs. 4."

The *Arakan News*, Akyab.

(*a*) "We have to thank Mr. Clancey for a copy of his work on surveying. The general arrangement and get-up of the book is excellent. The plans, geometrical figures, scales, and topographical drawings are well executed. It may be safely said that Burma owes its knowledge of the science of geodesy to Mr. Clancey and his books.

This is the only reliable book on surveying published in Burma, and the subject has been so thoroughly treated that there seems to be scarcely room for another work. That it is a work of great merit is the opinion of more than one competent authority in India and Burma. The lucid statements of the principles and methods of surveying, and the numerous problems, illustrations and examination questions make it a book admirably adapted for Survey students. The present, or third, edition of the Anglo-Vernacular *Aid to Land-Surveying* has been published with the approbation and help of the Government. The work has been closely scrutinized and brought up to date in all essential respects. Considerable additions and improvements have been effected in it, while, for convenience of reference, a copious index has likewise been inserted The above extract from the preface shows clearly the improvements and additions made to the present work. The Burmese portion of the text has also been revised and enlarged.

(*b*) "We have received from Mr. J C. Clancey, Superintendent of Land Records, Burma, a copy of a new and revised edition of his *Aid to Land-Surveying*, the authorized text-book in Burma for examinations for Surveyors, Myoôkships, and Thugyiships. The book has been most favourably commented upon by several well-known officers, the chief of whom are Messrs. Hordern, Fryer, Hodgkinson, Macrae, and Colonels Plant, Duff, Scott-Moncrieff and others. The book has been in such constant demand by students and officers of all grades in Burma, that Mr. Clancey has just issued a new and enlarged edition which has been most carefully prepared and which we would recommend to all those interested or engaged in surveying work."

The *Indo-European Correspondence*, Calcutta.

"We received a copy of the fourth edition (in English only) of the *Aid to Land-Surveying* by J. C. Clancey, F.R.G S. Even a cursory glance at the chapters which form the didactic part of it will suffice to observe that it is the work of a man whose life's experience has been spent in the profession which he purposes to aid. The Rules being so clear, we are not surprised that the work has been made an Authorized Text-book. As to the illustrations which are as many as 236, divided over 16 full-size pages, nothing better could be desired in point of neatness and practical suggestiveness."

The *Madras Mail*, Madras.

"We have received from Mr. J. C. Clancey, F.R.G.S., the fourth edition (in English only) of the *Aid to Land-Surveying* (small edition). The object of this treatise is to provide a simple and elementary work on land-surveying in English for the use of surveying classes in India and Burma. Judging by the number of editions through which the book has already gone, and the high opinion expressed of it by well-known Survey officials, Engineers and experts, it has more than fulfilled its object. Messrs. Newman and Company, Calcutta, are the printers and publishers."

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The *Ceylon Observer*, Colombo.

"The small edition of the *Aid to Land-Surveying* (in English only, accompanied by 236 illustrations, by J. C. Clancey, F.R.G.S., fourth edition) which we are noticing to-day, is by no means a puny production. It contains about 150 pages of foolscap size, laden with much useful information in several chapters. It is a small edition only with reference to a larger text-book by the same author, which we presume contains fuller and more advanced information. Quite apart from his Fellowship, Mr. Clancey possesses qualifications for the task he has set himself, through his long connection with India and Burma—being now Assistant Director of Land Records and Agriculture of Burma, and having been a member of the Great Trigonometrical Survey of India. His object has been to provide a simple and elementary work on land-s[illegible]ying in English and Burmese for the use of students; and this object he has furthered by the publication of several treatises, large and small, in English and in Burmese, some of which are out of print; while the further fact that this is a fourth edition is evidence that Mr. Clancey's work is appreciated and serves a very useful purpose. When we recently noticed the magnitude of the work of Land-Settlement, Forestry, and Surveying in the Southern Presidency, we saw how the aid of all classes and grades of public servants was being utilized to push on the work. The treatise before us cannot fail to be of the greatest help to students qualifying for such work, and also, so far as we can judge by a dip into the very full and well-arranged contents—even to Professional and District Officers, as also to planters. There are altogether 10 chapters with appendices, and a copious index. They deal with Practical Geometry, its First Principles and Definitions, Geometrical Theorems and Geometrical Problems; Mensuration of Surfaces, including definitions, lengths and areas; Mathematical Drawing Instruments, with a description of the various instruments in use and their uses; Scales with definitions, examples in simple scales, diagonal scales with examples, exercises in the construction of simple and diagonal scales, protractors, plotting instruments, etc.; Map-drawing with useful hints in every department, including the use and projection of materials, etc.; calculation of areas by acre-squares, by casting, and by the planimeter. The remaining chapters deal fully with chain-surveying, compass-surveying, and plane-table surveying; so that more advanced information, touching theodolite surveying, levelling, and practical astronomy, evidently finds a place only in the larger edition. The value of this edition is greatly enhanced by examples, problems, and theorems under every separate chapter, which must be very helpful to the practical surveyor. There are also several pages of illustrations and figures to serve as an aid and stimulus to practical work. Now that our own Government has recognized the need of protecting the public from the eccentricities of unqualified surveyors, by requiring every private one to take out a license, we have little doubt the book will serve some purpose outside India; and there should be a demand for Mr. Clancey's work for students and surveyors here too. The publishers are Messrs, W. Newman and Company, Caxton Press, Calcutta, and the book is for sale in Rangoon, Bombay, Lahore, and as near to us as Madras, at Messrs. Addison and Company."

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In two Parts: Part I (*Survey Section*) and Part II (*Revenue Section*).

DEDICATED, BY PERMISSION, TO SIR FREDERIC FRYER, K.C.S.I., I.C.S., BARRISTER-AT-LAW, LATE LIEUTENANT-GOVERNOR OF BURMA.

Price,—Part I (English), Fifth Edition, Re. 1; Parts I and II (English and Burmese), Fourth Edition, Rs. 2.

OBJECT OF BOOK BRIEFLY STATED.

The above book (fourth edition) was prepared at the instance of Government, with the object of helping Survey, Settlement and Revenue Officers in the calculations connected with their work. It clearly filled a required want as the several editions through which it has passed testify. The Tables include those of Logarithms, Traverse, Levelling, Interest, Exchange, Loans, and Advances, Income, Wages and Rent, Exhaustive Calendar, Equation of Time, Weights and Measures, etc. The work will be equally useful to bankers, merchants, brokers, lawyers, tradespeople and others. For the convenience of those employing them the tables have been subdivided into two sections of "Survey" (Professional) and "Revenue" (General). Part I relates to purely professional subjects and will therefore chiefly be of service to Survey Officers, while Part II appertains to matters of a more general character and would thus primarily concern Revenue Officers alone. Several of the tables in both the parts, however, are of a nature to suit others besides the classes of officers named. Some of the opinions of professional and other officers and of the Press on the book are recorded below.

EXTRACTS FROM OPINIONS.

(*Of Revenue and Professional Officers.*)

Sir **CHARLES EDWARD BERNARD**, K.C.S.I., Revenue Secretary at the India Office, London, late Chief Commissioner of Burma.

(*See* opinion against the large "Aid to Land-Surveying.")

Colonel **C. H. E. ADAMSON**, C.I.E., S.C., Deputy Commissioner, Moulmein Town District.

"......... The tables of logarithms, etc., will be found very useful."

(*Of the Press.*)

The *Madras Mail*, Madras.

"The other day we noticed in these columns a book called *Aid to Land-Surveying*, by Mr. J. C. Clancey, Assistant Director of Land Records and Agriculture, Burma. We have now received another book by the same author. The *Calculating Tables* under notice were first issued at the instance of the Burma Government in 1884 with the object of helping Survey, Settlement and Revenue Officers of the Province in calculations connected with their work. The utility of the book can best be described by stating that this is the third edition, and it has been very much enlarged and revised. In the province for which it is primarily intended it will be very useful indeed, the tables, rules and explanations being in Burmese as well as in English. Messrs. Addison and Company are the Madras agents for the *Calculating Tables*.

The *Bombay Gazette*, Bombay.

"What the Indian Penal Code is to the District Magistrate, these *Calculating Tables*, compiled by Mr. Clancey, F.R.G.S., of the Land Records and Agricultural Department, Burma, should be to Survey, Settlement, and Revenue Officers in the calculations connected with their work. The volume, which is printed by authority at the Government Press, Rangoon, is obtainable in Bombay of Messrs. Thacker and Company, and has now reached a third edition. It is printed not only in English, but also in Burmese, thus bringing the tables within the intellectual compass of subordinate members of the surveying staff. The first edition was published in 1884, and the second in 1890, but this, the third edition, is without doubt the best, since it has been thoroughly revised, and numerous checks have been applied to render it accurate and complete. It must not be supposed that the work is merely of local value to the province of Burma, for many of the tables and calculations are of a general character, suitable for use in any part of the Empire."

The *Civil and Military Gazette*, Lahore.

"We have received a copy of a set of *Calculating Table* by Mr. J. C. Clancey, Assistant Director of Land Records and Agriculture, in Burma. The tables deal with such subjects as land-surveying, the calculation of revenue, weights and measures, and questions relating to income, wages, rent, and exchange. They are elaborately and exhaustively compiled, and should save much mental wear and tear to the student. The merits of the book appear to have been already recognized, as it is now in its third edition. It can be obtained at Messrs. Gillon and Company, Lahore."

The *Ceylon Observer*, Colombo.

"We have received from the author a copy of the *Calculating Tables*, in English and Burmese, for use in Burma, by J. C. Clancey, F.R.G.S., Assistant Director of Land Records and Agriculture, Burma. Authorized edition (third) revised and enlarged. Though primarily intended for use in Burma, this volume should prove of great assistance to Surveyors, P. W. D. Officers, planters, etc., in Ceylon."

The *Rangoon Times*, Rangoon.

"We have received a copy in English and Burmese, bound in one volume, of this well known work by Mr. J. C. Clancey, F.R.G.S., Assistant Director of Land Records and Agriculture, Burma. We should think that it is the best practical guide to the study of surveying in use in the East. What makes it more valuable than other works of a similar nature is the practical way in which the entire matter is dealt with, from the elementary principles of surveying up to the higher branches of the art which terminate in astronomy. It is specially compiled for the use of Survey, Settlement, Engineer and Revenue Officers of the province and will be equally useful to bankers, merchants, brokers, lawyers, tradespeople, and others, and to all engaged in revenue work, so its demand amongst these classes should be very great. To the planter and others to whom a knowledge of surveying is useful the work will be found a great help. Being illustrated copiously with numerous practical examples and plates, the student of surveying can easily master the art by a study of the work alone and without the assistance of an instructor. The tables include interest, exchange, loans and advances, income, wages and rent, exhaustive calendar, equation of time, weights and measures, and numerous calculations connected with revenue matters, surveying and astronomy. The printing and general get-up are excellent, while the price is ridiculously small for a work of its nature."

The *Rangoon Gazette*, Rangoon.

"A new edition (third), revised and enlarged, has been issued of Mr. J. C. Clancey's *Calculating Tables* in English and Burmese. It is twelve years since the first edition was issued, and its great value to Revenue, Settlement and Survey Officers in the province soon became apparent. The present edition has been much enlarged and remodelled. The book is issued by the Government Press and costs only a nominal sum."

The *Moulmein Advertiser*, Moulmein.

"Mr. J. C. Clancey, F.R.G.S., Assistant Director of Land Records and Agriculture, Burma, has published another edition of his "Calculating Tables" The book before us is carefully got up and admirably arranged. It is in both English and Burmese, and we have no doubt it will fully meet the expectation of the author in proving a most useful book not only to Survey, Settlement and Revenue Officers, but to a large section of the enlightened public who could appreciate its sphere of usefulness. The *Calculating Tables* were originally prepared in 1884 at the instance of the Local Government. The second edition was published in 1890 and the present book is the third edition of the kind which upholds the necessity and usefulness of the work."

"Notes on Supplementary Survey and Registration," in English (*out of print*). Price Rs. 4.

"Notes on Supplementary Survey and Registration," in Burmese (*out of print*). Price Rs. 4.

"Revenue Law Notes," in English (*out of print*) Price Rs. 2.

"Brief Instructions for the prosecution of Local Traverse and Cadastral Surveys," in English and Burmese (*out of print*). Price Re. 1.

"Conspectus of Insect Pests which affect crops in Burma," in English and Burmese (*out of print*). Price Re. 1.

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IN ENGLISH AND BURMESE

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Issued with the approbation of Government.

SECOND EDITION (AUTHORIZED)—REVISED AND ENLARGED.

(*Out of print.*)

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OBJECT OF BOOK BRIEFLY STATED.

The object of the above compilation (second edition) has been to provide, in a convenient and systematic form, for the requirements of candidates preparing for the Thugyiship, Clerkship, Myoôkship, and other kindred examinations that are periodically held under the auspices of the Educational Syndicate of the country, in order that the book may prove a true guide to their studies merely of the students going up for the examinations referred to. The arrangement of the chapters in the book has been made to correspond with that of the small edition of the "Aid to Land-Surveying," the subject heads of the appointed examinations being shown under those chapters according to relation borne to them respectively. It is hoped that the volume will continue to be utilized by those for whom it has been primarily designed and that the purpose of the compilation, namely, the encouragement of the spread of scientific education, may be thereby furthered. Some opinions of the Press, etc., on the book are given below.

EXTRACTS FROM OPINIONS.

(*Of Revenue and Professional Officers.*)

Sir CHARLES EDWARD BERNARD, K.C.S.I., Revenue Secretary at the India Office, London, late Chief Commissioner of Burma.

(*See* opinion against the large " Aid to Land-Surveying.")

(*Of the Press.*)

The *Rangoon Times*, Rangoon.

" We have received a copy of the second edition of this compilation by Mr. J. C. Clancey, of the Land Records and Agricultural Department. As most of our readers in Burma no doubt know, the compilation consists of a collection of the various question-papers in surveying which have been set from time to time in this province, the object being to assist candidates to pass the Thugyiship, Clerkship, Myoôkship, and other kindred examinations which are held by the Educational Syndicate of Burma, by arranging the question-papers in a convenient and serviceable form and so affording them some guide as to their studies. A second edition has been issued as the first edition of the work fully verified the hopes of its compiler. The compilation is in both English and Burmese. The letter-press is excellent, but that is to be expected as the book has been turned out under the eye of the Superintendent, Government Printing, Burma."

The *Rangoon Gazette*, Rangoon.

" Mr. Clancey has published a book of questions on surveying, which will be found invaluable to candidates who have to take up that subject in the various Government examinations. The price is the small one of a rupee. The book ought to command a ready sale."

The *Mandalay Herald*, Mandalay.

" A second edition of that exceedingly useful work, *Examination Questions in Surveying*, up to the Thugyiship, Clerkship, and Myoôkship standards for Burma, compiled by J. C. Clancey, Assistant Director of Land Records and Agriculture, Burma, has been published, and should be welcomed by candidates preparing for either of the above appointments. As explained by the compiler of the book, the contents are not intended to introduce a system of cram, but to provide for the use of students going up for the examinations, a true guide for their studies. The course of study, which is described as both theoretical and practical, embraces no less than ten subjects, and the very marked improvement that followed the first publication has been such as to offer a guarantee for the success of the second improved and enlarged edition. As an encouragement to the spread of scientific education—a most valuable acquisition in the present year of grace, when competitors for Government appointments tread upon each other's heels in the struggle to reach the first rung of the ladder which is fondly supposed to lead to the summit of their ambition, Mr. Clancey's work cannot be too highly recommended. Besides a vast number of important questions, as they relate to the various subjects dealt with, the value of the book is enhanced by the introduction of numerous diagrams, plans, and maps, the latter being especially useful as showing the student the different methods of drawing with accuracy and upon given scales. The student will also peruse with attentive interest the sets of question-papers in surveying, the description of examinations, and the number of marks necessary to be obtained in order to ensure success. We may mention that the work is published with the approval of Government and that it is brought up in all material respects to date. It is printed in the excellent style characteristic of the Burma Government Press, and we have little doubt but that the *Examination Questions on Surveying* will duly find its way into the hands of, and be deservedly appreciated by, those for whose information the work is intended."

The *Moulmein Advertiser*, Moulmein.

" Mr. J. C. Clancey has laid the youth of the province under another obligation by the publication, with the approbation of Government, of a volume of *Examination Questions in Surveying* up to the Thugyiship, Clerkship, and Myoôkship standards for Burma. Mr. Clancey's two works on surveying, published many years ago, are so well known and have so fully established his reputation as an author on a subject he thoroughly understands, that it is not necessary for us to say that the work before us is of that excellent quality that we now have the right to expect from so thorough a Surveyor as Mr. Clancey.

It will, however, be useful to those for whom the book is intended, if we give briefly the contents of the volume which will be of the greatest assistance to those who are desirous of passing examinations. For many years past the compiler of the volume has been engaged in collecting the various question-papers in surveying which have been set from time to time in the province prior to 1881 by the various District Committees of Public Instruction and the Education Department, and subsequently by the Educational Syndicate of Burma. The object in view was the eventual publication, in a convenient and systematic form, of a compilation, which would, it was thought, prove serviceable to candidates preparing for the Thugyiship, Clerkship, Myoôkship, and other kindred examinations that are now periodically held under the auspices of the Educational Syndicate of the country. This book, Mr Clancey says, is not intended to introduce a system of cram—indeed it could not do so—but to provide students with a true guide to their studies. This it will undoubtedly do and every student should arm himself with a copy."

The *Arakan News*, Akyab.

"We have to acknowledge with thanks the receipt of the .... and *Examination Questions in Surveying* compiled by J. C. Clancey, second edition, revised and enlarged."

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OBJECT OF THE BOOK BRIEFLY STATED.

The above compilation contains, in a convenient form, questions in surveying, revenue law, and land records work which have been set at various times in connection with the examination of the Superior Land Records Officers, of candidates for admission into the Land Records Department, of Junior Civilians and of Probationary Myoôks, and is designed to assist officers of these classes who may be preparing for similar examinations. Some Press opinions on the merits of the book are given below.

EXTRACTS FROM OPINIONS OF THE PRESS.

The *Rangoon Times*, Rangoon.

"Land Records Examination Manual.—This little compilation, by Mr. J. C. Clancey, contains in a convenient form questions in surveying, revenue law and land records which have been set at various times in connection with the examinations of the Superior Land Records Officers, of candidates for admission into the Land Records Department and of Junior Civilians, and will be found by these officials of great use as a guide in cramming for similar examinations. In the next edition we would recommend the introduction of specimen answers alongside the questions contained in the examination papers, for many a candidate is plucked from inability to condense his answers in proper and easily intelligible form. It is often not the candidate who has a knowledge of the subject that passes, but he who is able to express his knowledge in a clear and concise style. This little compilation costs but a small sum."

The *Rangoon Gazette*, Rangoon.

"Another of Mr. Clancey's useful compilations has also been received. It is a ***Land Records Examination Manual*** containing questions in surveying, revenue law and land records work, which have been set at various times in examinations on these subjects, and should be of very great service to those for whose use it is intended."

The *Moulmein Advertiser*, Moulmein.

"The Land Records Examination Manual for Burma, compiled by Mr. J. C. Clancey, F.R.G.S., Assistant Director of Land Records and Agriculture, Burma, is an exceedingly cheap pamphlet which should be invaluable to students in general, containing as it does questions concisely put together upon surveying, revenue law, and land records work."

EXTRACTS FROM OPINIONS

ON THE

(i) Aid to Land-Surveying, large edition (3rd), in English.
(ii) Calculating Tables, Part I, 5th edition, in English.
(iii) Aid to Land-Surveying, small edition (5th), in English.
(iv) Aid to Land-Surveying, small edition (4th), in Burmese.
(v) Calculating Tables, Parts I and II, 4th edition, in English and Burmese.

(*Of Revenue Officers and others.*)

Sir WALTER LAWRENCE, K.C.I.E., Private Secretary to His Excellency Lord CURZON of KEDLESTON, G.C.S.I., G.C.I.E., Viceroy and Governor-General of India.

"I am directed to acknowledge, with thanks, the receipt of your books on surveying."

G. B. H. FELL, Esq., I.C.S, Private Secretary to His Honour Sir HUGH BARNES, K.C.S.I., K.C.V.O., Lieutenant-Governor of Burma.

(*a*) "I am directed by His Honour the Lieutenant-Governor to acknowledge, with many thanks, the receipt of the copies of your *Aid to Land-Surveying* and *Calculating Tables* which you have been good enough to present to him. I congratulate you on the completeness of the works which must be invaluable to surveyors here and elsewhere.

(*b*) "I have been looking through the *Calculating Tables* and am much struck with the monumental industry which you must have applied to the undertaking. I congratulate you on the accuracy of the Table of Burmese New years. It's a subject which I understand bristles with difficulties."

Sir FREDERIC FRYER, K.C.S.I., late Lieutenant-Governor of Burma.

"I have received a copy of the *Calculating Tables* with many thanks."

From H. THOMPSON, Esq., I.C.S., Officiating Revenue Secretary to the Government of Burma, to the Financial Commissioner, Burma.

"...........I am directed to ask that you will be so good as to convey to Mr. J. C. Clancey, Assistant Director of Land Records and Agriculture, the thanks of the Local Government for the valuable assistance which he has given to Government in preparing the revised editions of the *Aid to Land-Surveying*."

H. M. S. MATHEWS, Esq., Revenue Secretary to the Government of Burma.

"............I have looked through the books with interest and think they form a very successful issue. I always like to have a book of surveying with me."

A. GAITSKELL, Esq., I.C S., Director of Land Records and Agriculture, Burma.

"............They are both very full and complete, and a great help to those who have need to use them in their work. The printing and the designs are very clear. You have added, I see, some useful tables in the *Calculating Tables*. In the large *Aid to Land Surveying*, I see you have considerably amplified the traverse section on theodolite surveying"

The Hon'ble W. T. HALL, B.A., I.C.S., LL.B., Financial Commissioner, Burma.

(*a*) "I consider that these books will be of great use in Burma.

(*b*) "I am well aware of the excellence of these books and of the great benefit which the province has gained from the earlier editions. A great deal of labour has been spent on the present editions, which are beautifully got up. They will be of the greatest use."

T. C. WILSON, Esq., I.C.S., Settlement Commissioner, Burma.

".........They will no doubt prove very useful. The *Calculating Tables* should render references to Committees of Burmans regarding date of Burmese festivals, etc., unnecessary in the future."

H. THOMPSON, Esq., I.C.S., Revenue Secretary to the Government of Burma.

".........There can be no question as to the time and labour which you must have given to the revision. I wish your efforts may be given a fitting reward."

O. S. PARSONS, Esq., I.C.S., Director of Land Records and Agriculture, Burma.

(*a*) "The publication of Mr. Clancey's books has been of great service to the province. They are in constant use in all offices."

(*b*) "You deserve great credit for getting the books through."

M. **LAURIE**, Esq., M.V.O., I.C.S., President, Rangoon Municipality.

"........I am sure that they will be of the greatest value, both to the student and the expert surveyor. Your set of illustrations in the Aids is unusually instructive and interesting."

W. F. **NOYCE**, Esq., K.-I-H., Superintendent of Stamps, Burma.

"........It has been very neatly got up. Few, if any, can conceive the labour which the tables must have involved. The 'Thingyan' period calculations in your book are correct to the second."

(*Of Professional Officers.*)

Revd. **ANTRED BERG**, S.J., in charge, St. Xavier's College Observatory, Calcutta.

"........If they are of great practical use for survey work, surely they contain plenty of valuable matter for other Mathematical Departments, which it would be difficult to find elsewhere. I congratulate you warmly for the useful work you have done, and which required undoubtedly much energy and patient work to be brought to an end.

JULIAN C. **ROGERS**, Esq., F.S.I., Secretary to the Surveyors' Institution, London.

"The work is very exhaustive and interesting."

Revd. J. **BAKKER**, S.J., Professor in charge, St. Xavier's College Observatory, Calcutta.

These books, so brimful of useful matter, must prove an invaluable boon to all who are engaged in surveying and other branches of the public service; and any candidate wishing to enter the service would certainly find in them most if not all of what he requires."

Dr. A. M. W. **DOWNING**, D.Sc., F.R.S., F.R.A.S., Superintendent, H.M. Nautical Almanac Office, London.

"The chapter on the Burmese Calendar of the *Calculating Tables* is on a subject of such interest to Astronomers that it seems a great pity that it should be buried in a publication that would not be seen by one per cent. of them........."

(*Of the Press.*)

The *Rangoon Gazette*, Rangoon.

"The new edition of Mr. J. C. Clancey's *Aid to Land-Surveying* just issued from the Government Press, has been thoroughly revised. The text and diagrams have been much augmented with new matter and considerable portions of the old matter greatly modified in places, the whole being condensed, re-arranged and rendered up-to date in all particulars. The logarithmic, traverse and other tables which existed in the previous edition have now been transferred to the author's *Calculating Tables*, Part I, a revised and enlarged edition of which is simultaneously published. It is worthy of notice that the details in the recent Government notification with regard to the Thingyan period of the Buddhist year 1260 correspond exactly with Mr. Clancey's calculations made two years ago, which are to be found on page 151 of the 'Tables.' The present editions of these valuable works are sure to increase their usefulness and popularity."

The *Rangoon Times*, Rangoon.

"We have to thank Mr. J. C. Clancey, F.R.A.S., F.R.G.S., F.S.I., Assistant Director of Land Records and Agriculture, Burma, and lately a member of the Great Trigonometrical Survey of India, for a copy of his excellent and exhaustive treatise entitled *Aid to Land-Surveying*; as also a ponderous volume full of figures which is styled *Calculating Tables*. It is quite impossible for any one except a professional or highly trained Land Surveyor and Mathematician to do justice to these two valuable volumes, but it is quite evident to the most ordinary person that they must have necessitated a thorough knowledge and grasp of the subjects treated of as well as enormous patience and sustained labour on the part of the author in their preparation and revision. The first-named work is stated to be a 'small edition' in English accompanied by 239 illustrations and containing numerous useful examples and exercises for practice. From this it may be inferred that the volume is an instructive text-book and the inference would be correct, it having been prescribed by the Government of Burma as a text-book in surveying. At the very commencement of the volume is a quaint picture of 'Surveying in India and Burma.' This appears a misnomer, as the survey that is depicted is all of Burma without a hint of anything in it to suggest the country across the Bay. There is a range of hills in the distance and a pagoda on the summit of one hill. In the valley beneath, where some Burmans are surveying, there is a Burmese Monastery and a pagoda, whilst the top-knotted, bare-footed chief surveyor who is using a patent Burmese theodolite, level, or compass is unmistakeably a child of the land of the golden pagodas. However, this is a detail; further on the illustrations are less interesting and considerably more puzzling to the layman observer. The work deals with practical Geometry from first principles and definitions to mensuration of surfaces and questions and answers on geometrical subjects. There are chapters devoted to mathematical drawing instruments and their use; to scales and the manner of drawing them, etc., on the application of the protractor to use; map drawing and how to draw maps; lithographic work and typing, and an essay on reducing, copying and enlarging plans. Then we have a chapter on calculation of areas by acre-squares; problems in casting; chain surveying; compass surveying; plane-table surveying; questions for examination and finally a most copious index. With such a book it is to be imagined that one might be constructed into a very fair theoretical surveyor without having gone into a field or handled a chain, compass, level or theodolite. Mr. Clancey, it will be observed from an advertisement in this volume, is an author of some nine other works. Most of them refer to surveying, but one—'Conspectus of Insect Pests which affect crops in Burma' indicates that Mr. J. C. Clancey devotes his time and attention to other subjects of importance. The *Calculating Tables* appear to refer chiefly to calculations necessary for correct deductions in survey matters, but there is also much in this volume relating to the Burmese Calendar, Burmese Chronology and other scientific and astronomical subjects. The Land Revenue Act and Rules also receive attention as well as calculations of use to agriculturists. In short, it is a most valuable work dealing with an infinity of subjects involving calculations, and a study of it cannot fail to prove both interesting and instructive. The works, we understand, have been through several editions and are procurable from the leading book-sellers in Rangoon and India."

The *Mandalay Herald*, Mandalay.

"We have been favoured by Government with a copy of each of (1) *Aid to Land-Surveying* (small edition) and (2) *Calculating Tables* in Burmese and in English, both by Mr. J. C. Clancey, K.-I-H., F.R.A.S., F.R.G.S., F.S.I., M.R.Ag.S., etc., Assistant Director of Land Records and Agriculture, Burma, and a late member of the Great Trigonometrical Survey of India. The *Aid to Land-Surveying* is now an authorized text-book, and the one issued is in its fifth edition, revised and enlarged. The reputation of Mr. Clancey's *Aid to Land Surveying* is so widely known and appreciated that any comments on our part would be superfluous. We know for a fact that the wish of every student in land-surveying is to possess himself of a copy of Clancey Thakin's Myedaing Saôk, so high is it held in estimation by them. Personally we had known Mr. Clancey's abilities as a mathematician and a surveyor in days long past—and our expectations in him have exceeded all bounds. His second work *Calculating Tables* is another authorized edition (fourth), in the compilation of which the author must have devoted much time and attention. The calculations are, as far as we have carefully gone through, correct. As confirming our assertion the 'Rangoon Gazette' of the 5th August 1903, made allusion to the receipt of a letter from the Mandalay Commissioner, forwarding a report on the Thingyan of 1266 B.E. by the *ex*-Kinwun Mingyi, C.S.I., after the latter had called a meeting of the Sayadaws, *ex*-ministers, and other well informed residents of Mandalay, among whom were a number of Pônnas, to work out the date on which King Thingyan would appear. This the latter and a few others worked out, and the date fixed corresponded with the date which Mr. Clancey had worked out some two years ago and which can be found at page 151, Part I, of the *Calculating Tables*."

The *Arakan News*, Akyab.

"We have received a copy of the fourth edition of the *Calculating Tables* in English and Burmese bound in one volume, by Mr. J. C. Clancey, F.R.A.S., F.R.G.S., F.S.I., M.R.Ag.S., etc., Assistant Director of Land Records and Agriculture, Burma. The book is divided into two parts, *viz.*, first a survey section, which deals with professional subjects, and second a revenue section, dealing with subjects of a general character. Apart from the tables of income, wages, rent, exchange, weights and measures, logarithms, etc., there are, an exhaustive calendar, and calculations of various matters connected with surveying, astronomy and revenue. In fact, the book comprises almost all manner of calculations, from the elementary principles of surveying up to its higher branch of astronomy, and from calculations of land revenue up to the conversion of village baskets into merchants' standard baskets. It is profusely illustrated with practical examples and plates which are beautifully executed. We should think that it is not only the best practical guide to the study of surveying, and useful not only to survey and revenue officers, but equally so to merchants, tradesmen, brokers, bankers, lawyers, and the general public, and as such, should find a place on every one's table as a book of reference. The general get-up and printing are excellent, and considering the amount of time and labour spent over the compilation, the price is infinitesimally small."

[*Survey Manual for Land Records Officers, in English and Burmese* (*with Illustrations*), *Price, Re. 1.*]

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